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Time interval statistics of a symmetric two-peaked spectrum

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Abstract. A complete and an exact analysis of the time interval statistics of *pure* Brillouin spectrum (a symmetric two-peaked spectrum) is provided here, by utilizing the *unique* analytic-form of the generating-function recently given by the present author [1]. Unfortunately, the earlier approximate results due to Blake and Barakat [3] are found to violate even the well known equation (28) for the time-interval-distribution for the conditional photocount.

Introduction

One of the long standing problems in the field of photon-counting-statistics (PCS) of chaotic light is the exact analysis of the time-interval-statistics (TIS) of the *pure* Brillouin spectrum (symmetric two-peaked spectrum). This had been mainly due to the non-availability of the required *exact* one-fold generating-function (g.f.) characterising this multiple-peaked spectrum. However, this task has recently been accomplished by the present author in Ref. 1, wherein the usage of the Hadamard's Theorem for the infinite-products [2] coupled with the following boundary conditions, help us in obtaining the unique analytic form of the g.f. for the symmetric two-peaked spectrum:

$$0 \leq Q(s, T) \leq 1 \quad (1)$$

$$\left\{ Q(s, T) = \sum_{n=0}^{\infty} (1-s)^n P(n, T) \right\} \Big|_{s=0} = \sum_{n=0}^{\infty} P(n, T) = 1 \quad (2)$$

$$\{(-1) \partial Q(s, T) / \partial s\} \Big|_{s=0} = \langle n \rangle \quad (3)$$

The central idea behind such a formalism being that if any useful spectral information is to be obtained via the one-fold photon counting analysis like the TIS here, the relevant g.f., *first* ought to satisfy the boundary conditions stated in equations (1)–(3).

We show in this paper (Section 3) how the mathematical rigor of [1] now helps us in capturing the exact details of the various Time-Interval-Distributions (TIDs). Unfortunately, the earlier analysis due to Blake and Barakat [3] based on

the numerical scheme suggested in Ref. 4, has completely failed on this account as far as the TID of registering a conditional photo-count is concerned (see Fig. 5).

The Time-Interval (TI) analysis offers us a simple and direct way of studying the spectral features of an optical beam, as the experimental set-up here, requires a combination of Time-to-Amplitude-Converter (TAC) and Pulse-Height-Analyser (PHA). Sensitive TI measurements with the resolution $\sim 10^{-6}$ – 10^{-7} s have been reported in literature (see Refs. 5, 6 for example). Reference is also made to 'Time-of Arrival-Correlator' recently constructed by Dhadwal et al. [7].

For a theoretical study of the TIDs, we require the g.f. characterizing the spectrum under study, as shown by Glauber in Ref. 8. A simple analysis of the TIDs is then offered via the one-fold g.f. where the underlying assumption is [8],

$$\int_t^{t+\delta t} I(t') dt' \sim I(t) \delta t \quad (4)$$

It must be mentioned here that for a rigorous analysis of the TIDs (where the field strength is arbitrary), we require a third-order g.f. as shown by Barakat and Blake in their review article [9]. This task has been accomplished for the case of Gaussian–Lorentzian (G–L) light in [10], by employing the exact third-order g.f. given in Ref. 11.

2. The generating-functions

The desired one-fold analysis of the present symmetric two-peaked spectrum is facilitated by solving the following Fredholm integral equation of the second-kind:

$$s \int_0^T g |t - t'| \phi_k(t') dt' = \lambda_k \phi_k(t) \quad (5)$$

where $g |t - t'|$ is the field-correlation of the spectrum under study and ϕ_k s and λ_k s are the eigen-functions and eigen-values respectively. The field-correlation $g(\tau)$ for this symmetric two-peaked spectrum is given by [3]:

$$\begin{aligned} g(\tau) &= \alpha e^{-\delta |\tau|} \cos \Delta \tau \\ &= \frac{\alpha}{2} (e^{-\beta |\tau|} + e^{-\beta^* |\tau|}) \quad (\alpha = 1 \text{ here}) \end{aligned} \quad (6)$$

