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## Theoretical calculation of the resistivity of the High-Tc superconductors

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**Abstract :** Working within the Anderson lattice model, we calculate the first correction to the mean field theory and evaluate the self-energy  $\Sigma$  of the carriers. The resistivity is then given by Drude's formula  $\rho = m^*/ne^2\tau$  where  $\tau^{-1} = 2 \operatorname{Im} \Sigma$ .

### 1. Introduction

As shown by the observations, the interaction between the oxygen and copper atoms, within their two-dimensional layers, seems to play the leading role for a new microscopic mechanism leading to the pairing of the holes. We consider then the typical Anderson lattice model, used in heavy fermion systems [1], which can describe all electronic structure effects.

### 2. Description of the model and some ideas about the calculations

The Anderson Hamiltonian, in the slave boson representation, writes [2, 3] :

$$H = \sum_{k\sigma} \epsilon_k c_{k\sigma}^\dagger c_{k\sigma} + \sum_i E_0 b_i^\dagger b_i + \sum_{i\sigma} E_1 d_{i\sigma}^\dagger d_{i\sigma} + \sum_i E_2 a_i^\dagger a_i + \sum_{ik\sigma} V_k [b_i d_{i\sigma}^\dagger c_{k\sigma} e^{ikri} + a_i d_{i\sigma}^\dagger c_{k\sigma}^\dagger e^{-ikri} \operatorname{sgn}\sigma + h.c.] \quad (1)$$

Here the  $c_{k\sigma}$  are fermion operators for holes of spin  $\sigma$  in the o-band;  $E_1$ ,  $E_0$  and  $E_2$  are the energies of the  $d^9$ ,  $d^{10}$  and  $d^8$  Cu-states respectively represented by the localised fermion  $d_{i\sigma}$  and by the bosons  $b_i$  and  $a_i$  respectively. The matrix elements  $V_k = \langle \text{cod} | V | k\alpha \rangle$  define the hopping amplitude from an atomic d-state at the origin to an itinerant o-state in the band  $\alpha$ . Newns and Rasolt [2] worked out a mean field theory of this model. They could estimate the critical temperature and its dependence in the doping. Performing a  $1/N$  expansion, where  $N = 2\sigma+1$  is the spin degeneracy, we consider the first order correction and calculate the self-energy of the holes.

On a microscopic level, the resistivity is due to the scattering of the holes by many different processes but in our model, the most important one appears to be the scattering by the a-boson[4]. The main contribution to the self-energy writes :

$$\Sigma(p, \omega_n) = -V^2/\beta \sum_{i\omega_n} \sum_q D(q, i\omega_n) G(p+q, i\omega_n + i\omega_n) \quad (2)$$

where  $G$  is the quasi-hole propagator ( $G^{-1}(p, \omega) = \omega - \xi(p)$  where  $\xi(p) = p^2/2m^* - \mu$ )

and  $D$  is the a-boson propagator, given by

$$D^{-1}(q, i\omega_n) = -i\omega_n + U_{\text{eff}} - NV^2/\beta \sum_{i\omega_n} \sum_k [G_{dd}(q-k, i\omega_n - i\omega_n) G_{cc}(k, i\omega_n) + G_{cd}(q-k, i\omega_n - i\omega_n) G_{dc}(k, i)] \quad (3)$$

Performing the integration over the momentum and frequencies of the localised d- and itinerant c-electrons, we find [4], in the approximation  $\omega_n, v^* F q \ll 4\pi K T$  :

$$D^{-1}(q, i\omega_n) = A [B \ln(T/T_c) - i\omega_n + C q^2] \quad (4)$$

$$\text{where } A = \frac{s_o^2}{s_o^2 + \epsilon_d^2} \frac{NV^2 N_s p_o}{8\pi K T} \quad B = \frac{4\pi K T}{\pi^3} \quad C = \frac{7\zeta(3)}{\pi^2} \frac{v^* F^2}{4\pi K T}$$

The parameters  $\epsilon_d$  and  $s_0$  were calculated self-consistently in the mean field theory [2] while  $V \approx 2\text{eV}$  and  $\rho_0 \approx 0.25\text{eV}^{-1}$  are given by band structure calculations [2]. The effective Fermi velocity is  $v^*_F = m/m^* v_F \approx 0.2 v_F$  and  $N_s$  is the number of unit cells. Introducing (3) in (2) and performing the usual integrals over momentum and energy, one finds :

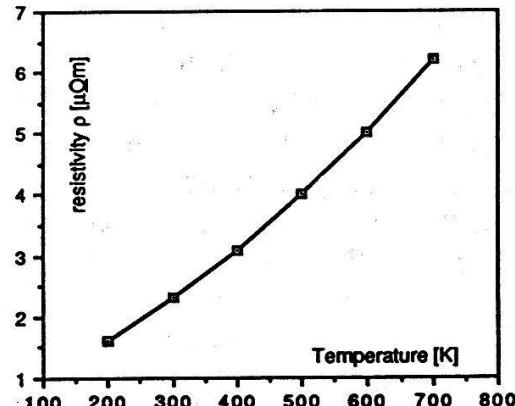
$$\rho = \frac{m^*}{ne^2 \tau_a} = \frac{m^*}{ne^2} \frac{2}{\hbar} \operatorname{Im} \Sigma = \frac{m^*}{\hbar ne^2} \frac{4V^2}{(hv^* F)^2 A} E(\bar{x})$$

$$\text{where } E(\bar{x}) = \int_{-\mu}^{\infty} dz \frac{e^z}{(1+e^z)^2} \int_0^{\bar{x}} dx \int_{-x-z}^{x-z} dy \frac{xy}{[(0.5 \ln T/T_c + x^2/4\pi)^2 + y^2]} \frac{[n_F(y+z) + n_B(y)]}{\sqrt{x^2 - (y+z)^2}}$$

with  $z = \omega / KT$ ,  $y = \omega' / KT$ ,  $x = v^* F q / KT$  and  $\bar{x} = v^* F \bar{q} / KT$

### 3. Results and discussion

In the above expression, there is one single free parameter  $\bar{x} = v^* F \bar{q} / KT$  which means that nothing in the theory fixes the value of the cut-off  $\bar{q}$  of the a-boson momentum. The temperature dependence of the resistivity is very much sensitive to the value of this parameter, but if we choose the value  $\bar{x} = 1$ , which satisfies all the approximations done in the calculations, the resistivity is almost linear in the temperature (see graph) as shown in the experiments [5]. Moreover, for  $n=3 \times 10^{27} \text{ m}^{-3}$  [5], its value is very close to the experimental one [5]  $\rho_{\text{exp}}(T=300) \approx 4 \mu\Omega\text{m}$ . We should of course understand the meaning of the relation  $\bar{q} = KT/v^* F$  if it has any and also analyse carefully Hall experiments to know more about the density  $n$ , taken here as a constant. Besides the results, the above calculation is interesting since it considers one of the many theories on the market of the High-Tc and we have to understand the normal state in order to know more about the superconducting state.



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