Superconductivity in quasi-two-dimensional systems : theoretical aspects

Superconductivity in Quasi-Two-Dimensional Systems: Theoretical Aspects.

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Abstract. Vortex fluctuations for 2D superconductors are reviewed together with the Ginzburg-Landau Coulomb gas model and the 2D Coulomb gas scaling concept. Some aspects of the 2D vortex-unbinding transition are clarified. Consequences for the non-linear IV-characteristics and the resistance are briefly recapitulated. The effects of the phasecoupling between superconducting planes and its possible consequences for high-T_c superconductors are discussed. The linear term in the 2D vortex-antivortex interaction, caused by the interplane coupling, is described, together with the related existence of an intrinsic critical current and the resulting functional form of the non-linear IV-characteristics. The dramatic 3D to 2D crossover just above T_c for the anisotropic 3D XY-model is described and its possible significance for layered superconductors is pointed out. Results from MC-simulations for the anisotropic 3D XY-model, its dynamical counterpart the RSJ-model, and an analysis revealing finestructure in the resistance data for YBCO/PBCO superlattices are presented.

Introduction

The present survey focuses on vortex-fluctuations in connection with 'quasi'-2D superconductors. This subject started with the realization[1] that a 2D superconductor can be associated with a Kosterlitz-Thouless transition.[2,3] For a review see e.g. ref.[4]. With the discovery of the high-T_c materials and the possible 'quasi'-2D character of the superconductivity for these materials, the subject has again attracted a lot of interest.[5] Especially the new superlattice structures of YBCO/PBCO, where the interplane coupling between superconducting planes can systematically be varied, lead to questions of when and how the 2D vortex-fluctuations are reflected in the experiments.[6] However, even the subject of 2D vortex-fluctuations in connection with 'quasi'-2D superconductors is by now fairly extensive.[4] I will hence narrow down the subject further and discuss some concepts (and perhaps misconceptions) which I think will be useful to be aware of, in particular in view of the new development stemming from the high-T_c materials. This selection is to some extent based on personal preference and conviction. I hope that it will anyway be of some use to the reader.

Coulomb Gas Scaling

Let me first recapitulate a simple description of a 2D superfluid (see e.g. ref.[4]): A neutral 2D superfluid can be characterized by an order parameter $\psi(\mathbf{r}) = |\psi(\mathbf{r})| \exp[i\theta(\mathbf{r})]$ where the magnitude of the order parameter is related to the superfluid (areal) density ρ_S, $|\psi(\mathbf{r})|^2 = \rho_S(\mathbf{r})$, and the phase is related to the superfluid velocity v_S, $v_S(\mathbf{r}) = \frac{\hbar}{m^*}\nabla\theta(\mathbf{r})$ where m^* is the mass of the superfluid particle. This gives a phenomenological description of the superfluid characterized in terms of the

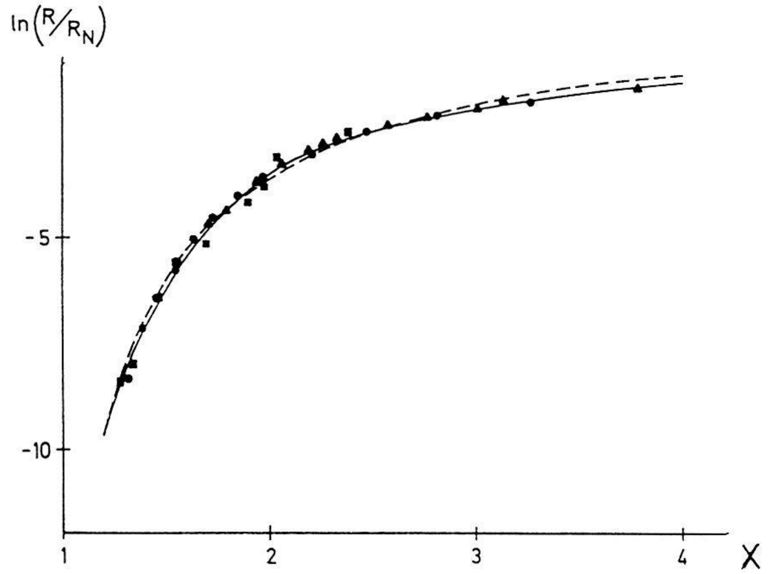

Figure 1: Resistance scaling function: Data from five superconducting films are plotted against the scaling variable X and collapse onto a single curve (from ref.[7]). This resistance scaling curve is a manifestation of the GLCG-model.[8]

