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# SUPERCURRENT WITHOUT AUXILIARY FIELDS

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**Abstract.** The supercurrent multiplet is constructed for the Wess-Zumino supersymmetric model of a self-interacting chiral multiplet in four-dimensional spacetime. The theory is quantized without auxiliary fields. The known supercurrent anomaly, which yields the dilatation anomaly, is recovered.

## 1. Introduction

The aim of the present work is twofold. It first offers the opportunity to testify the gratitude of one of the authors (O.P.) to Gérard Wanders who has guided his first steps in the beautiful field of theoretical physics.

Secondly, as the title lets it understand, we wish to treat the problem of the supercurrent multiplet [1] in a formulation of supersymmetry without auxiliary fields. The difficulty of the task follows from the fact that, in this formulation, supersymmetry is realized in a nonlinear way. We shall consider the simplest four-

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dimensional model, namely the Wess-Zumino model [2] of a chiral supermultiplet with a cubic interaction. Although we know that the model itself is renormalizable in the formulation without auxiliary fields [3], the construction of the supercurrent and the study of its anomalies has only been achieved up to now in the linear realization, i.e. with auxiliary fields [4].

Our first motivation for undertaking this research was the need of understanding the supercurrent anomaly – directly related to the scale or trace anomaly – in the context of the three-dimensional Chern-Simons topological field theory [5]. This model, taken in the Landau gauge, possesses indeed a supersymmetry of the same kind as the one we are going to discuss here [6]. But no auxiliary field formulation is known in this case: the use of the techniques we intend to demonstrate in the following is then unavoidable. Let us recall that they have led to the proof of the full scale invariance of the Chern-Simons theory [7]. For the Wess-Zumino model, we shall simply recover the well-known [8] relation of the Callan-Symanzik $\beta$-function with the anomalous dimension of the field.

The iterative method we shall apply to the perturbative construction of the theory is algebraic in character [9], [10], [11]. It makes use of general theorems of renormalization theory – in particular of the quantum action principle [12], [13] – which are based on power counting and locality. The power of the method results from the remarkable fact that it allows one to reduce the discussion of the radiative corrections to that of polynomials of classical fields. These fields are the arguments of the generating functional of the vertex functions, i.e. they are fast decreasing $C_\infty$ functions. The method is thus independent of the particular renormalization scheme one chooses, be it linked or not with a regularization.

It was also one of our motivations, to illustrate the "algebraic renormalization" method in a simple but nontrivial case.

The plan of the paper is the following: after a short summary of known facts about the Wess-Zumino model without auxiliary fields, given in Section 2, we present the supercurrent and its properties at the classical level in Section 3, whereas Section 4 is devoted to the construction of the supercurrent to all orders of perturbation theory and to the determination of its anomalies.

## 2. The Wess-Zumino model without auxiliary fields

The Wess-Zumino model [2] involves a chiral supermultiplet consisting of a complex scalar field $A(x)$ and of a complex two-component Weyl spinor field $\psi_\alpha(x)$, $\alpha = 1,2$, their complex conjugates being noted $\bar{A}(x)$ and $\bar{\psi}_{\dot\alpha}$. The infinitesimal supersymmetry transformations read

$$\begin{aligned}
&\delta_\alpha A = \psi_\alpha, && \bar\delta_{\dot\alpha}\bar A = \bar\psi_{\dot\alpha},\\
&\delta_\alpha \psi^\beta = 2g\delta_\alpha^\beta \bar A^2, && \bar\delta_{\dot\alpha}\bar\psi^{\dot\beta} = -2g\delta_{\dot\alpha}^{\dot\beta} A^2,\\
&\delta_\alpha \bar A = 0, && \bar\delta_{\dot\alpha} A = 0,\\
&\delta_\alpha \bar\psi_{\dot\beta} = 2i\partial\!\!\!/_{\alpha\dot\beta}\bar A, && \bar\delta_{\dot\alpha}\psi_\beta = 2i\partial\!\!\!/_{\beta\dot\alpha} A,
\end{aligned} \tag{2.1}$$

where $\partial\!\!\!/_{\alpha\dot\beta} = \partial_\mu(\sigma^\mu)_{\alpha\dot\beta}$, $\sigma^0$ being the unit $2\times 2$-matrix and $\sigma^i$, $i = 1,2,3$, the three Pauli matrices. Lowering and raising of spinor indices are performed with the help of the antisymmetric matrices $\epsilon_{\alpha\beta}$ and $\epsilon^{\alpha\beta} = -\epsilon_{\alpha\beta}$. The notations and conventions are those of [3] and [4].

