

**Zeitschrift:** Helvetica Physica Acta

**Band:** 62 (1989)

**Heft:** 6-7

**Artikel:** Speeding-up simulations of interacting systems near critical points by the multigrid Monte Carlo method

**Autor:** Stoll, E.P.

**DOI:** <https://doi.org/10.5169/seals-116171>

### **Nutzungsbedingungen**

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften auf E-Periodica. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Das Veröffentlichen von Bildern in Print- und Online-Publikationen sowie auf Social Media-Kanälen oder Webseiten ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. [Mehr erfahren](#)

### **Conditions d'utilisation**

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. La reproduction d'images dans des publications imprimées ou en ligne ainsi que sur des canaux de médias sociaux ou des sites web n'est autorisée qu'avec l'accord préalable des détenteurs des droits. [En savoir plus](#)

### **Terms of use**

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. Publishing images in print and online publications, as well as on social media channels or websites, is only permitted with the prior consent of the rights holders. [Find out more](#)

**Download PDF:** 13.12.2025

**ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>**

SPEEDING-UP SIMULATIONS OF INTERACTING SYSTEMS NEAR CRITICAL POINTS  
BY THE MULTIGRID MONTE CARLO METHOD

E.P. Stoll, IBM Research Division, Zurich Research Laboratory, CH-8803 RÜSCHLIKON, Switzerland

Abstract: With multigrid coarse-to-fine transformations, loss of computer performance due to very slow long-wavelength relaxations can be eliminated. For systems with short-range interactions, effective interactions can however be modified according to renormalization-group considerations. With coarse-to-fine grid transformations alone, deviations indicate whether fine-to-coarse transformations are also necessary.

1. Introduction

Computer simulations have become very effective for many problems which cannot be treated by analytic methods. However, in complex systems, very slow relaxations at long wavelength restrict the application of vector and parallel computers. For the numerical solution of partial differential equations, the multigrid method [1] has demonstrated how it is possible to speed up slow long-wavelength relaxations by many orders of magnitude. A condition for the solution of such systems by this method is their selfsimilarity, i.e. the couplings between grid points remain unchanged in the mapping procedure from fine-to-coarse grids. For equilibrium systems with short-range interactions, renormalization-group theory [2] shows that effective interactions have to be recalculated at each fine-to-coarse mapping except at fixed points corresponding to critical points.

2. Multigrid theory

Multigrid procedures [1] usually start with fine grids, iterate, transform to coarser grids, iterate again and return to the fine grid. This algorithm can be extended to several levels. For a short-range interaction, the interaction has to be renormalized as demonstrated for the Ising model at  $T > T_c$  in [3]. The mapping from fine-to-coarse grids specifies the "melting" of blockspins on the fine grid. In our simulations, large blockspins with renormalized interactions were used at the outset and were allowed to melt only during successive iterations. The entropy was kept constant in each melting step.

3. Results

Figure 1 shows thermodynamic quantities for the two-dimensional Ising model normalized to exact values [4-6] computed with the multigrid Monte Carlo method. Deviations from

1 close to the critical point indicate that if the information of additional multigrid levels is used, blockspin melting at constant entropy is by itself unsatisfactory. Therefore, fine-to-coarse mappings are also needed, in which renormalized interactions can be estimated as demonstrated in [3].

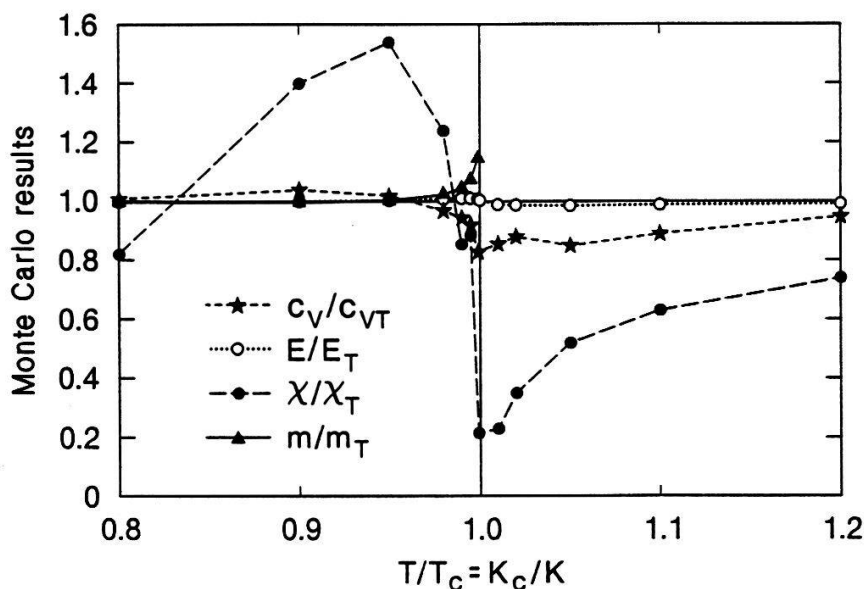


Figure 1: Plot of specific heat  $c_V$ , energy  $E$ , susceptibility  $\chi$  and order parameter  $m$  normalized to their exact theoretical values [4-6] for two-dimensional Ising models with  $1024 \times 1024$  and  $4096 \times 4096$  spins.

#### 4. Conclusions

Renormalization-group multigrid Monte Carlo calculations are very useful to improve the performance of numerically intensive calculations. Interactions have to be renormalized if the fine-to-coarse decimation step cannot be treated via a mapping to a percolation problem [7]. Fine-to-coarse mappings have to be combined with coarse-to-fine mappings for adequate blockspin melting.

#### 5. References

- [1] W.L. Briggs, *A Multigrid Tutorial* (Philadelphia: Society for Industrial and Applied Mathematics, 1987)
- [2] K.G. Wilson, *Rev. Mod. Phys.* **47**, 773 (1975)
- [3] H.H. Hahn and T.S.J. Streit, *Physica A* **154**, 108 (1988)
- [4] C.N. Yang, *Phys. Rev.* **85**, 808 (1952)
- [5] J.W. Essam and M.E. Fisher, *J. Chem. Phys.* **38**, 802 (1963)
- [6] M.F. Sykes, D.S. Gaunt, P.D. Roberts and J.A. Wyles, *J. Phys. A: Math. Gen.* **5**, 624 (1972)
- [7] D. Kandel, E. Domany, D. Ron, A. Brandt and E. Loh Jr., *Phys. Rev. Lett.* **60**, 1591 (1988)