density and velocity of the fluid. We assume that the groundstate of the fluid corresponds to zero velocity and constant density. The energy associated with a thermal fluctuation out of the groundstate may then be estimated by the kinetic energy associated with the corresponding velocity field, $H_S = \rho_0 \int d\mathbf{r}[v_S(\mathbf{r})]^2/2$, where ρ_0 is the superfluid density in the absence of currents. The occurrence of a velocity fluctuation is controlled by the corresponding Boltzmann factor $\exp[-H_S/k_BT]$. The velocity field may be separated into a rotation free part $\mathbf{v}_{||}$ and a divergence free part $\mathbf{v}_\perp$. $\mathbf{v}_\perp$ describes the thermal vortex excitations; each $\mathbf{v}_\perp(\mathbf{r})$ can be totally specified in terms of a vortex configuration. We simplify the phenomenology further and assume that we can separate out the vortex part in such a way that without vortices we have a T-dependent ρ_0 coming from the $v_{||}$-part. The vortex fluctuations are then associated with the effective energy $H_S = \rho_0(T) \int d\mathbf{r}[v_\perp(\mathbf{r})]^2/2$.

The Ginzburg-Landau Coulomb gas model (GLCG) for vortex fluctuations in 2D superconductors is constructed along these lines (see e.g. ref.[4]): The superconductor in the absence of vortices is assumed to be well described by a Ginzburg-Landau theory. The vortex shape and energy is obtained by minimizing the Ginzburg-Landau equations and the energy of a vortex configuration is estimated by superposition of single vortices. The vortex part of this explicit model is controlled by two effective parameters i.e. an effective dimensionless temperature variable $T^{CG} = k_BT/[2\pi\rho_0(T)(\frac{\hbar}{m^*})^2]$ and the Ginzburg-Landau coherence length $\xi(T)$. In practice the standard Ginzburg-Landau approach usually suffices, so that $\rho_0(T) = \rho_0(0)[1 - T/T_{c0}]$ and $\xi(T) = \xi(0)/\sqrt{1 - T/T_{c0}}$, where T_{c0} is the Ginzburg-Landau temperature. The name Coulomb gas derives from the fact that the vortices may be viewed as Coulomb gas particles. The Coulomb gas particles which constitute the GLCG-model have a particular single particle charge distribution (they are not point charges), chemical potential and interaction; all coming from the Ginzburg-Landau equations.[4]

The Coulomb gas scaling concept is just the observation that a dimensionless quantity related to the vortices can only be a function of the effective variable T^{CG}. Such a quantity is the resistance ratio R/R_N where R is the resistance caused by thermally created vortices. Thus all 'quasi'-2D