The nonlinearity of these transformation laws is due to the elimination of the auxiliary fields $F$ and $G$ which appear in the usual formulation [2] of the model. The classical invariant action reads:

$$\Gamma_{\rm inv} = \int d^4x \left[\partial_\mu A\partial^\mu \bar A + \frac{i}{2}\psi^\alpha\sigma^\mu_{\alpha\dot\beta}\partial_\mu\bar\psi^{\dot\beta} + \frac{g}{2}(\psi^\alpha\psi_\alpha A + \bar\psi_{\dot\alpha}\bar\psi^{\dot\alpha}\bar A) - g^2(A\bar A)^2\right] . \tag{2.2}$$

(We consider the massless theory.)

In order to be able to write Ward identities expressing the invariance of the theory under the nonlinear supersymmetry transformations (2.1), one must introduce a complex scalar source $u(x)$ for the composite operator $A^2$. Thus the complete classical action reads:

$$\Gamma^{\rm class} = \Gamma_{\rm inv} + \int d^4x\, 2g(uA^2 + \bar u\bar A^2) , \tag{2.3}$$

and it obeys the Ward identities

$$\begin{aligned}
W_\alpha(\Gamma^{\rm class}) \equiv \int d^4x \Big\{ &\psi_\alpha \frac{\delta\Gamma^{\rm class}}{\delta A} + \left(\frac{\delta\Gamma^{\rm class}}{\delta\bar u} - 4u\right)\frac{\delta\Gamma^{\rm class}}{\delta\psi^\alpha}\\
&- 2i(\partial\!\!\!/_{\alpha\dot\beta}\bar A)\frac{\delta\Gamma^{\rm class}}{\delta\bar\psi_{\dot\beta}} + 2iu\partial\!\!\!/_{\alpha\dot\beta}\bar\psi^{\dot\beta}\Big\} = 0 ,\\
\overline{W}_{\dot\alpha}(\Gamma^{\rm class}) \equiv \int d^4x \Big\{ &\bar\psi_{\dot\alpha} \frac{\delta\Gamma^{\rm class}}{\delta\bar A} - \left(\frac{\delta\Gamma^{\rm class}}{\delta u} - 4\bar u\right)\frac{\delta\Gamma^{\rm class}}{\delta\bar\psi^{\dot\alpha}}\\
&- 2i(\partial\!\!\!/_{\beta\dot\alpha} A)\frac{\delta\Gamma^{\rm class}}{\delta\psi_\beta} - 2i\bar u\partial\!\!\!/_{\beta\dot\alpha}\psi^\beta\Big\} = 0 .
\end{aligned} \tag{2.4}$$

Note the presence of terms linear in the quantum field $\bar\psi$ $(\psi)$.

The corresponding quantum theory is described by the generating functional of its Green functions or, more conveniently for our purposes, by the vertex functional $\Gamma(A,\psi,u)$ which coincides with the classical action (2.3) in the limit $\hbar\to 0$:

$$\Gamma=\Gamma^{\rm class}+O(\hbar)\ . \tag{2.5}$$

The renormalizability of the theory defined by the *nonlinear* Ward identities

$$W_\alpha(\Gamma)=0\ ,\qquad \overline{W}_{\dot\alpha}(\Gamma)=0\ , \tag{2.6}$$

was proven in [3] for all orders of perturbation theory – considered as a formal power series in $\hbar$.

