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QUANTUM HALL EFFECT WITHOUT LANDAU QUANTIZATION

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In the framework of the scaling hypothesis of localisation of 2D electrons in a magnetic field B and random potential it is shown that a discrete set of extended states, characteristic for Quantum Hall Effect, does not disappear at vanishing B, when Landau levels overlap. Energies of these states E_n in weak fields did not follow Landau levels going to zero, but, contrary, are increasing for decreasing field, floating up like bubbles and leaving Fermi sea. As a result, at weak enough magnetic fields all states below chemical potential are localised. For case of two overlapped Landau levels (two layers system or two overlapped spin subbands) a new phenomenon of spontaneous splitting of extended states is predicted.

1. The quantization of the Hall conductance in two dimensional electron gas discovered experimentally in 1980 [1] was almost immediately explained as a result of the Anderson localisation of electronic states in a random impurity potential [2,3]. It was necessary to suggest [2] that, provided disorder is weak, i.e. $\Omega\tau >> 1$ (Ω = eB/mc is the cyclotron frequency and τ is the mean free time), at least one state should be delocalised at each single Landau level. This picture is relevant when Landau levels are not overlapped, i.e. at $\Omega\tau >> 1$ or at Larmore radius R_L much larger than mean free path l ($R_L >> l$). From the other hand, the theory of weak localisation [4] in a random potential is valid at weak enough fields when $l << \lambda_H << R_L$ ($\lambda_H = (ch/eB)^{1/2}$). Therefore, there is a huge gap in the parameter space, inside which the Landau levels are disappeared already because of broadening and overlapping, but weak localisation in not working yet. We are going to describe what scaling theory of localisation tell us on magnetotransport under these conditions or how Quantum Hall Effect (QHE) looks like without Landau quantization. Surely, there are many different cases, when overlapping of the Landau levels has to be taking into account. Some of them will be discussed here.

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This action contains two parameters: dissipatieve conductance σ_{xx} and Hall conductance σ_{xy}[1]. Therefore, the dependencies of σ_{xy} and σ_{xx} on the scale L are given by the renormalization group equations

$$\frac{d\sigma_{xx}}{d\xi} = \beta_{xx}(\sigma_{xx}, \sigma_{xy}),$$
$$\frac{d\sigma_{xy}}{d\xi} = \beta_{xy}(\sigma_{xx}, \sigma_{xy}), \xi = \ln L$$

where β_{xx} and β_{xy} are periodic functions of σ_{xy} with period equal to unity. Main properties of the system are reflected in the phase diagram of renormalization group equations [5], shown in Fig 1.

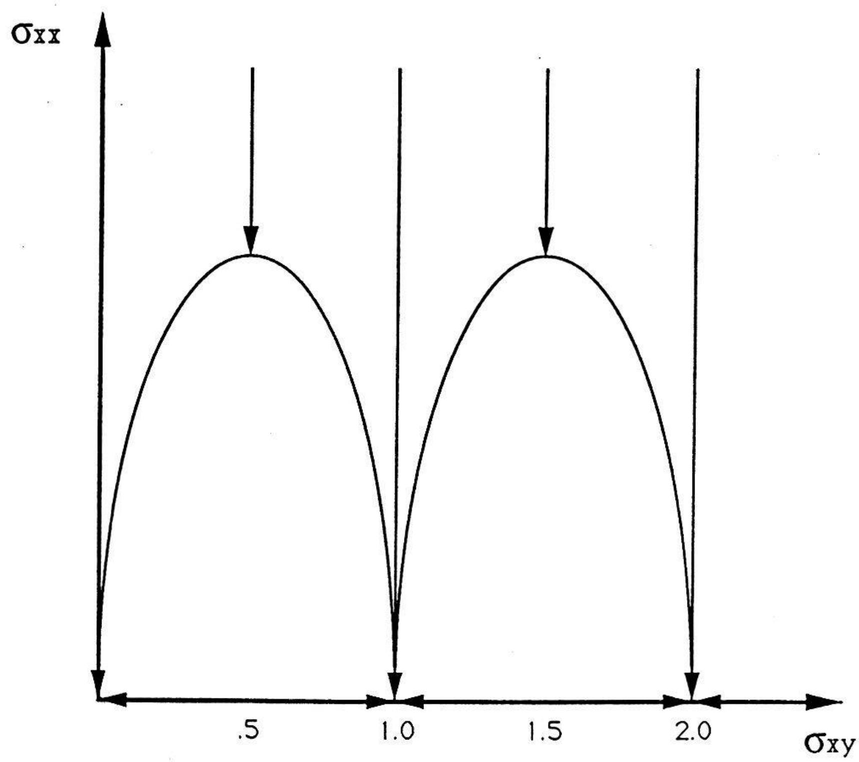

Fig. 1

Unstable fixed points at this diagram are corresponding to semi-integer values of Hall conductance ($\sigma_{xy} = (n + 1/2)$) and non-zero value of dissipative conductance σ_{xx}. Existence of such fixed points was pointed out first by Levine, Libby and Pruisken [7]. They correspond to extended states associated with semi-integer values σ_{xy} and leading to quantization of the Hall conductance. It is necessary to note here that two parameter renormalization group