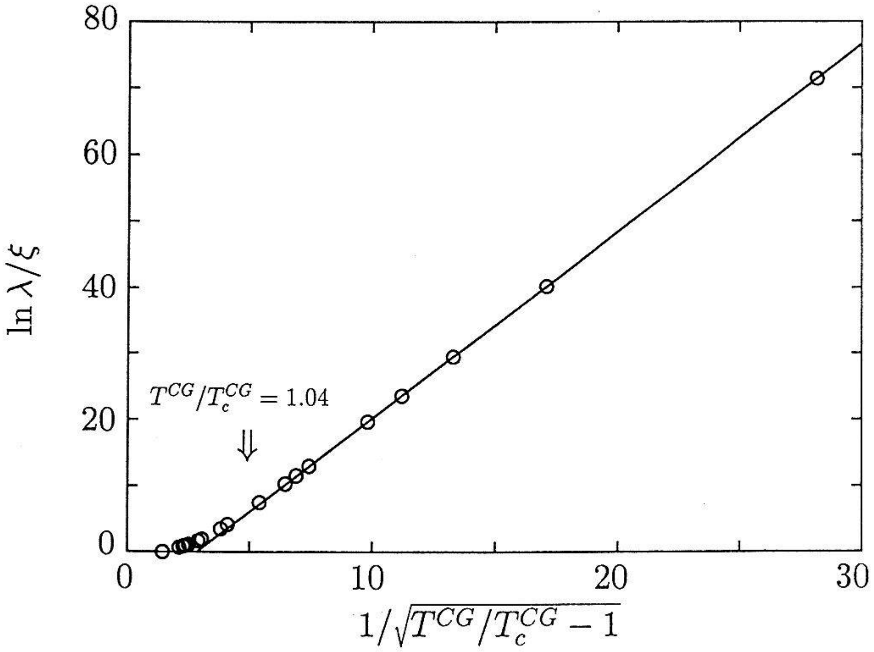

Figure 2: Estimate of the critical region for the KT-transition based on a numerical solution of a set of self-consistent equations for the Coulomb gas screening length λ (from ref.[10] where details and parameter choices are given). The true KT-signature shows up for $T^{CG}/T_c^{CG} < 1.04$ and $\lambda/\xi > 10^3$.

superconductors which can be well described by the GLCG-model should produce a unique R/R_N-curve provided the data is plotted against T^{CG}. This is illustrated in fig.1. In fig.1 the data is plotted against the variable $X = T^{CG}/T_c^{CG}$ where T_c^{CG} is the phase transition temperature for the GLCG-model. This test of the Coulomb gas scaling concept hence presumes that T_{c0} and T_c (=the temperature where the vortex system has a phase transition) can be extracted from the data in a convincing way.

I would like to emphasize the following points: The Coulomb gas scaling is well borne out for 'quasi'-2D superconductors.[4] The Coulomb gas scaling curve obtained from the data can be explicitly linked to to the GLCG-model through Monte Carlo simulations.[8] So the success of the scaling is not accidental; it really reflects that thermal vortex fluctuations for 'quasi'-2D superconductors are often well described by the GLCG-model.

All relevant data obtained so far is for $T^{CG}/T_c^{CG} > 1.1$ (or equivalently for $R/R_N > 10^{-6}$).[4] These means that the majority of the data do *not* reflect any true critical property of the phase-transition.[4] This is contrary to many claims in the literature and appears to be of some importance for the high-T_c materials, so I will try to clarify this point further.

Vortex-Unbinding Transition

The 2D Coulomb gas undergoes a charge(=vortex)-unbinding transition at T_c^{CG}. Pictorially expressed, all vortices are bound together into vortex-antivortex pairs below T_c^{CG} whereas above some pairs are broken. More precisely the Coulomb gas screening length λ is infinite below T_c^{CG} but finite above. The signature of the phase transition is (at least for low enough Coulomb gas particle densities[9,4]) given by the Kosterlitz RG equations[3]. A phase transition with this particular signature is usually referred to as a Kosterlitz-Thouless(KT) transition. λ diverges as as $\ln(\lambda) \propto 1/\sqrt{T^{CG}/T_c^{CG} - 1}$ for the KT transition. Since $R/R_N \propto \lambda^{-2}$, this also means that

$\ln(R/R_N) \propto 1/\sqrt{T^{CG}/T_c^{CG}-1}$. Successful fits of R/R_N-data to this functional form is very commonly (but most likely incorrectly) taken as strong evidence for a KT-transition. Obviously the KT-interpretation only makes sense provided the data is really inside the critical region of the vortex unbinding transition.

The width of the critical region can be estimated from a set of self-consistent equations for the linearly screened Coulomb gas interaction $V_L(r)$.[9] From these equations λ can be calculated numerically. The equations are approximate but go beyond Kosterlitz RG equations.[9] Furthermore (and contrary to Kosterlitz RG equations) they are also valid in the high-temperature phase.[9] Fig.2 illustrates the result.[10] Fig.2 gives an estimated critical region $1 < T^{CG}/T_c^{CG} < 1.04$ corresponding roughly to a critical resistance region $0 < R/R_N < 10^{-6}$. This suggests that most of the measured resistance data are well outside the true critical region. An additional complication for the KT-interpretation is that the critical region $0 < R/R_N < 10^{-6}$ involves very large lengthscales and so the data in this region will in practice easily be contaminated by finite- size, current, and magnetic field effects.[4]