where ' δ ' represents the half-width of the Brillouin lines, ' Δ ' the frequency-shift and β , $\beta^* = \delta - i\Delta$, $\delta + i\Delta$. As fair estimates of the line-width ' δ ' of the Brillouin spectrum are available in literature (see Mountain [12]), the quantity of main interest in such a study is the frequency-shift ' Δ ' – for it is directly related to the speed of sound in a fluid or the condensed-matter [12]. Multipass Fabry–Pérot interferometer with contrast of the order of 10^{12} have been found extremely

useful to study these Brillouin frequencies [13]. Interesting details of the various experimental techniques employed to study the Brillouin spectrum are provided in the review article of Borovick-Romanov and Kreines [14] and the recent papers [15, 16]

It is well known that the one-fold g.f. for the Gaussian light can be expressed as follows:

$$Q(s) = \prod_k (1 + s\lambda_k \langle I \rangle)^{-1} \quad (7)$$

where $\langle I \rangle$ represents the mean count rate and $\lambda_k s$ are the eigen-values of equation (5). Blake and Barakat [3] had obtained the following expression for the one-fold g.f. for this two-peaked spectrum by utilizing the numerical-scheme suggested in [4]:

$$Q(s, T) = \exp \sum_{r=1}^{\infty} \frac{[(-\langle I \rangle s)^r]}{r} I_r(T) \quad (8)$$

where

$$I_r(t) = \int_0^T g_r(t', t') dt' \quad (9)$$

and $g_r(t', t')$ is the r th iterated kernel of the integral equation (5), defined recursively through the following relations:

$$g_1(t', t'') \equiv g(t', t'')$$

and

$$g_r(t', t'') = \int_0^T g(t', t) g_{r-1}(t, t'') dt \quad (r \geq 2) \quad (10)$$

Obviously, no functional form for the g.f. $Q(s, T)$ is possible in this formalism, unlike the g.f. given in [1], where we can clearly see its exact relation with the parameters ' δ and Δ ' characterising this two-peaked spectrum under study.

We had obtained the *unique* analytic form of the g.f. for this spectrum [1] by first rendering the Fredholm-determinant (obtained on solving equation (5)) analytic in nature to obtain an entire-function $P(\xi)$ ($\xi = \lambda^{-1}$) and then by applying Hadamard's Theorem [2], we could compare the zeros of $P(\xi)$ with those of $Q(s, T)$ in equation (7), so as to finally arrive at the following important relation:

$$Q(s, T) = P(0)/P(-s\langle I \rangle) \quad (11)$$

where $P(0)$ is a constant given by:

$$P(0) = \beta^* \beta (\beta^{*2} - \beta^2)^2 \quad (12)$$

and,

$$\begin{aligned}
 P(-s\langle I \rangle) = & e^{-(\beta^* + \beta)T} \{ \beta\beta^* [Z^{*2}(p_1)Z^{*2}(p_2)F(p_1)F^*(p_2) \\
 & + Z^2(p_1)Z^{*2}(p_2)F^*(p_1)F(p_2)] \\
 & - 2[\delta^2 G(p_1)G(p_2) + \Delta^2]Z(p_1)Z^*(p_1)Z(p_2)Z^*(p_2) \} (p_1^2 - p_2^2)^{-2}
 \end{aligned} \tag{13}$$

where

$$\begin{aligned}
 Z(p) &= (\beta^2 - p^2), & Z^*(p) &= (\beta^{*2} - p^2), \\
 F(p) &= [\cosh pT + \tfrac{1}{2}(\beta/p + p/\beta) \sinh pT], \\
 F^*(p) &= [\cosh pT + \tfrac{1}{2}(\beta^*/p + p/\beta^*) \sinh pT], \\
 G(p) &= \left[\cosh pT + \left(\frac{\beta^*\beta + p^2}{(\beta^* + \beta)p} \right) \sinh pT \right]
 \end{aligned} \tag{14}$$

and the values of 'p' are determined from the following equation:

$$p = \pm \sqrt{A \pm \sqrt{A^2 - 4b}} / \sqrt{2}$$

where

$$A = [(\beta^* + \beta) + s\langle I \rangle(\beta^* + \beta)],$$

and

$$B = \beta\beta^*[\beta^*\beta + s\langle I \rangle(\beta^* + \beta)] \tag{15}$$