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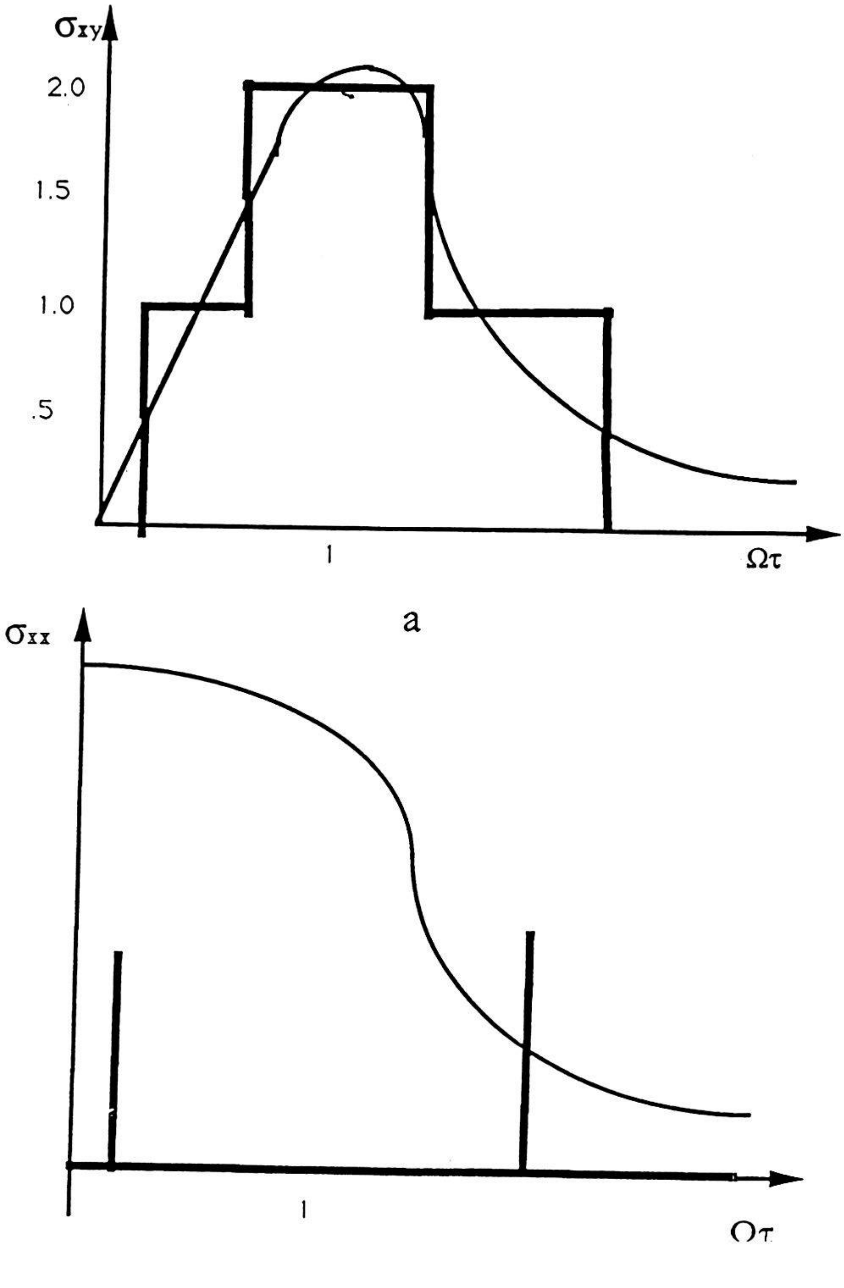
[1]We use natural units e^2/h for conductance

equations are natural generalization of single parameter scaling theory of the Anderson localisation in the absence of magnetic field [4].

To show how this theory explains QHE and to answer the question we started, it is necessary to look at renormalized dependencies of both σ_{xy} and σ_{xx} on magnetic field B. These dependencies at short distances are given by Drude-like expressions. In particular, it is convenient to use the following interpolating formula for $\sigma_{xy}^{(0)}$

$$\sigma_{xy}^{(0)} = \frac{ne^2\tau}{m}\frac{\Omega\tau}{1+(\Omega\tau)^2}$$

The results of the renormalization of both σ_{xy} and σ_{xx}, i.e. the field dependence of observed values of σ_{xy} and σ_{xx} at $T = 0$ are show on Fig 2a and 2b.