The Coulomb gas resistance scaling function turns out to have a functional form which is well approximated by
$\ln(R/R_N) \approx \text{const}_1/\sqrt{T^{CG}/T_c^{CG}-1} + \ln(T^{CG}/T_c^{CG}) + \text{const}_2$
over a substantial interval.[4] However this fact should not be confused with the critical property of a KT transition; it happens to be a non-critical property of the GLCG-model.

Coupled Layers

How does the vortex-unbinding change when instead of one superconducting plane we have many parallel superconducting planes weakly coupled together? This may be the situation for some of the high-T_c materials where the superconducting planes are associated with the CuO_2 planes. I will discuss this on the level of a 3D anisotropic XY-model. The hamiltonian is given by

$$H_{XY}^{3D} = -\sum_{<ij>_{\parallel}} J_{\parallel}\cos(\theta_i-\theta_j) - \sum_{<ij>_{\perp}} J_{\perp}\cos(\theta_i-\theta_j)$$

where the sums are over the nearest-neighbor pairs on a cubic lattice and $< ij >_{\parallel}$ denotes nearest-neighbor pairs belonging to the same superconducting plane and $< ij >_{\perp}$ denotes pairs belonging to two adjacent planes. So in short we have put the superconducting order parameter $\psi = |\psi|\exp(i\theta)$ on a cubic lattice, suppressed the magnitude variations, and assumed that the interplane coupling can be described as a phasecoupling between adjacent planes.

Fig.3 shows a vortex-antivortex pair associated with one particular plane. As illustrated in the figure a 2D vortex-antivortex pair for a 2D superconductor goes over into a vortex loop where the loop only cuts one plane. The 2D vortex-antivortex interaction is logarithmic for large separations. This logarithmic interaction is a prerequisite for the KT-transition. The vortex-antivortex interaction for coupled layers is linear for large separations.[11,12] The variational estimate in ref.[11] for the the coefficient in front of this linear term is $\pi^2 J_{\parallel}\sqrt{2J_{\perp}/J_{\parallel}}$.

The linearly screened interaction $V_L(r)$ for the vortex-antivortex interaction, obtained from MC-simulations, is in case of the anisotropic 3D XY-model to good approximation of the form[13]

$$V_L(r) = k_0\ln(r) + kr + \text{const} \tag{1}$$

The coefficient k vanishes precisely at T_c as $k \propto \sqrt{m}$ where m is the order parameter (=the magnetization per spin for the XY-model).[13] The screening length of the effective vortex interaction

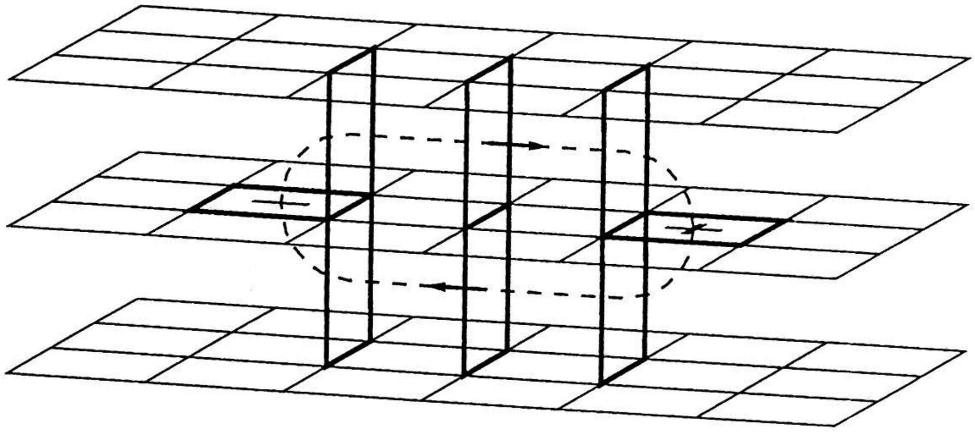

Figure 3: Vortex-antivortex pair for coupled layers. The pair is associated with a vortex loop which only cuts one plane (middle plane in the figure, from ref.[13]).

is finite above T_c.[13] Above T_c the superconducting planes are, from the point of view of the vortex-antivortex pairs, effectively decoupled.[14,15] The phase transition can just as in the pure 2D case be associated with a vortex-unbinding transition. Above T_c broken pairs associated with planes generate flux-flow resistance. However, this vortex-unbinding transition does not have a KT-character. It is quite different; the phase transition of the anisotropic XY-model is of second order.

