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Non-existence of path space measure for local $(QED)_1$*

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Abstract. We study the interaction of a "charged" particle with an oscillator. On the classical level holds $m\dot{x} = p - eA$, where $x, A \in \mathbb{R}$. In QM we let x move on the circle S to have a proper ground state. The imaginary time Green's functions exist, satisfy OS-like axioms and, for $e \neq 0$, are complex valued. They define a normalized quasimeasure $d\lambda$ on path space $Q \times A$. Our main result is the proof of $\|\lambda\| = +\infty$, due to a theorem of Yngvason. Integrating out the oscillator variable A we find some probability measure $d\mu$ on Q (given by the effective action for the particle). Because of memory it allows us to recover the Hamiltonian semigroup for the coupled quantum system.

1. Introduction

On a heuristic level the idea of path integral was introduced by Feynman [1]. After reformulation of QFT in terms of Euclidean Green's functions [2] its existence became a challenge for mathematicians [3].

In particular Yngvason [4] obtained the following result: Given those Green's functions then strong OS-positivity implies that a measure exists and must be real. He used an argument of Fröhlich. However from QED we know that the interaction of charged matter with gauge fields is given by a complex phase factor and, if Θ denotes time reflection, one has combined $PC\Theta$-symmetry. To understand the crux we looked for some caricature of electromagnetism in standard QM avoiding any troubles with Fermions [5].

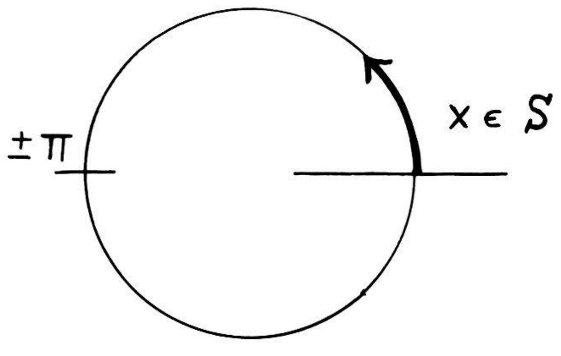

Fig. 1.

*) Seminar given at the Workshop "white noise analysis: New results and their impact on quantum physics", ZiF Bielefeld Sept. 24–29, 1990

Let x be the angular coordinate of a particle moving on the circle as drawn above. The stationary states of the system are described by eigenfunctions $\Psi = \exp\{i \cdot (kx)\}$, $k \in \mathbb{Z}$, of $p = -id/dx$. Indeed imposing periodic boundary conditions at $x = \pm\pi$ the momentum p defines a selfadjoint operator [6] in $L^2(S)$. When x couples to a homogeneous magnetic field of flux $\Phi = 2\pi A$ then p acquires a shift by $\alpha = eA$, where e was the charge. We claim that for $\alpha \notin \mathbb{Z}$ (otherwise the effect would be unvisible) the propagator leads to a complex-valued normalized cylinder measure $d\omega_\alpha$ of infinite total variation [7].

So contrary to general believe the formal substitution $t \to -it$ does not resolve all problems with the path integral.

2. Model

Below we consider $A \in \mathbb{R}$ as dynamical degree of freedom describing a quantum oscillator. On the classical level our model is given by the coupled equations of motion

$$\left.\begin{aligned} m\ddot{x} &= eE \\ \ddot{A} + \beta^2 A &= e\dot{x} \end{aligned}\right\}, \tag{1}$$

where $E = -\dot{A}$. Clearly, $p = m\dot{x} + eA$ and total energy H are conserved. We will fix m equal to one. In QM we realize

$$H = \frac{(p - eA)^2}{2} + I_\beta(E, A), \tag{2}$$

where $I_\beta = 1/2(E^2 + \beta^2 A^2)$, as a Hermitean operator in the separable Hilbert space $\mathscr{H} = L^2(S \times \mathbb{R})$. Of course $p \in \mathbb{Z}$ so that H has discrete spectrum and a proper ground state Ω. The variable $x \in S$ gives a bounded operator with norm $\|x\| = \pi$. This affects the classical identity $m\dot{x} = p - eA$. Indeed, we find the singular anomalous commutation relations