A Callan-Symanzik equation was also shown to hold:

$$[\mu\partial_\mu+\beta\partial_g-\gamma N]\,\Gamma=0\ , \tag{2.7}$$

where $\mu$ is the normalization mass and

$$N\Gamma\equiv\int d^4x\ \left\{\left[\psi\frac{\delta}{\delta\psi}+\bar\psi\frac{\delta}{\delta\bar\psi}+A\frac{\delta}{\delta A}+\bar A\frac{\delta}{\delta\bar A}-u\frac{\delta}{\delta u}-\bar u\frac{\delta}{\delta\bar u}\right]\Gamma+8u\bar u\right\}\ , \tag{2.8}$$

is the "counting" operator. The differential operators $\mu\partial_\mu$, $\partial_g$ and $N$ are supersymmetric, i.e. their action on $\Gamma$ defines – through the quantum action principle [12], [13] – vertex insertions which are invariant under the $\Gamma$-dependent *linearized Ward operators*

$$\begin{aligned}\mathcal{W}_\alpha\equiv\int d^4x\ &\left\{\psi_\alpha\frac{\delta}{\delta A}+\left(\frac{\delta\Gamma}{\delta\bar u}-4u\right)\frac{\delta}{\delta\psi^\alpha}\right.\\&\left.\quad+\frac{\delta\Gamma}{\delta\psi^\alpha}\frac{\delta}{\delta\bar u}-2i(\partial\!\!\!/_{\alpha\dot\beta}\bar A)\frac{\delta}{\delta\bar\psi_{\dot\beta}}\right\}\ ,\\ \overline{\mathcal{W}}_{\dot\alpha}\equiv\int d^4x\ &\left\{\bar\psi_{\dot\alpha}\frac{\delta}{\delta\bar A}-\left(\frac{\delta\Gamma}{\delta u}-4\bar u\right)\frac{\delta}{\delta\bar\psi^{\dot\alpha}}\right.\\&\left.\quad-\frac{\delta\Gamma}{\delta\bar\psi^{\dot\alpha}}\frac{\delta}{\delta u}-2i(\partial\!\!\!/_{\beta\dot\alpha}A)\frac{\delta}{\delta\psi_\beta}\right\}\ .\end{aligned} \tag{2.9}$$

The model is also invariant under the Abelian symmetry R of Fayet [14]. The infinitesimal R-transformations are given by

$$\delta_R\varphi = n_\varphi \varphi \ , \qquad \delta_R\bar{\varphi} = -n_\varphi \bar{\varphi} \ , \tag{2.10}$$

for $\varphi = A,\ \psi,\ u$. The R-weights $n_\varphi$ are given in Table 1 together with the dimensions in mass units.

| $\varphi$ | $A$ | $\psi$ | $u$ |
|---|---|---|---|
| $d_\varphi$ | 1 | 3/2 | 2 |
| $n_\varphi$ | −2/3 | 1/3 | 4/3 |

**Table 1.** *Dimensions $d_\varphi$ and R-weights $n_\varphi$*

The R-invariance of the theory is expressed by the Ward identity

$$\mathcal{W}_R\Gamma \equiv \int d^4x \ w_R(x)\Gamma \equiv \sum_{\phi=A,\psi,u,\bar{A},\bar{\psi},\bar{u}} \int d^4x \ \delta_R\phi \frac{\delta\Gamma}{\delta\phi} = 0 \ . \tag{2.11}$$

Also introducing the translation Ward identity

$$\mathcal{W}_\mu\Gamma \equiv \int d^4x \ w_\mu(x)\Gamma \equiv \sum_{\phi=A,\psi,u,\bar{A},\bar{\psi},\bar{u}} \int d^4x \ \partial_\mu\phi \frac{\delta\Gamma}{\delta\phi} = 0 \ , \tag{2.12}$$

one easily checks that the linear Ward operators defined in (2.9), (2.11) and (2.12) fulfill the superalgebra