Critical Current

A supercurrent I_S parallel to the superconducting planes gives rise to a Lorentz force $F_L \propto I_S$ which tries to pull a thermally created vortex-antivortex pair apart. This pulling is counteracted by a binding force (see eq.1) $F_B = k_0/r + k$. The threshold condition for pairbreaking is consequently given by $F_L = k$ which in turn corresponds to a critical current I_c.[12,16] This pairbreaking mechanism, generating a finite critical current below T_c, does not involve any pinning. It is a consequence of the interplane coupling of the material. There are indications for such an 'intrinsic'(=material characteristic) value of I_c in case of high quality epitaxially grown YBCO-films.[16,17] The simple estimate from ref.[11] gives[16]

$$I_c = 5.83 \cdot 10^{20} \frac{1}{\lambda_L(0)^2 d} \sqrt{\frac{m_{\parallel}^*}{m_{\perp}^*}} \left[1 - \frac{T}{T_{c0}}\right] \tag{2}$$

provided I_c is in Am^{-2}, λ_L(=London penetration lenght) and d(=layer spacing) is in Å. $m_{\parallel}^*/m_{\perp}^*$ is the mass-anisotropy. The temperature dependence stems from a Ginzburg-Landau assumption. A linear temperature dependence is borne out by data for YBCO-films and the simple estimate for the magnitude is reasonable.[17]

The vortex-antivortex pair breaking can be viewed as an escape over a barrier problem balanced by a two-particle recombination process.[18] In the present case this leads to a non-linear IV-characteristics of the form $V \propto I[I - I_c]^{a-1}$ (to be compared to $V \propto I^a$ for the pure 2D-case).[16]

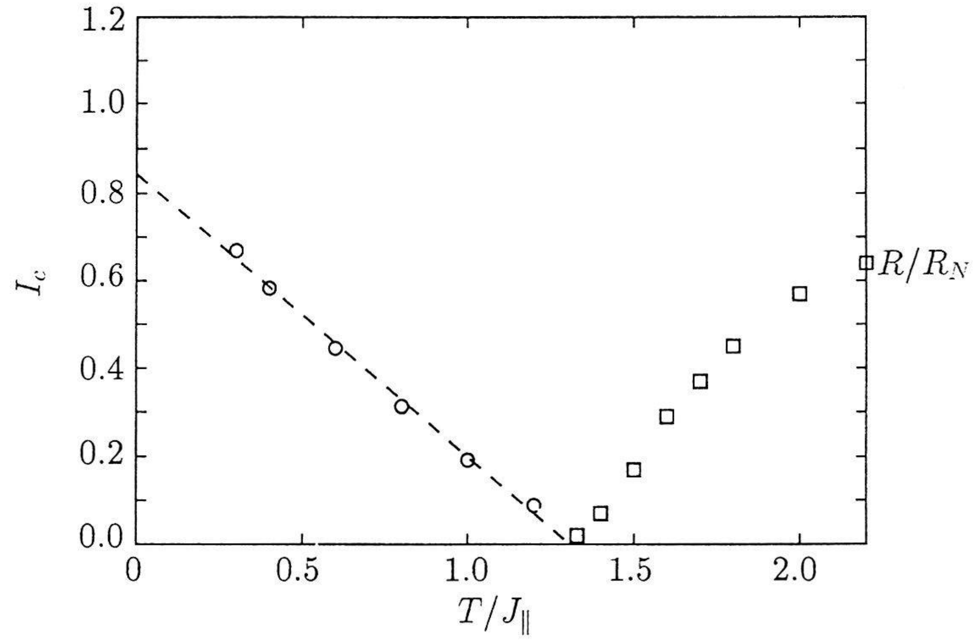

Figure 4: Critical current and resistance for the anisotropic RSJ-model obtained by simulations (from ref.[19]). $J_\perp/J_{||} = 0.1$ and the dashed straight line is a fit to the I_c-data subject to the additional condition that it cuts the horizontal axis precisely at T_c.