As seen from Fig 2b, besides the Shubnikov-tipe oscillations with maxima at fields

$$B_n^{(1)} = \frac{mc}{eh}\frac{E_F}{n + 1/2}$$

there is the same amount of additional oscillations with maxima of σ_{xx} at

$$B_n^{(2)} = \frac{mch}{e}\frac{1}{E_F\tau^2(n + 1/2)}$$

This result may be interpreted as follows: at $\Omega\tau \geq 1$ a delocalised level is associated with each Landau subband. When its energy E_n coincides with the Fermi level, σ_{xx} has a pick. At $\Omega\tau < 1$ broadening of Landau levels is larger then the distance between them. Under these conditions, the energies of delocalised states increase with decreasing field and once more coincide with the Fermi energy, resulting in additional oscillations. The number of delocalised states below the Fermi energy coincides with the Hall conductance, which works as their counter. Using interpolation for Hall conductance at short distances and expressing electron concentration through E_F one can obtain the following interpolation for the energy of the n-th delocalised state [8,9]

$$E_n = h\Omega(n + 1/2)[1 + \frac{1}{(\Omega\tau)^2}]$$

The lowest delocalised level crosses Fermi level under the condition when $\lambda_H \propto 1$. So, the gap, we have mentioned are full filled now.

3. Similar situation arises when Landau levels are strongly separated, but there is some extra quantum number and levels with the same Landau numbers but different additional one are overlappped. It is possible, if there are two layers, hybridized by tunneling, or subbands, corresponding opposite spin orientation, became overlapped because of low Zeemann energy. If there is, for instance, strong spin-orbit interaction, then spin orientation is not good quantum number and we need to consider the Anderson localisation in doubled Landau subband. The main prescription how to do this is the same: we have to draw dependence of both σ_{xy} and σ_{xx} on, say, Fermi energy and then to renormalize it according general rules. For particular case of doubled Landau subband it is shown in Fig 3a and 3b.

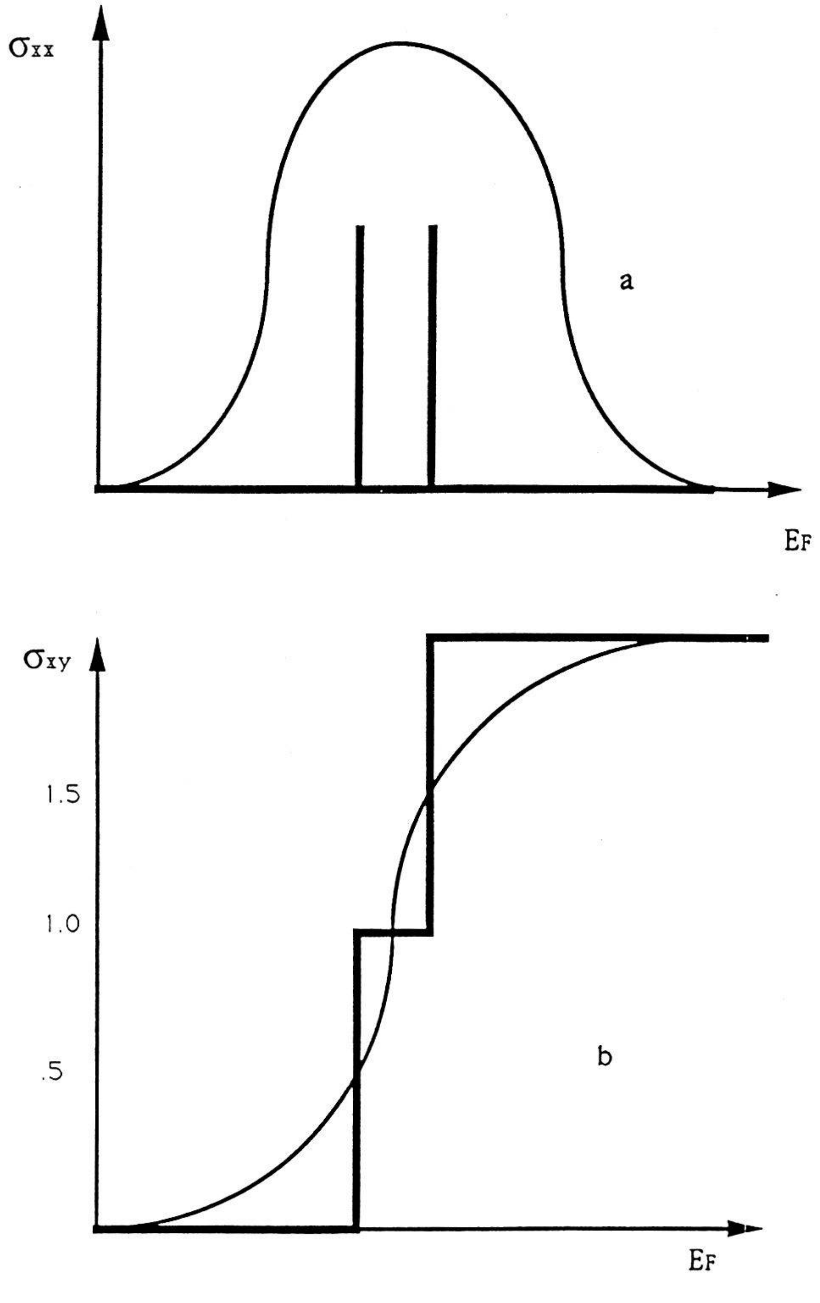


Fig 3

One can see from Fig 3 that localisation leads to splitting of initially doubled jump of σ_{xy} what manifests an spontaneous splitting of extended states, associated with this subband.

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