$$\begin{aligned} &\{\mathcal{W}_\alpha, \overline{\mathcal{W}}_{\dot\beta}\} = 2i\sigma^\mu_{\alpha\dot\beta}\mathcal{W}_\mu \ , \qquad \{\mathcal{W}_\alpha, \mathcal{W}_\beta\} = \{\overline{\mathcal{W}}_{\dot\alpha}, \overline{\mathcal{W}}_{\dot\beta}\} = 0 \ , \\ &[\mathcal{W}_\alpha, \mathcal{W}_R] = -\mathcal{W}_\alpha \ , \qquad [\overline{\mathcal{W}}_{\dot\alpha}, \mathcal{W}_R] = \overline{\mathcal{W}}_{\dot\alpha} \ , \\ &[\mathcal{W}_\mu, \mathcal{W}_R] = [\mathcal{W}_\mu, \mathcal{W}_\alpha] = [\mathcal{W}_\mu, \overline{\mathcal{W}}_{\dot\alpha}] = 0 \ , \end{aligned} \tag{2.13}$$

as a consequence of $\Gamma$ obeying the Ward identities (2.6), (2.11) and (2.12). One recognizes here the Poincaré supersymmetry algebra enlarged with the automorphism $\mathcal{W}_R$. Note that the present convention for the Ward identity operators differs from that of [3] by a factor $i$.

## 3. The supercurrent and the trace identity: classical approximation

The invariance of the classical action $\Gamma^{\rm class}$ under the R-transformations (2.10) implies the existence of a conserved classical current defined uniquely by:

$$\partial^\mu R_\mu(x) = w_R(x)\Gamma^{\rm class} \ . \tag{3.1}$$

For $R^\mu$ one gets:

$$R^{\alpha\dot\alpha} = \ \frac{1}{3}\left[\, A(\partial\!\!\!/^{\alpha\dot\alpha}\bar A) - (\partial\!\!\!/^{\alpha\dot\alpha}A)\bar A \,\right] \ + \ \frac{i}{6}\psi^\alpha\bar\psi^{\dot\alpha} \ , \tag{3.2}$$

whith

$$R^\mu = \ (\sigma^\mu)_{\alpha\dot\alpha}R^{\alpha\dot\alpha} \ . \tag{3.3}$$

Repeated application of the supersymmetry transformations – given by the linearized Ward operators (2.9) – on the R-current $R_\mu$ generates a supermultiplet called the supercurrent [1]. The physically relevant components of this multiplet are, besides the R-current, the conserved spinor current

$$\mathcal{J}_{\mu\alpha} = \mathcal{W}^{\rm class}_\alpha R_\mu \ , \tag{3.4}$$

and the conserved energy momentum tensor $\theta_{\mu\nu}$:

$$\begin{aligned}\theta_{\mu\nu} =& \frac{1}{16i}\left(\ (\sigma_\mu)^{\alpha\dot\alpha}(\sigma_\nu)_{\beta\dot\beta} + \ (\sigma_\nu)^{\alpha\dot\alpha}(\sigma_\mu)_{\beta\dot\beta}\ \right)\left[\mathcal{W}^{\rm class}_\alpha \ , \ \overline{\mathcal{W}}^{\rm class}_{\dot\alpha}\right] R^{\beta\dot\beta} \\ & - \frac{4}{3}g_{\mu\nu}u\bar u \ ,\end{aligned} \tag{3.5}$$

where $\mathcal{W}^{\rm class}_\alpha$, $\overline{\mathcal{W}}^{\rm class}_{\dot\alpha}$ are defined as in Eq. (2.9), but with $\Gamma$ replaced by $\Gamma^{\rm class}$.