This offers a possible signature of the vortex-antivortex breaking mechanism for I_c which can be tested against experimentally.[16,17]

The critical current due to vortex-antivortex breaking can be studied in some further detail on the level of an anisotropic RSJ-model (resistance-shunted-junction-model).[19] This model may be viewed as the dynamical counterpart of the anisotropic 3D XY-model; the coupled dynamical equations are of the Langevin form (compare H_{XY}^{3D})

$$V_{<ij>_{||(\perp)}} = \frac{d(\theta_i - \theta_j)}{d\tau} = -J_{||(\perp)} \sin(\theta_i - \theta_j) + \eta_{<ij>_{||(\perp)}}$$

where $V_{<ij>_{||(\perp)}}$ is a measure of the voltage across a link in the parallel (perpendicular) direction on a cubic lattice and the thermal noise current $\eta_{<ij>_{||(\perp)}}$ is subject to the white noise condition

$$< \eta_{<ij>_{||(\perp)}}(\tau)\eta_{<ij>_{||(\perp)}}(\tau') >= 2k_B T\delta(\tau - \tau')$$

A drive current I corresponds to a particular boundary condition imposed at the sides of the cubic lattice. Fig.4 gives the results from simulations for the RSJ-model.[19] A critical current I_c, linear with temperature, is obtained. The figure also gives the flux-flow resistance above T_c.

3D to 2D Crossover

Fig.5 shows the vortex density per superconducting plane plotted against the Coulomb gas temperature T^{CG} for the anisotropic 3D XY-model obtained through MC-simulations.[14] The highest curve is for the 2D XY-model (i.e. $J_\perp = 0$) and the other two are for non-vanishing interplane couplings ($J_\perp/J_{||} = 0.02$, and 0.1, respectively). The critical temperatures for these latter two cases are marked with arrows (the one to the left in the figure corresponds the smaller of the two interplane coupling constants). The figure suggests that the high-temperature phase in all cases is to very good

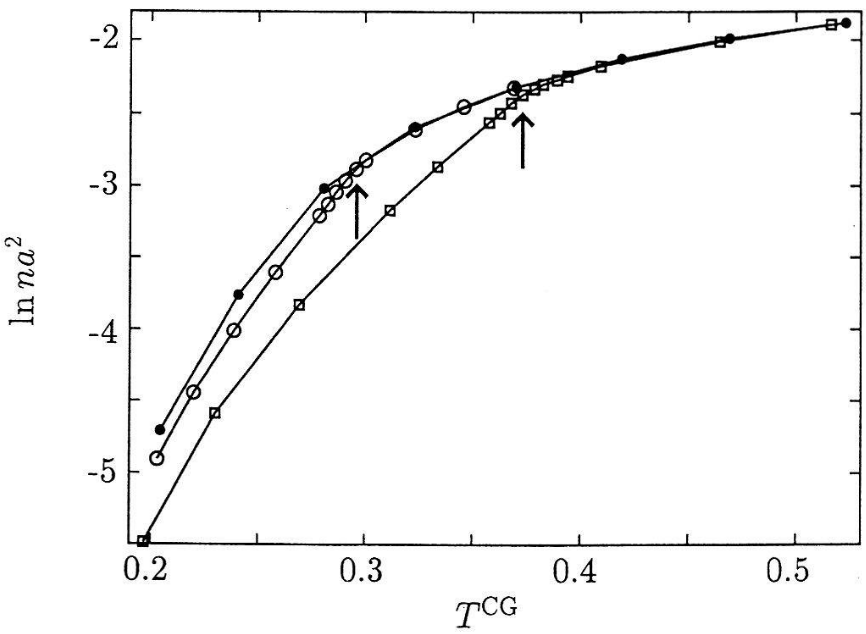

Figure 5: 3D to 2D crossover. Vortex density per plane, n, plotted against T^{CG} for the anisotropic 3D XY-model (from ref.[14]). (a is here the lattice constant, for further explanation see text).