The probability of zero-counts defined below gives us a measure of this g.f.:

$$P(0, T) = Q(s, T)|_{s=1} \tag{16}$$

3. Time interval statistics

3.1. Definitions and general features

Several important and interesting details of the TIS are given in the review article of Barakat and Blake [9]. We briefly recapitulate the following important details required for discussing the TIS of the chaotic light having *any spectral shape*.

Recollecting that the g.f. can be written as,

$$Q(s, T) = \langle \exp [-sE(T)] \rangle \tag{17}$$

where

$$E(T) = \int_t^{t+T} I(t') dt' \tag{18}$$

and $I(t)$ is the instantaneous intensity of the scattered-field, the TIDs of registering

the first photocount, $V(T)$, and the conditional photocount, $P(T)$, are respectively given by:

$$V(T) = \langle I(T) \exp[-sE(T)] \rangle, \quad (19)$$

$$P(T) = \langle I(0)I(T) \exp[-sE(T)] \rangle, \quad (20)$$

if we take $t=0$ in equation (18). From equation (17), it is easy to see the following relations of these TIDs with the one-fold g.f.,

$$V(T) = -\partial Q(s, T)/\partial T|_{s=1} \quad (21)$$

and

$$P(T) = \frac{1}{\langle I \rangle} [\partial^2 Q(s, T)/\partial T^2]|_{s=1} \quad (22)$$

For a completely polarized light, Barakat and Blake [9] have derived the following interesting expressions for $V(T)$ and $P(T)$, when the interval times are of short duration but the mean count rate $\langle I \rangle$ can be anything,

$$V(T) = \langle I \rangle / (1 + \frac{1}{2} \langle I \rangle T)^2 \quad (23)$$

and

$$P(T) = 2\langle I \rangle / (1 + \langle I \rangle T)^3 \quad (24)$$

However, the above expressions for $V(T)$ and $P(T)$ are obviously insensitive to the spectral-shape of the polarized chaotic light.

For $T=0$, we have from equation (23),

$$V(0) = \langle I \rangle \quad (25)$$

and from equation (24), we get,

$$P(0) = 2\langle I \rangle \quad [\text{or } 2V(0)] \quad (26)$$

Thus, at the origin, the conditional TID is always twice the value of the TID for registering the first photo-count, *irrespective of the spectral shape of the chaotic light.*

On the other hand, when $\langle I \rangle T \gg 1$, we have,

$$P(T)/V(T) \approx (2\langle I \rangle T)^{-1} \quad (27)$$

which implies that $P(T)$ decays-off faster than $V(T)$ for large count rates and the crossing point is at about $T = 0.088$ for $\langle I \rangle = 5.0$ as can be easily seen from eqs. (23)–(24). It is also instructive to note that for $\langle I \rangle T \ll 1$, the conditional TID, $P(T)$, behaves like the correlation function $C(T)$ (bunching of photons in a chaotic light!):

$$P(T) = \langle I \rangle [1 + g^2(T)] = \langle I \rangle C(T) \quad (28)$$

This important fact has recently been used by Dhadwal et al. [7], while constructing the ‘Time-of-Arrival-Correlator’ – operative in the μs regime.

At $T=0$, equation (28) also gives $P(0) = 2\langle I \rangle$ as $g(0) = 1$.

It is interesting to observe that whereas Barakat and Blake in [9] confirm equation (25) (Fig. 6.3) and equation (26) (Fig. 6.4) for the G–L light, the same authors report values of $P(T)$ for the symmetric two-peaked spectrum in [3] (or see Figs. 6.7 and 6.8 in ref. [9]), which neither follow equation (26) (for any $\langle I \rangle$) nor the well-known equation (28) for $\langle I \rangle T \ll 1$ (see Fig. 5).

