

**Zeitschrift:** Helvetica Physica Acta  
**Band:** 62 (1989)  
**Heft:** 6-7

**Artikel:** A mean-field description of the antiferromagnetism of CuO<sub>2</sub> layers  
**Autor:** Arrigoni, E. / Strinati, G. / Castellani, C.  
**DOI:** <https://doi.org/10.5169/seals-116063>

#### **Nutzungsbedingungen**

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften auf E-Periodica. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Das Veröffentlichen von Bildern in Print- und Online-Publikationen sowie auf Social Media-Kanälen oder Webseiten ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. [Mehr erfahren](#)

#### **Conditions d'utilisation**

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. La reproduction d'images dans des publications imprimées ou en ligne ainsi que sur des canaux de médias sociaux ou des sites web n'est autorisée qu'avec l'accord préalable des détenteurs des droits. [En savoir plus](#)

#### **Terms of use**

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. Publishing images in print and online publications, as well as on social media channels or websites, is only permitted with the prior consent of the rights holders. [Find out more](#)

**Download PDF:** 23.02.2026

**ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>**

A MEAN-FIELD DESCRIPTION OF THE ANTIFERROMAGNETISM  
OF CuO<sub>2</sub> LAYERS

E. Arrigoni and G. Strinati, Scuola Normale Superiore,  
I-56100 Pisa, Italy

C. Castellani, Dipartimento di Fisica, Università "La Sapienza",  
I-00185 Roma, Italy

**Abstract:** Antiferromagnetic (AF) ground state properties of CuO<sub>2</sub> layers are calculated at the mean-field level within a slave-boson approach to a multiband Hubbard model which includes a minimal number of Cu and O orbitals.

### 1. Introduction

Purpose of this work is to provide a description of the antiferromagnetism of CuO<sub>2</sub> layers via a mean field which is suited to describe strongly correlated systems with partly itinerant electrons.

Specifically we consider a two-dimensional square lattice of Cu atoms bridged by O atoms, and describe the electronic phenomena occurring within the plane by a multiband Hubbard model that includes the  $d_{x^2-y^2}$  orbital for Cu and the  $p_x$  and  $p_y$  orbitals for O:

$$\begin{aligned} H = & \sum_{ij} \sum_{\nu\sigma} t_1 \left( d_{i\sigma}^\dagger p_{j\nu\sigma} + H.c. \right) + \sum_{jj'} \sum_{\nu\nu'\sigma} t_2 \left( p_{j\nu\sigma}^\dagger p_{j'\nu'\sigma} + H.c. \right) \\ & + \varepsilon_d \sum_{i\sigma} d_{i\sigma}^\dagger d_{i\sigma} + \varepsilon_p \sum_{j\nu\sigma} p_{j\nu\sigma}^\dagger p_{j\nu\sigma} \\ & + U_d \sum_i d_{i\uparrow}^\dagger d_{i\uparrow} d_{i\downarrow}^\dagger d_{i\downarrow} + U_p \sum_{j\nu} p_{j\nu\uparrow}^\dagger p_{j\nu\uparrow} p_{j\nu\downarrow}^\dagger p_{j\nu\downarrow} \end{aligned} \quad (1)$$

Here, the lattice sums are limited to neighboring sites, the index  $\nu$  distinguishes the  $p$  orbitals,  $\sigma$  is the spin projection,  $\varepsilon_p - \varepsilon_d = \Delta$  is the separation between  $p$  and  $d$  hole energy levels, and  $U_d$  and  $U_p$  are the Coulomb repulsions at the Cu and O sites, respectively. The Cu-O hopping  $t_1$  and the O-O hopping  $t_2$  can be obtained by a tight-binding fit to a conventional band structure. For La<sub>2</sub>CuO<sub>4</sub> one gets  $t_1 \cong 1.4$  eV

and  $|t_2/t_1| \cong 1/4 \div 1/2$ . The value of  $\Delta$  is rather controversial: reported values for  $\text{La}_2\text{CuO}_4$  differ by an order of magnitude ranging from .5 eV to 5 eV. We shall take  $\Delta$  as a free parameter in the following. Reasonable estimates for  $U_d$  and  $U_p$  are 10 eV and 2 eV, respectively.

These estimates suggest that a conventional Hartree-Fock decoupling, which replaces the two-body interaction term in (1) by a one-body operator upon factorization of its average value, is reliable only for the O sites where  $U_p/t_1 \cong 1$ . A different approach is, however, required for the Cu site since  $U_d/t_1 \gg 1$ . We shall limit ourselves to consider a mean-field solution in the following.

Before describing the approach, it is worthed to comment on its limitations and the way to overcome them. Since we consider a strictly two-dimensional system, our results will be meaningful at zero temperature although the mean-field approximation provides an AF solution even at finite temperature. We should mention in this context that there have appeared recently several arguments in favor of the existence of AF order in two dimensions for spin 1/2 at zero temperature. In addition, quantum fluctuations are also neglected by our mean-field solution that assumes a Néel state. To overcome the limitations of our two-dimensional mean-field approach one could (i) include the inter-layers coupling to obtain an AF temperature which compares favorably with the experimental value even at the mean-field level, and (ii) include the fluctuations about the mean field by relying on its definition as the saddle point of a functional.