$$\begin{aligned} L &= px - xp \\ &= i \cdot \sum_{k \neq 0} (-1)^k e^{ikx}, \end{aligned} \tag{3}$$

and hence

$$[H, x] = \frac{1}{2i}\{(p - eA)L + L(p - eA)\}. \tag{4}$$

3. Propagator

To obtain the propagator one may start from the Lagrangean [8], calculate the action along a trajectory $t \to (x(t), A(t))$, $t_1 \leqslant t \leqslant t_2$, and then go to imaginary time. Because of the restriction $x \in S$ this seems to be a doubtful venture. Instead we rewrite

$$P^t = \exp(-tH) = \exp\left(\frac{-tp^2}{2M}\right) \cdot V^* K^t V, \quad t \geqslant 0, \tag{5}$$

where $V = \exp\{ie/\gamma^2(pE)\}$, $\gamma = \sqrt{\beta^2 + e^2}$ and M is an effective mass. Moreover we introduced $K_\gamma^t = \exp(-tI\gamma)$, governing the oscillator [9]. The unitary V commutes with momentum p. So if

$$L^2(S \times \mathbb{R}) = \bigoplus_{k \in \mathbb{Z}} \mathscr{H}_k, \tag{6}$$

on wave functions $\Psi = \Psi(x, A)$ from some fixed sector $\mathscr{H}_k$ the operator V induces a shift of A to $A_k = A - ek/\gamma^2$. Using Poisson's formula [10] we get

$$P^t = \sum_{l \in \mathbb{Z}} \frac{\exp\left(-\frac{z_l^2}{2\tau}\right)}{\sqrt{2\pi\tau}} \cdot K_\gamma^t(A, B), \tag{7}$$

where

$$z_l = (y - x) + i\delta(t)\,\frac{e(A+B)}{2} + 2\pi l, \tag{8}$$

$\tau(t) = \gamma^{-2} \cdot (\beta^2 t + e^2 \delta)$ and $0 \leqslant \delta(t) \leqslant 2/\gamma$. Hence, for $e \neq 0$, in the Schrödinger representation the imaginary time propagator of the model $(\mathrm{QED})_1$ is complex-valued. We may hardly associate a genuine stochastic process with trajectories $t \to (x(t), A(t))$ on path space

$$Q \times A = \bigtimes_{t \in (-\infty, \infty)} (S \times \mathbb{R}). \tag{9}$$

4. Quasimeasure

Let us renormalize the Hamiltonian so that $H\Omega = 0$ and perform a unitary transformation on $L^2(S \times \mathbb{R})$ which brings the ground state vector Ω into the function equal one.

Then for any $t_0 \leqslant t_1 \leqslant t_2 \leqslant \cdots \leqslant t_n$ the iteration of the propagator defines a normalized complex-valued measure $d\lambda_n(xA, yB)$ on the space $\bigtimes_{j=0,1,\ldots,n} (S \times \mathbb{R})$. For $n = 1$ we obtain

$$d\lambda_1 = d\rho(xA) \cdot P^\varepsilon(xA, yB)\, dy\, dB, \tag{10}$$

where $\varepsilon = t_1 - t_0$ and $d\rho = \Omega^2 \cdot dx\, dA$ is the measure on $S \times \mathbb{R}$ defining the new scalar product in the "physical" Hilbert space. Its extension to some σ-additive measure on $Q \times A$ yields a nontrivial problem. Unfortunately we cannot use [11]. But the simple structure of our model allows us to control the total variation of $d\lambda_n$ in the limit $n \nearrow \infty$, directly.

We return to the situation of a particle x moving on the circle S in presence of magnetic flux Φ. The evolution operator

$$R_\alpha^t = \sum_{k \in \mathbb{Z}} \frac{e^{ik(y-x)}}{2\pi} \cdot \frac{e^{-t(k-d)^2}}{2} \frac{1}{2\pi} \sum_{k \in \mathbb{Z}} \exp\left\{ik(y-x) - \frac{t}{2}(k-\alpha)^2\right\}, \quad t \geqslant 0, \quad (11)$$

satisfies $R_\alpha^t 1 = \exp\left(-\frac{t\alpha^2}{2}\right) \cdot 1$, where $\alpha = e\Phi/2\pi$ and the Chapman equation. Via the Kolmogorov construction we find a normalized cylinder measure $d\omega_\alpha$ on Q. We claim that for given $\varepsilon > 0$

$$Y_\alpha(\varepsilon) = \exp\left(\frac{\varepsilon\alpha^2}{2}\right) \cdot \int_S |R_\alpha^\varepsilon(x, y)| \, dy \geqslant 1. \quad (12)$$