Thus, to give a direct feeling of how the exact TIDs behave for the present case, we now present their explicit functional form (Section 3.2). The direction dependence of these TIDs on the parameters ' δ and Δ ' which characterize this spectrum, comes through the g.f. given in equations (11)–(15).

3.2. TIDs for the symmetric two-peaked spectrum

Rewriting the g.f. as,

$$Q(s, T) = A(0)e^{(\beta^* + \beta)T}/A(p_1, p_2, T) \quad (29)$$

where

$$A(0) = P(0) = \beta\beta^*(\beta^{*2} - \beta^2)^2 \quad (30)$$

and

$$\begin{aligned} A(p_1, p_2, T) = & \{\beta\beta^*[Z^{*2}(p_1)Z^2(p_2)F(p_1)F^*(p_2) \\ & + Z^{*2}(p_2)Z^2(p_1)F^*(p_1)F(p_2)] - 2[\delta^2 G(p_1)G(p_2) + \Delta^2] \\ & \times Z(p_1)Z^*(p_1)Z(p_2)Z^*(p_2)\}(p_1^2 - p_2^2)^{-2} \end{aligned} \quad (31)$$

We first determine from equations (29), (30) and (21), the following functional form of the TID for the first photo-count, $V(T)$:

$$\begin{aligned} V &= -\partial Q(s, T)/\partial T|_{s=1} \\ &= P(0, T) \left[A^{-1}(p_1, p_2, T) \frac{\partial A}{\partial T}(p_1, p_2, T) \right] \Big|_{s=1} \end{aligned} \quad (32)$$

where $P(0, T)$ is the probability of zero-counts (equation (16)), and

$$\begin{aligned} \frac{\partial A}{\partial T}(p_1, p_2, T) = & \{\beta\beta^*[Z^2(p_1)Z^2(p_2)(p_1\dot{F}(p_1)F^*(p_2) \\ & + p_2F(p_1)\dot{F}^*(p_2)) + Z^{*2}(p_2)Z^2(p_1) \\ & \times (p_1\dot{F}^*(p_1)F(p_2) + p_2F^*(p_1)\dot{F}(p_2))] \\ & - 2[\delta^2(p_1\dot{G}(p_1)G(p_2) + p_2(G(p_1)\dot{G}(p_2)) \\ & \times Z(p_1)Z^*(p_1)Z(p_2)Z^*(p_2)\}(p_1^2 - p_2^2)^{-2} \end{aligned} \quad (33)$$

and

$$\begin{aligned} \dot{F}(p) &= [\sinh pT + \tfrac{1}{2}(\beta/p + p/\beta) \cosh pT], \\ \dot{F}^*(p) &= [\sinh pT + \tfrac{1}{2}(\beta^*/p + p/\beta^*) \cosh pT], \\ \dot{G}(p) &= \left[\sinh pT + \frac{(\beta^*\beta + p^2)}{(\beta^* + \beta)p} \cosh pT \right] \end{aligned} \quad (34)$$

Similarly, from equations (30)–(33) and (22), we have the following explicit expression for $P(T)$:

$$P(T) = \frac{1}{\langle I \rangle} \left\{ \frac{2V^2(T)}{P(0, T)} - P(0, T) \left[A^{-1}(p_1, p_2, T) \frac{\partial^2 A}{\partial T^2}(p_1, p_2, T) \right] \right\} \Big|_{s=1} \quad (35)$$

where

$$\frac{\partial^2 A}{\partial T^2}(p_1, p_2, T) = \{(p_1^2 + p_2^2)A(p_1, p_2, T) + 2p_1 p_2 A(p_1, p_2, T)\} \quad (36)$$

with

$$\begin{aligned} A(p_1, p_2, T) = & \{\beta\beta^*[Z^{*2}(p_1)Z^2(p_2)\dot{F}(p_1)\dot{F}^*(p_2) \\ & + Z^{*2}(p_2)Z^2(p_1)\dot{F}^*(p_1)\dot{F}(p_2)] \\ & - 2[\delta^2\dot{G}(p_1)\dot{G}(p_2) + \Delta^2]Z(p_1)Z^*(p_1)Z(p_2)Z^*(p_2)\} \\ & \times (p_1^2 - p_2^2)^{-2} \end{aligned} \quad (37)$$