The basic Ward identities obeyed by the supercurrent are the "supertrace identities":

$$\begin{aligned}&\bar A\ \frac{\delta\Gamma^{\rm class}}{\delta\bar\psi_{\dot\alpha}} + 2u\bar\psi^{\dot\alpha} + \frac{3}{2i}\mathcal{W}^{\rm class}_\alpha R^{\alpha\dot\alpha} = 0 \ , \\ &A\ \frac{\delta\Gamma^{\rm class}}{\delta\psi_\alpha} - 2\bar u\psi^\alpha - \frac{3}{2i}\overline{\mathcal{W}}^{\rm class}_{\dot\alpha} R^{\alpha\dot\alpha} = 0 \ .\end{aligned} \tag{3.6}$$

Repeated application of $\mathcal{W}_\alpha^{\rm class}, \overline{\mathcal{W}}_{\dot\alpha}^{\rm class}$ and use of the anticommutation relations (2.13) lead indeed to the conservation law (3.1) of the R-current and to the conservation of the energy-momentum tensor:

$$\partial^\mu \theta_{\mu\nu} = w_\nu(x)\Gamma^{\rm class} + \partial^\mu \hat{w}_{\mu\nu}(x)\Gamma^{\rm class} , \tag{3.7}$$

with $w_\nu(x)$ defined as in (2.12) and

$$\begin{aligned} \hat{w}_{\mu\nu} = & -g_{\mu\nu}\frac{1}{3}\left(A\frac{\delta}{\delta A} + \bar{A}\frac{\delta}{\delta \bar{A}} + 2u\frac{\delta}{\delta u} + 2\bar{u}\frac{\delta}{\delta \bar{u}} + \psi^\alpha\frac{\delta}{\delta\psi^\alpha} + \bar\psi^{\dot\alpha}\frac{\delta}{\delta\bar\psi^{\dot\alpha}}\right) \\ & -\frac{i}{4}\psi^\alpha(\sigma_{\mu\nu})_\alpha{}^\beta\frac{\delta}{\delta\psi^\beta} + \frac{i}{4}\bar\psi^{\dot\alpha}(\bar\sigma_{\mu\nu})_{\dot\alpha}{}^{\dot\beta}\frac{\delta}{\delta\bar\psi^{\dot\beta}} \; . \end{aligned} \tag{3.8}$$

This also leads to the trace identity:

$$\theta^\mu{}_\mu(x) = w^{\rm trace}(x)\Gamma^{\rm class} , \tag{3.9}$$

where

$$w^{\rm trace} = -\frac{1}{3}\left(A\frac{\delta}{\delta A} + \bar{A}\frac{\delta}{\delta \bar{A}} - \frac{1}{2}\psi^\alpha\frac{\delta}{\delta\psi^\alpha} - \frac{1}{2}\bar\psi^{\dot\alpha}\frac{\delta}{\delta\bar\psi^{\dot\alpha}} + 2u\frac{\delta}{\delta u} + 2\bar{u}\frac{\delta}{\delta \bar{u}}\right) . \tag{3.10}$$

Let us remind that the differential operators $w_R$, etc. acting on $\Gamma^{\rm class}$ and which appear in the right-hand side of all these Ward identities represent nonphysical "contact" terms, as one can see if one writes the corresponding identities in terms of Green functions instead of the vertex functional.

The above expressions show that the energy-momentum tensor $\theta_{\mu\nu}$ defined in (3.5), being symmetric, traceless and conserved, is indeed the improved one.

The classical dilatation current $\mathcal{D}_\mu$ can now be defined as

$$\mathcal{D}_\mu = x^\nu\theta_{\mu\nu} - x^\nu\hat{w}_{\mu\nu}\Gamma^{\rm class} . \tag{3.11}$$

It obeys the conservation Ward identity

$$\partial^\mu\mathcal{D}_\mu = w_D(x)\Gamma^{\rm class} , \tag{3.12}$$

where

$$w_D = \sum_{\text{all fields}} (x\partial\phi + d_\phi \phi)\frac{\delta}{\delta\phi} , \tag{3.13}$$

is the local operator for the scale transformations and $d_\phi$ is the canonical dimension of the field $\phi$ (see Table 1). Finally, integrating Eq. (3.12) we get the exact Ward identity for the classical scale invariance:

$$\mathcal{W}_D \Gamma^{\text{class}} \equiv \int d^4x \; w_D(x)\Gamma^{\text{class}} = 0 . \tag{3.14}$$

Let us summarize this section by stressing that the supercurrent and the supertrace equations (3.6) are the basic ingredients we shall need in order to explore the behaviour of the model under the scale transformations.