approximation described by the very same 2D Coulomb gas.[14] From the point of view of the vortices the superconducting planes appear to be effectively decoupled.[14,15] This result carried over to layered superconductors suggests that the thermally induced flux-flow resistance just above the transition effectively reduces to the pure 2D case. In other words the resistance scaling curve from the GLCG-model should be applicable also for the layered superconductors and high-T_c materials. Evidence for this has been reported for BSCCO [20] and YBCO/PBCO-superlattices[21].

Fig.6 shows resistance data for four YBCO/PBCO-superlattices plotted against the Coulomb gas scaling variable $X = T^{CG}/T_c^{CG}$.[21] The data corresponds in order of increasing interplane coupling to the superlattice structures N_Y unit-cell-thick YBCO/N_P unit-cell-thick PBCO with $(N_Y, N_P) = (1,16)$, $(2,16)$, $(3,16)$, and $(3,4)$. The full curve is the 2D Coulomb gas resistance scaling curve. At the small resistance end of this curve the data for (3,16) and (3,4) show a slight but significant deviation from the 2D scaling curve. This deviation increases with increasing interplane coupling and reflects a corresponding increase of T_c.[21] The same effect is seen in Fig.5. Above T_c the data rapidly collapse onto the 2D scaling curve suggesting a dramatic crossover from 3D to an effectively 2D vortex description associated with decoupled planes. BSCCO resistance data suggest that this material has effectively the same interplane coupling as the $(3,16)$ YBCO/PBCO-superlattice.[21]

According to the GLCG-description for a 2D superconductor, the exponent a for the non-linear IV-characteristics below T_c ($V \propto I^a$), is another Coulomb gas scaling function and $a = 3$ precisely at the phase transition.[4] However, there is no effective 3D to 2D decoupling of the vortex system below T_c for layered superconductors. Consequently $a(T^{CG})$ for layered superconductors should depend on the interplane coupling. This is illustrated in Fig.7. The figure gives $a(T^{CG})$-data for two realizations of a 2D superconductor. The 2D Coulomb gas scaling is well borne out.[4,22] The full curve represents BSCCO-data (close to T_c so that I is small but still $I \gg I_c$) and is significantly different.[22] There are indications that $a(T^{CG})$ also in this latter case is a scaling function e.g. in the sense that a magnetic field dependence perpendicular to the superconducting planes is absorbed into the Coulomb gas temperature T^{CG}.[22] The existence of such a scaling