From above, it is interesting to observe that the TID for the first count, $V(T)$, is related to the probability of zero counts, $P(0, T)$ (equation 32) and the TID for the conditional photo-count, $P(T)$, is related to both the probability of zero counts $P(0, T)$ as well as the TID for the first photo-count $V(T)$ equation 35).

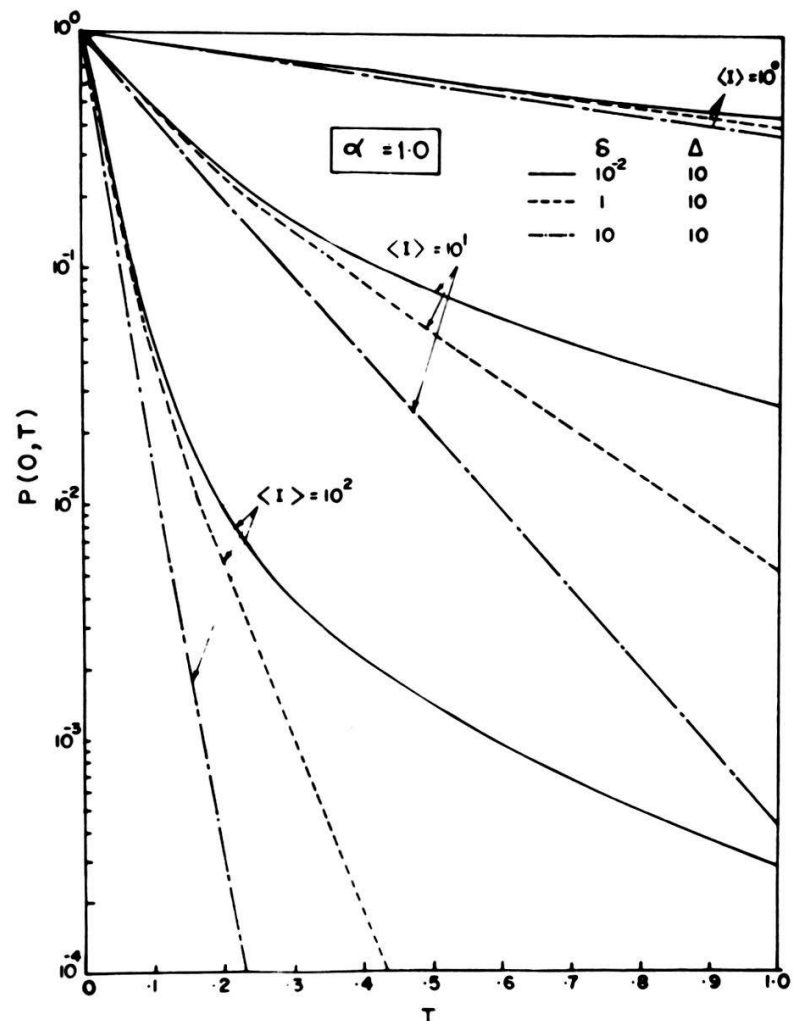


Figure 1
The probability of zero counts.

3.3. Results and discussion

It is natural to expect that the probability of zero counts $P(0, T)$ at $T = 0$ should be 1 and as T grows, it should fall-off. This is depicted in Fig. 1 and we also see that as $\langle I \rangle$ increases, $P(0, T)$ decreases. In other words, the probability of registering a photocount gets enhanced at a high count rate.