Equality holds if and only if $\alpha \in \mathbb{Z}$. By inspection the QM amplitude $(\cos x, R_\alpha^\varepsilon \sin x)$ is not real. We remark that $R_\alpha^\varepsilon(x, y)$ coincides with Jacobi's theta function [12] at $z = (y - x) + i\varepsilon\alpha$ and imaginary time parameter. Using the fact that the value of the integral does not depend on $x \in S$ we get

Lemma.

$$\|\omega_\alpha\|(S \times S^n) = Y_\alpha(\varepsilon)^n, \quad n = 0, 1, 2, \ldots \quad (13)$$

For large n the total variation of $d\omega_\alpha$ diverges. Conversely, let us fix $t > 0$, so that $\varepsilon = t/n \searrow +0$ if n tends to infinity. Then from $Y_\alpha(\varepsilon) \leqslant \exp(\varepsilon\alpha^2/2)$ and translation invariance we conclude that on a finite time interval the total variation stays bounded. We easily check Daletzki's condition. We observe the symmetry $d\omega_\alpha \cdot \Theta = (d\omega_\alpha)^*$. Complex conjugation is equivalent to changing α by $-\alpha$. How our lemma can be applied to the full propagator?

For small $\varepsilon \geqslant 0$ one may substitute $P^\varepsilon(xA, yB)$ by $R_\alpha^\varepsilon(x, y) \cdot K_\gamma^\varepsilon(A, B)$, with $\alpha = e \cdot (A + B)/2$. In the case $n = 1$ for the total variation of $d\lambda$ on $\times_{j=0,1} (S \times \mathbb{R})$ we obtain approximately

$$\iint_{\mathbb{R}^2} Y_{e(A+B)/2}(\varepsilon) \cdot d\varphi_\gamma(A, B), \quad (14)$$

where $d\varphi_\gamma$ denotes the oscillator measure. Finally we combine the above estimate with $e \cdot (A + B)/2 \notin \mathbb{Z}$, for a.e. $(A, B) \in \mathbb{R}^2$, and the fact that $d\varphi_\gamma$ was normalized. Similarly we proceed when $n = 2, 3, \ldots$. Hence $d\lambda$ acquires unbounded total variation. We shortly write $\|\lambda\| = +\infty$.

5. OS-axioms

If $e = 0$ the Hamiltonian is the sum of $P^2/2m$ and I_β. Its ground state is given by the vector $1 \otimes \Omega_\beta \in L^2(S \times \mathbb{R})$, where Ω_β stands for the oscillator vacuum.

At imaginary time we have a two-dimensional Markov process $t \to (x(t), A(t))$ on the large probability space

$$\left(Q \times A, \sum \times \mathfrak{A}, d\omega_0 \otimes d\varphi_\beta\right). \quad (15)$$

The measure $d\omega_\alpha$, for $\alpha = 0$, describes free Brownian motion in the circle S and $d\varphi_\beta$ governs an Ornstein-Uhlenbeck process. The moments factorize. Using $x \in S$ and the fact that $d\varphi_\beta$ is Gaussian we easily derive the estimate

$$\left|\langle x(s_1)x(s_2) \cdots x(s_m) \cdots A(t_n)\rangle_0\right| \leqslant \pi^m \frac{(n!)^{1/2}}{(2\beta)^n}, \tag{16}$$

valid for $m, n = 0, 1, 2, \ldots$. Of course for odd m or n this vanishes. As an exercise let us calculate the correlation function of $d\omega_0$. Expanding $f(x) = x$ in a Fourier series on $(-\pi, \pi)$ we get

$$\langle x(s_1)x(s_2)\rangle_0 = \sum_{k \neq 0} k^{-2} \cdot \exp(-\varepsilon k^2/2), \tag{17}$$

where $\varepsilon = |s_2 - s_1|$. In the limit $\varepsilon \searrow +0$ we recover the variance of the normalized Lebesque measure $dx/2\pi$ on S.