## 2. The method

The method we consider extends to a multiband Hubbard model the treatment by Kotliar and Ruckenstein for a one-band Hubbard model [1]. According to the method, slave bosons which specify empty ( $e_i$ ) and singly occupied ( $s_{i\sigma}$ ) states are assigned to each Cu site  $i$  as additional degrees of freedom to take into account the local environment. [The doubly occupied states need not be considered in the limit  $U_d \approx \infty$  we shall assume]. To avoid multiple bosonic occupation at a given Cu site, the physical subspace is selected by the requirement that the operators

$$Q_i^I = e_i^\dagger e_i + \sum_{\sigma} s_{i\sigma}^\dagger s_{i\sigma} \quad (2a)$$

$$Q_{i\sigma}^{II} = d_{i\sigma}^\dagger d_{i\sigma} - s_{i\sigma}^\dagger s_{i\sigma} \quad (2b)$$

have eigenvalue 1 and 0, respectively. A convenient way to enforce

this requirement is to resort to a functional integral approach for the coupled Bose-Fermi system, where one introduces Lagrange multiplier fields to project onto the physical subspace [1]. The approach provides in turn a mean-field description of the antiferromagnetism of the CuO<sub>2</sub> layers by selecting the saddle-point for the bosonic part of the functional.

The introduction of slave bosons at Cu sites requires one to map the original operator  $d_{i\sigma}$  of (1) onto the product  $d_{i\sigma}z_{i\sigma}$  where  $z_{i\sigma}$  is an appropriate bosonic operator. At the saddle point the introduction of slave bosons results thus into a renormalization of the hopping integral  $t_1$  owing to correlations. This procedure has to be contrasted with the Hartree-Fock decoupling that we have adopted for the O sites, which affects the interaction terms but leaves the hopping  $t_2$  unchanged. The Hamiltonian (1) is then replaced by

$$H = \sum_{ij} \sum_{\nu\sigma} t_1 (z_{i\sigma}^\dagger d_{i\sigma}^\dagger p_{j\nu\sigma} + H.c.) + \sum_{jj'} \sum_{\nu\nu'\sigma} t_2 (p_{j\nu\sigma}^\dagger p_{j'\nu'\sigma} + H.c.) + \epsilon_d \sum_{i\sigma} d_{i\sigma}^\dagger d_{i\sigma} + \epsilon_p \sum_{j\nu\sigma} p_{j\nu\sigma}^\dagger p_{j\nu\sigma} + U_p \sum_{j\nu\sigma} [n_0 p_{j\nu\sigma}^\dagger p_{j\nu\sigma} - \frac{1}{2} n_0^2] \quad (3)$$

with the condition  $n_0 = \langle p_{j\nu\sigma}^\dagger p_{j\nu\sigma} \rangle$  at the O sites.

The identification of the bosonic operator  $z_{i\sigma}$  in terms of  $e_i$  and  $s_{i\sigma}$  is not unique within the physical subspace. Although the choice of  $z_{i\sigma}$  would not be an issue for an exact calculation, attention must be paid to this choice when dealing with approximations. In particular, the arbitrariness of  $z_{i\sigma}$  must be resolved within the mean-field approximation so as to reproduce known results and to avoid spurious outcomes. It turns out from our calculation that the choice [1]

$$z_{i\sigma} = e_i^\dagger (1 - s_{i\sigma}^\dagger s_{i\sigma})^{-1/2} (1 - e_i^\dagger e_i - s_{i\bar{\sigma}}^\dagger s_{i\bar{\sigma}})^{-1/2} s_{i\sigma} \quad (4)$$

reproduces the AF energy of a Néel state with one hole per Cu site, namely,

$$E_{AF} = N(-4 t_1^2 / \Delta + 16 t_1^4 / \Delta^3 + \dots) \quad (5)$$

( $N$  being the number of Cu sites) in the limit of large  $\Delta$  (i.e., for  $\Delta/t_1 \gtrsim 10$ ), when comparison with perturbation theory in  $t_1/\Delta$  is meaningful [2]. In addition, the slave-boson method with the choice (4) develops a molecular-like AF state for increasing  $\Delta$  by suitably renormalizing the hopping for the two spin projections. Specifically, for

an atom of the 'spin up' sublattice  $z_\uparrow \sim 1$  and  $z_\downarrow \sim 0$  for increasing  $\Delta$ , so that a hole with spin up tends to be confined near the corresponding site. For these reasons, we adopt confidently the choice (4) for all values of  $\Delta$ .