## 4. The supercurrent and the trace identity: radiative corrections

We have now to extend the construction of the preceding Section to all orders of perturbation theory. The starting point is again the Ward identity (2.11) for R-invariance, as well as those for supersymmetry (2.6) and for translation invariance (2.12). The vanishing of the left-hand side of (2.11) implies that its integrand is a total derivative insertion. This defines – uniquely – the R-current insertion $R_\mu \cdot \Gamma$ through the conservation Ward identity

$$\partial^\mu R_\mu(x) \cdot \Gamma = w_R(x)\Gamma . \tag{4.1}$$

The supertrace identities (3.6) take now the form

$$A\,\frac{\delta\Gamma}{\delta\psi_\alpha} - 2\bar{u}\psi^\alpha - \frac{3}{2i}\overline{W}_{\dot\alpha} R^{\alpha\dot\alpha} = \Delta^\alpha \cdot \Gamma = \Delta^\alpha + O(\hbar\Delta) , \tag{4.2}$$

(and complex conjugate equation).
The insertion in the right-hand side represents the effect of the radiative corrections we want to study, and $\Delta^\alpha$ is their lowest order contribution. We know from the quantum action principle [12], [13] that $\Delta^\alpha$ is a local field polynomial of

dimension 7/2 (the dimension of the left-hand side of (4.2)), and its R-weight is -1.

It is not difficult to deduce from the specific form of the left-hand side of (4.2) and from the algebra (2.13) that the insertion $\Delta^\alpha$ satisfies the consistency conditions

$$\begin{aligned} &\mathcal{W}^\alpha(\Delta_\alpha\cdot\Gamma)-\overline{\mathcal{W}}_{\dot\alpha}\left(\overline{\Delta}^{\dot\alpha}\cdot\Gamma\right)=0\,,\\ &\mathcal{W}^\beta\mathcal{W}_\beta\left(\overline{\Delta}^{\dot\alpha}\cdot\Gamma\right)=0\,,\quad \overline{\mathcal{W}}_{\dot\beta}\overline{\mathcal{W}}^{\dot\beta}(\Delta^\alpha\cdot\Gamma)=0\,, \end{aligned} \tag{4.3}$$

or, at the lowest nonvanishing order:

$$\begin{aligned} &\mathcal{W}^{\mathrm{class}\,\alpha}\Delta_\alpha-\overline{\mathcal{W}}^{\mathrm{class}}_{\dot\alpha}\overline{\Delta}^{\dot\alpha}=0\,,\\ &\mathcal{W}^{\mathrm{class}\,\beta}\mathcal{W}^{\mathrm{class}}_\beta\overline{\Delta}^{\dot\alpha}=0\,,\quad \overline{\mathcal{W}}^{\mathrm{class}}_{\dot\beta}\overline{\mathcal{W}}^{\mathrm{class}\,\dot\beta}\Delta^\alpha=0\,. \end{aligned} \tag{4.4}$$

One finds by an explicit search that the general solution of (4.4) is of the form:

$$\Delta_\alpha=r\overline{\mathcal{W}}^{\mathrm{class}}_{\dot\alpha}\overline{\mathcal{W}}^{\mathrm{class}\,\dot\alpha}\mathcal{W}^{\mathrm{class}}_\alpha(A\bar A)\,, \tag{4.5}$$

where $r$ is a constant parameter, of order $\hbar$. The procedure iterates to give as the general solution of (4.3) the *supertrace anomaly*

$$\Delta_\alpha\cdot\Gamma=r\overline{\mathcal{W}}_{\dot\alpha}\overline{\mathcal{W}}^{\dot\alpha}\mathcal{W}_\alpha(\Phi\cdot\Gamma)\,, \tag{4.6}$$

where $r$ now is a power series in $\hbar$, and $\Phi$ some quantum extension of $A\bar A$:

$$\Phi\cdot\Gamma=A\bar A+O(\hbar)\,. \tag{4.7}$$

Some algebraic manipulations based on the anticommutation relations in (2.13) show that the supertrace anomaly does not spoil the conservation equations for the R-current, the spinor current and the energy-momentum tensor. The latter currents are defined as in (3.4) and (3.5), but now with $\mathcal{W}^{\mathrm{class}}_\alpha$ replaced by the full quantum operator $\mathcal{W}_\alpha$, and the classical current $R_\mu$ by the insertion $R_\mu\cdot\Gamma$. Thus, in particular:

$$\partial^\mu\theta_{\mu\nu}\cdot\Gamma=\;w_\nu(x)\Gamma+\partial^\mu\hat w_{\mu\nu}(x)\Gamma\,. \tag{4.8}$$

In the contrary the trace identity (3.9) becomes anomalous:

$$\theta^{\mu}{}_{\mu}(x)\cdot\Gamma = w^{\rm trace}(x)\Gamma - \frac{r}{3}\mathcal{W}^{\alpha}\overline{\mathcal{W}}_{\dot\alpha}\overline{\mathcal{W}}^{\dot\alpha}\mathcal{W}_{\alpha}\left(\Phi(x)\cdot\Gamma\right) \ . \tag{4.9}$$

From this follows the broken conservation Ward identity for the dilatation current (defined analogously to (3.11)):

$$\partial^{\mu}D_{\mu}(x)\cdot\Gamma = w_D(x)\Gamma - \frac{r}{3}\mathcal{W}^{\alpha}\overline{\mathcal{W}}_{\dot\alpha}\overline{\mathcal{W}}^{\dot\alpha}\mathcal{W}_{\alpha}\left(\Phi(x)\cdot\Gamma\right) \ , \tag{4.10}$$

and, through spacetime integration, the anomalous dilatation Ward identity:

$$\begin{aligned}\mathcal{W}_D\Gamma &= \frac{r}{3}\int d^4x\ \mathcal{W}^{\alpha}\overline{\mathcal{W}}_{\dot\alpha}\overline{\mathcal{W}}^{\dot\alpha}\mathcal{W}_{\alpha}\left(\Phi(x)\cdot\Gamma\right)\\ &= (\beta\partial_g - \gamma N)\,\Gamma \ .\end{aligned} \tag{4.11}$$

In order to get the second equality we have expanded the spacetime integral of the trace anomaly (4.9) in the basis

$$\{\partial_g\Gamma,\ \ N\Gamma\} \ , \tag{4.12}$$

for the supersymmetric integrated insertions of dimension 4. The differential operator $N$ was defined in (2.8).

What we have thus obtained is nothing else than the Callan-Symanzik equation (2.7), as one can infer from the dimensional analysis identity

$$\left(\mu\frac{\partial}{\partial\mu} + \mathcal{W}_D\right)\Gamma = 0 \ , \tag{4.13}$$

$\mu$ being the normalization mass.

We must note that, although the basis (4.12) has two elements, the trace anomaly consists effectively of the unique insertion given in the first of the equalities (4.11). This means that there is a relation between the cofficients $\beta$ and $\gamma$. This relation can be worked out at the lowest order by expressing the basis (4.12) in terms of $\Gamma^{\rm class}$. At zeroth order the right-hand side of the first of Eqs. (4.11) reads indeed

$$\int d^4x\ \mathcal{W}^{{\rm class}\ \alpha}\overline{\mathcal{W}}^{\rm class}_{\dot\alpha}\overline{\mathcal{W}}^{{\rm class}\ \dot\alpha}\mathcal{W}^{\rm class}_{\alpha}(A\bar A) = (4N - 12g\partial_g)\,\Gamma^{\rm class} \ . \tag{4.14}$$

One recovers in this way the well-known identity [8]

$$\beta = 3g\gamma + O(\hbar^2) . \tag{4.15}$$

The higher order corrections to (4.15) depend on the renormalization scheme.

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