We denote $x = x(0)$ and $u = dx(t)/dt$. Then by the Feynman–Kac formula $-\langle x \cdot u(\varepsilon)\rangle_0$ for small ε becomes equal to the divergent expression

$$\tfrac{1}{2}(x\Omega, p^2(x\Omega)) = +\infty. \tag{18}$$

Because of $x \in S$ also in the interacting case the existence of Green's functions is rather trivial. But, as we learned above, their time derivatives are singular at coinciding arguments [13].

Theorem I. *The moments* $\langle x(s_1)x(s_2) \cdots x(s_m) \cdots A(t_n)\rangle$, *where* $m, n = 0, 1, 2, \ldots$ *of the cylinder measure* $d\lambda$ *on* $Q \times A$ *exist and are*

(i) *integrable,*
(ii) *time translation invariant,*
(iii) OS-*positive,*
(iv) *complex for* $e \neq 0$. (19)

Proof. Within the famous reconstruction theorem of Osterwalder and Schrader (iii) is a consequence of QM □. We would like to check it looking just at the Green's functions of our model. The idea is simple.

The propagator which defines $d\omega_\alpha$ satisfies $R^s_\alpha(x, y)^* = R^s_\alpha(y, x)$, where $x, y \in S$ and $s \geqslant 0$. So for any bounded function $f = f(x(s))$ we get

$$\int_Q f^* \cdot \Theta f \, d\omega_\alpha = \exp(-s\alpha^2) \cdot \|\Psi\|^2 \geqslant 0, \tag{20}$$

where $\Psi = R^s_\alpha f \in L^2(S)$. We also check the inequality for cylinder functions say $f = f(x(s_1), \ldots, x(s_n))$ with $s_1, s_2, \ldots, s_n$ in $\mathbb{R}_+$. Strong OS-positivity requires it to hold for any exponential function measurable with respect to

$$\Sigma_+ = \sigma\left(\bigcup_{s \geqslant 0} \Sigma_s\right). \tag{21}$$

We observe that $\sigma(\Sigma_- \cup \Sigma_+) = \Sigma$, where Σ_- is the image of Σ_+ under reflection Θ.

This generalizes to the cylinder measure $d\lambda$. What about Yngvason's result? He proved that the above conditions are not compatible with the existence of $d\lambda$ as a *finite* measure. So the upper bound

$$\frac{1}{z}\iint_{Q\times A} |x(s_1)x(s_2)\cdots x(s_m)\cdots A(t_n)|\, d\omega_0 \otimes d\varphi_\beta \tag{22}$$

on the Green's functions, for all $m, n = 0, 1, 2, \ldots$, implies $Z = 0$. Indeed the free measure $d\omega_0 \otimes d\varphi_\beta$ is ergodic and hence [14] $d\lambda$ cannot be the perturbation by some phase factor.

6. White noise

We remark that

$$\left\langle \exp\left\{i \cdot \left(\sum_{j=1}^{m} k_j x(s_j)\right)\right\} A(t_1)\cdots A(t_n)\right\rangle, \tag{23}$$

for $k_1, k_2, \ldots, k_m \in \mathbb{Z}$, has an integral representation with respect to the measure $dx \otimes d\eta(u, A)$, where $t \to u(t) \in \mathbb{R}$ for $e = 0$ was white noise [15]. More precisely, let us consider the operator

$$C = G_\gamma^{1/2}\begin{bmatrix} \Delta & -ie \\ -ie & 1 \end{bmatrix} G_\gamma^{1/2} \tag{24}$$

in $D = L^2(\mathbb{R}) \oplus L^2(\mathbb{R})$. Above $\Delta = \beta^2 - d^2/dt^2$ and $G_\beta(\cdot, \cdot)$ will denote the kernel of its inverse Δ^{-1}. One easily shows that C defines the covariance of the desired cylinder measure on $U \times A$. Of course the restriction of $d\eta$ to $\mathfrak{A}$ should coincide with $d\varphi_\gamma$. To avoid confusion we introduce another symbol $\langle\!\langle \cdot \rangle\!\rangle$ for expectation with respect to $d\eta$. We find