### 3. Results

A similar calculation for the paramagnetic phase shows that the hopping renormalization parameter  $z$  vanishes when  $\Delta \geq \Delta_c$  with  $\Delta_c \cong 4\sqrt{3}t_1$ . This indicates that the system is undergoing a transition a la Gutzwiller to a Mott insulating state [1]. Although this result represents a nice qualitative feature of the slave-boson mean-field approach, which cannot be obtained within the Hartree-Fock decoupling, it is associated to the vanishing of the ground state energy for  $\Delta \geq \Delta_c$ , because the Gutzwiller approximation inhibits the electronic fluctuations in the localized phase. A direct numerical comparison of the ground state energies per Cu site in the two phases, to extract an effective exchange integral  $J_{eff} \equiv 2(E_P - E_{AF})/N$ , results then in too large values for  $J_{eff}$  when  $\Delta \gtrsim \Delta_c$ .

A better estimate for  $J_{eff}$  is obtained by interpolating the paramagnetic energy between its asymptotic expression, namely,

$$E_P = N(-4 t_1^2/\Delta + 18 t_1^4/\Delta^3 + \dots) \quad (6)$$

valid for  $\Delta \gtrsim \Delta_c$  when perturbation theory in  $t_1/\Delta$  can be applied [2], and our numerical results for smaller  $\Delta$ . The resulting curve  $J_{eff}(\Delta)$  has by construction the correct asymptotic form  $4 t_1^4/\Delta^3$  and presents a maximum at intermediate values  $\Delta/t_1 \approx 3$ , remaining positive down to vanishing  $\Delta$ . Comparison with the experimental value  $J_{exp} \cong .14$  eV for  $\text{La}_2\text{CuO}_4$  [3] shows that two values,  $\Delta/t_1 \cong .5$  and  $\Delta/t_1 \cong 4.5$ , are consistent with  $J_{exp}$  when  $|t_2/t_1| = 1/4$ . It is interesting to notice that these two values lie within the interval of the reported values for  $\Delta$ : they correspond possibly to an itinerant and to a localized description of magnetism, respectively. One may also suggest in this context that a direct experimental check on which side of the maximum of the curve  $J_{eff}(\Delta)$  the  $\text{CuO}_2$  layers of  $\text{La}_2\text{CuO}_4$  actually lie, could be obtained by measuring the Néel temperature upon applying uniaxial pressure to the layers so as to decrease the ratio  $\Delta/t_1$ .

Diagonalization of the Hamiltonian matrix (3) (with the addition of terms corresponding to the constraints (2)) results in an effective

one-particle picture whereby a system of independent fermions interacts with a set of classical fields. The associated band structure shows a gap  $\Delta_{AF}$  at half filling which separates the lowest filled band from the higher bands. In Fig.1

we report  $\Delta_{AF}$  vs.  $\Delta$  when  $t_2 = 0$ . The AF gap is found to be  $\approx \Delta$  (the charge excitation gap) for large  $\Delta$  and to remain finite for vanishing  $\Delta$  owing to the strong nesting condition in two dimensions at half filling which stabilizes the antiferromagnetism against charge fluctuations. In Fig.1 we also report the width  $W$  of the lowest (filled) band which is seen to decrease rapidly for increasing  $\Delta$  .

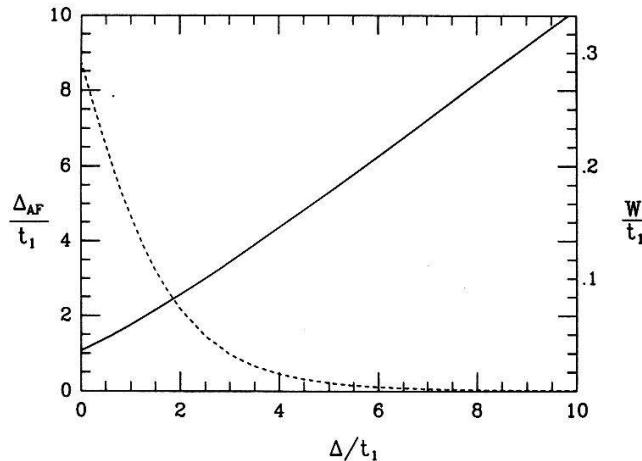


FIGURE 1  
AF gap  $\Delta_{AF}$  (full line) and width  $W$  of the lowest band (broken line) for  $t_2 = 0$

#### 4. Conclusions

We have calculated several AF ground state properties of  $\text{CuO}_2$  layers within a mean-field approach which is specifically conceived to deal with the strong correlation at Cu sites. Future work along these lines may concern the inclusion of fluctuations about the AF saddle point to obtain the spin-wave spectrum and the associated reduction factor of the Cu magnetic moment, as well as the calculation of a possible superconducting pairing interaction starting from the AF mean field.

#### 5. References

- [1] G. Kotliar and A.E. Ruckenstein, Phys. Rev. Lett. 57, 1362 (1986) .
- [2] J. Zaanen and A.M. Olés, Phys. Rev. B37, 9423 (1988) .
- [3] K.B. Lyons et al., Phys. Rev. B37, 2353 (1988) .