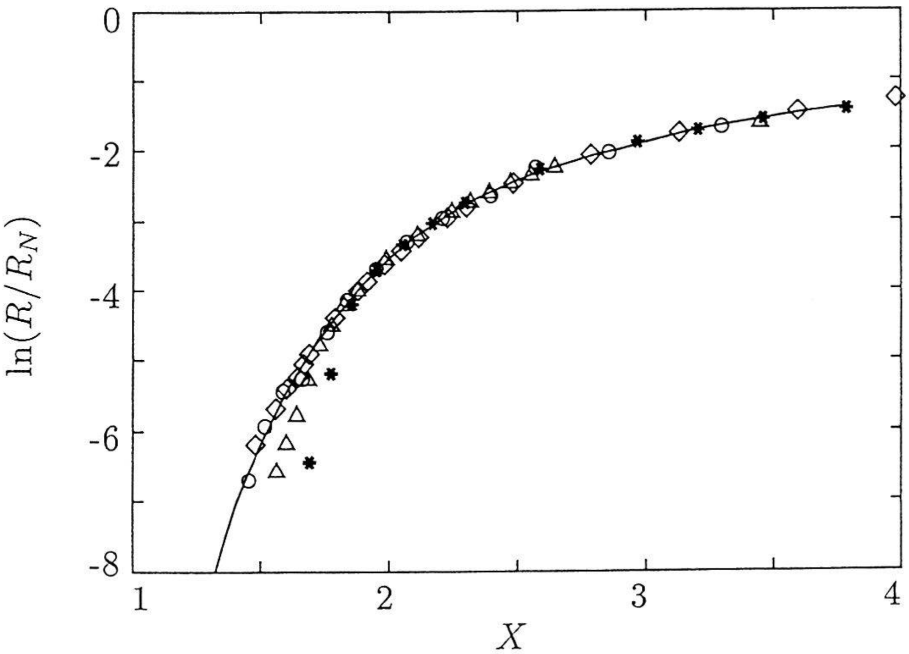

Figure 6: Resistance scaling for YBCO/PBCO-superlattices. Full curve is the 2D resistance scaling function and the four sets of data correspond to four superlattices with different interplane coupling (circles, diamonds, triangles and asterisks correspond to increasing interlayer coupling, for further explanations see text, from ref.[21]).

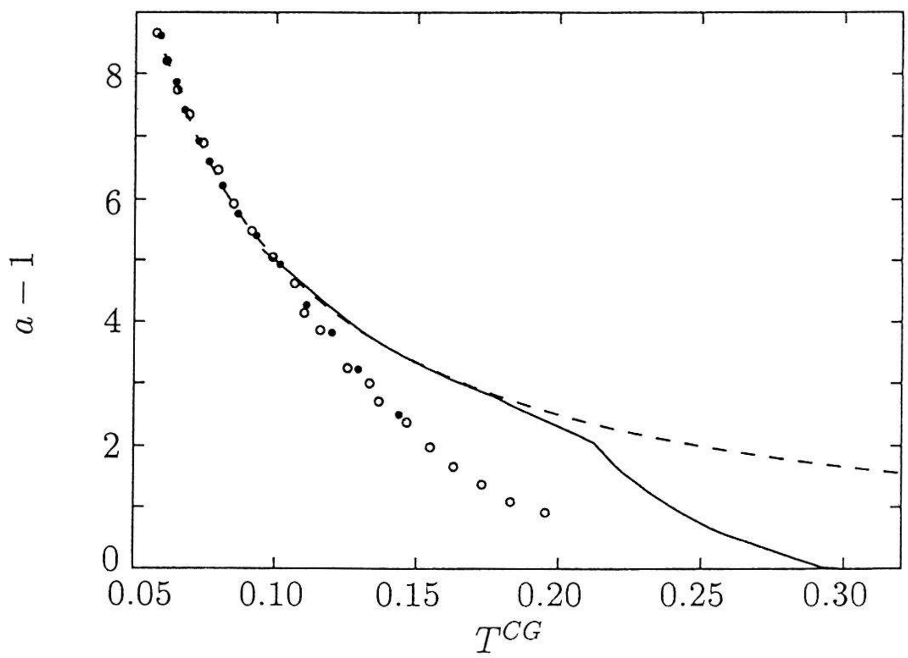

Figure 7: The exponent a plotted against T^{CG}. The filled and open circles corresponds to data from two different realizations of a 2D superconductor. The full curve is constructed from BSCCO-data. The dashed curve represents a in the absence of vortex-fluctuations (from ref.[22]).

would impose restrictions on the possible form of effective interplane coupling.[22]

Concluding Remarks

Thermal vortex-fluctuations for 2D superconductors has a direct counterpart in case of layered superconductors. However, the vortex-antivortex interaction is different due to the interlayer coupling. This difference gives rise to a critical current and a vortex-unbinding transition which is not of the KT-type. Some caution when interpreting resistance data has to be exercised because $|\ln R| \propto 1/\sqrt{T^{CG} - T_c^{CG}}$ can, in practice, not be taken as evidence for a KT-transition. Nevertheless, Coulomb gas scaling, the 2D resistance scaling function and the 2D GLCG-model is applicable above T_c due to an effective 3D to 2D decoupling. The Coulomb gas scaling concept appears to be applicable also below T_c. This suggests that data analysis based on the Coulomb gas scaling concept has the potential of revealing new interesting information. For example the size of the deviation from the 2D resistance scaling function for the smallest resistances in case of YBCO/PBCO-superlattices is directly related to the size of the interlayer coupling.

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