$$\langle\!\langle u(b)A(a) \rangle\!\rangle = -ie(b, G_\gamma a), \tag{25}$$

$$\langle\!\langle (u(b)A(a))^2 \rangle\!\rangle = -2e^2 \cdot (b, G_\gamma a)^2 + \left(b, \frac{1}{1 + e^2 G_\beta} b\right)(a, G_\gamma a), \tag{26}$$

etc. Provided $|e|^2 \leqslant \beta^2/2$, the last expression becomes non-negative. We claim that C is a sectorial operator [16] in D with half opening angle $\operatorname{arctg}(e/\beta)$. There is a striking analogy to the path space measure for the bosonized massless Schwinger model $(\mathrm{QED})_2$ [17]. Formally $d\eta$ is given by

$$e^{-z^2/2}\, dz \otimes d\varphi_\gamma \big|_{z = u + ieA}. \tag{27}$$

Of course the variable z was nothing but the imaginary time counter part of the canonical momentum $p = m\dot{x} + eA$, where m is fixed to one.

Let $t \to \xi(t) \in \mathbb{R}$ be one-dimensional Brownian motion mastering at time zero the slalom $\xi(0) \in (-\pi, \pi)$. This defines a Markov process on $(X, \Xi, d\nu)$, where X is the space of trajectories and Ξ the σ-algebra generated by cylinder sets.

$$dv = dx \otimes e^{-u^2/2}\, du, \quad x = \xi(0), \tag{28}$$

is an averaged conditional Wiener measure. If we close $(-\pi, \pi)$ to the circle and consider the real line as covering space [18] of S we may identify periodic sets

$$\pi^{-1}(M) = \bigcup_{l \in \mathbb{Z}} \{\xi \in X: \xi(t) - 2\pi l \in M\}, \tag{29}$$

for Borel M in S and $t \neq 0$, with elements of Σ. In other words π was the canonical projection from $\mathbb{R}$ onto $S = \mathbb{R}/\mathbb{Z}$. The lift π^{-1} in a natural way induces a measure isomorphism. All that generalizes to the coupled system.

Theorem II.

$$d\lambda = dx \otimes d\eta \,\big|_{\pi^{-1}(\Sigma \times \mathfrak{A})}. \tag{30}$$

We emphasize that $d\lambda$ is translation invariant whereas the measure $dx \otimes d\eta$ was not [19]. The above redefinition allows us to calculate expectation values as $\langle \exp\{i(kx)\} \cdot A(t)\rangle$, for $k \in \mathbb{Z}$. Integrating over $x \in S$ we obtain $\delta(k)$ and then we are left with a Gaussian. For non-integer k everything becomes more tricky. But "Gott kümmert sich nicht um unsre mathematischen Schwierigkeiten. Er integriert empirisch" [20].

7. Memory

As hidden in the title we have an alternative resolution to the problem of existence of a path space measure for $(\mathrm{QED})_1$. We claim that in the mixed representation where x and E are diagonal the QM propagator $P^t = \exp(-tH)$, $t \geqslant 0$, is positivity preserving [21].

Indeed given any bounded $f(x, E) \geqslant 0$, because of $\Omega(x, E) \geqslant 0$, the vector $\Psi = f \cdot \Omega$ is also represented by a non-negative function in the physical Hilbert space $L^2(S \times \mathbb{R})$. We now apply the factors V, K^t_γ, V^* and $\exp\{-t(p^2/2M)\}$ step by step. V shifts the variable x to $x + eE/\gamma^2$, V^* conversely. With the other two operators there is no trouble. Hence $P^t\Psi(x, E) \geqslant 0$. In particular this will be true for any $\Psi = (g \otimes 1) \cdot \Omega$ with $g(x) \geqslant 0$, $x \in S$. By induction

$$(\Omega, g \otimes 1\, e^{-(t_2 - t_1)H} g_2 \otimes 1 \cdots g_n \otimes 1 \cdot \Omega) \geqslant 0, \tag{31}$$

provided $g_j(x) \geqslant 0$ for all $j = 1, 2, \ldots, n$. Let $\mathscr{M}$ denote the Abelian algebra of those bounded multiplication operators $F = g \otimes 1$, $\|F\| < \infty$, acting in $L^2(S \times \mathbb{R})$. It is the completion of

$$\mathscr{M}_0 = \{F = e^{ikx} \otimes 1: k \in \mathbb{Z}\} \tag{32}$$

in norm and not maximal [22]. Instead we observe that the vacuum $\Omega = 1 \otimes \Omega\gamma$ is cyclic for the algebra $\mathscr{B}_0$ generated by P^t, $t \geqslant 0$, and the elements of $\mathscr{M}_0$. Of course $\mathscr{B}_0$ is dense in the algebra $\mathscr{B} = \mathscr{B}(L^2(S \times \mathbb{R}))$ of all bounded operators.

We claim that

$$\Psi(t) = P^t(e^{ikx} \otimes 1)\Omega, \tag{33}$$

$t \geqslant 0$, span $\mathscr{H}_k$ except in the case when $k = 0$. But the vacuum sector N of $L^2(S \times \mathbb{R})$ is spanned by the vectors $\Psi(t) = F^* P^t F\Omega$, $t \geqslant 0$, with any $F \in \mathscr{M}_0$ different from the unit element. This has a nice consequence.

Theorem III. *The triple*

$$L^2(S \times \mathbb{R}), \mathscr{M}_0, \{P^t, t \geqslant 0\} \tag{34}$$

together with the vacuum Ω *builds a generalized positive semigroup structure. Hence the above* QM *amplitude define the Fourier transform of a probability measure* $d\mu$ *on* Q.

Proof. See Klein's theorem [23] □. One can show that $d\mu$ is OS-positive. Given $0 \leqslant t_1 \leqslant t_2 \leqslant \cdots \leqslant t_n$ and $k_j \in \mathbb{Z}$ we find

$$\int_Q \exp\left\{i\left(\sum_{j=0}^{n} k_j x(t_j)\right)\right\} d\mu = \delta(k) \cdot \exp\left\{-\frac{1}{2}\left(\sum_{i,j=1}^{n}\sum k_i k_j \tau^{ij}\right)\right\}, \tag{35}$$

where

$$\tau^{ij} = \left(h_i, \frac{1}{1 + e^2 \cdot G_\beta} h_j\right), \quad i, j = 1, 2, \ldots, n. \tag{36}$$

Here h_j are the indicator functions of time intervals $0 \leqslant s \leqslant t_j$, $j = 1, 2, \ldots, n$ in $\overline{\mathbb{R}}_+$ and k stands shortly for the sum of all k_j's. The diagonal elements of the matrix τ^{ij} yield a modified function $t \to \tau(t)$, $t \geqslant 0$, satisfying

$$\tau(s + t) = \tau(s) + \tau(t) - \frac{e^2}{\gamma} \cdot \delta(s)\,\delta(t). \tag{37}$$

Let us introduce $L^2(Q)$ with scalar product given by the measure $d\mu$ and denote R the projection operator onto the subspace $L^2(Q_+)$ of functions which are measurable with respect to Σ_+.

Then [24]

$$\mathscr{K} = \overline{L^2(Q_+)/\ker W}, \tag{38}$$

where $W = +(R\Theta R)^{1/2}$, can be identified with the physical Hilber space. Since $d\mu$ violates the Markov property $\mathscr{K}$ is larger than $L^2(S)$. Indeed we find an isometry $J: \mathscr{K} \to \mathscr{H}$ so that

$$\begin{aligned}\Psi(t) &= JWT^t\, e^{ikx} \otimes 1 \\ &= c(t, k^2) \cdot \exp\{ik(x + \gamma^{-2} \cdot e\delta(t)E)\}\Omega,\end{aligned} \tag{39}$$

where E was the canonical conjugate to A. For details see [25]. On the classical level the elimination of the "unvisible" oscillator leads to the following integro-differential equation

$$m\ddot{x}(t) = -e^2\left\{x(t) + y \cdot \int_{t_1}^{t_2} C_\beta(s, t)x(s)\, ds\right\}. \tag{40}$$

Above $-\beta^2 C_\beta(\cdot, \cdot)$ stands for the periodic Green's function of the hyperbolic operator $\beta^2 + d^2/dt^2$ on (t_1, t_2) [26]. Of course one may choose other boundary conditions as well.

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