We see in Fig. 2, the behaviour of the TID for the first photo-count $V(T)$. At $T = 0$, we see that $V(0) = \langle I \rangle$ for all the values of $\langle I \rangle$, as it ought to be the case (equation (25)). Also we see that for low count rates ($\langle I \rangle = 0.5, 1.0$), $V(T)$ decays-off slowly whereas for relatively high count rates ($\langle I \rangle = 5.0, 10.0$) the decay is more pronounced (equation (23)). Unlike the Fig. 1, for $P(0, T)$, we now see the emergence of 'kinks' in this TID. At this juncture, we must mention that the results reported for $V(T)$ for a two-peaked spectrum characterizing the polydisperse medium [17], are in clear distinction with the present case of $V(T)$ for the *pure* Brillouin spectrum, as there were no 'kinks' in it. However, the other feature like $V(0) = \langle I \rangle$ and $V(T)$ decaying faster at higher count rates were also true for the two-peaked spectrum for the polydispersity case [17] (see Fig. 1).

Figure 3 shows the behaviour of the TID for the conditional photocount $P(T)$. At $T = 0$, we see that $P(0) = 2\langle I \rangle$, as suggested by equation (26) and/or equation (28). We now see that the mere 'kinks' of $V(T)$ have now turned into

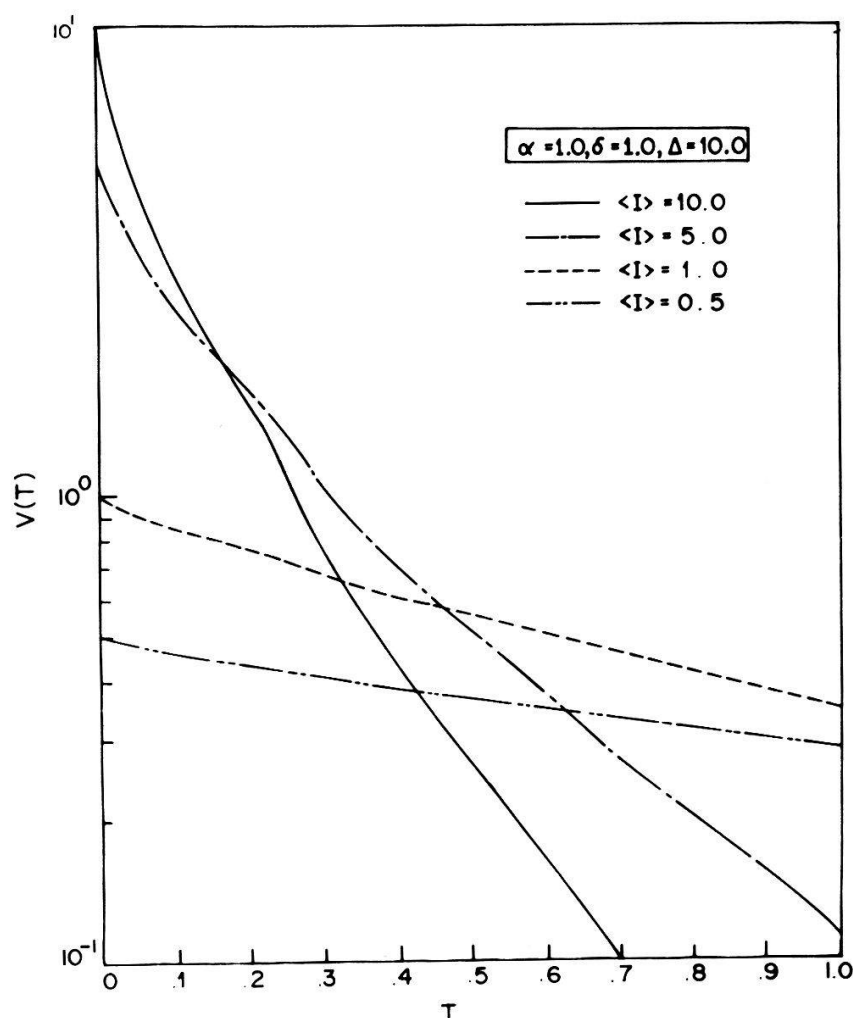


Figure 2
The time-interval-distribution
for the first photocount.

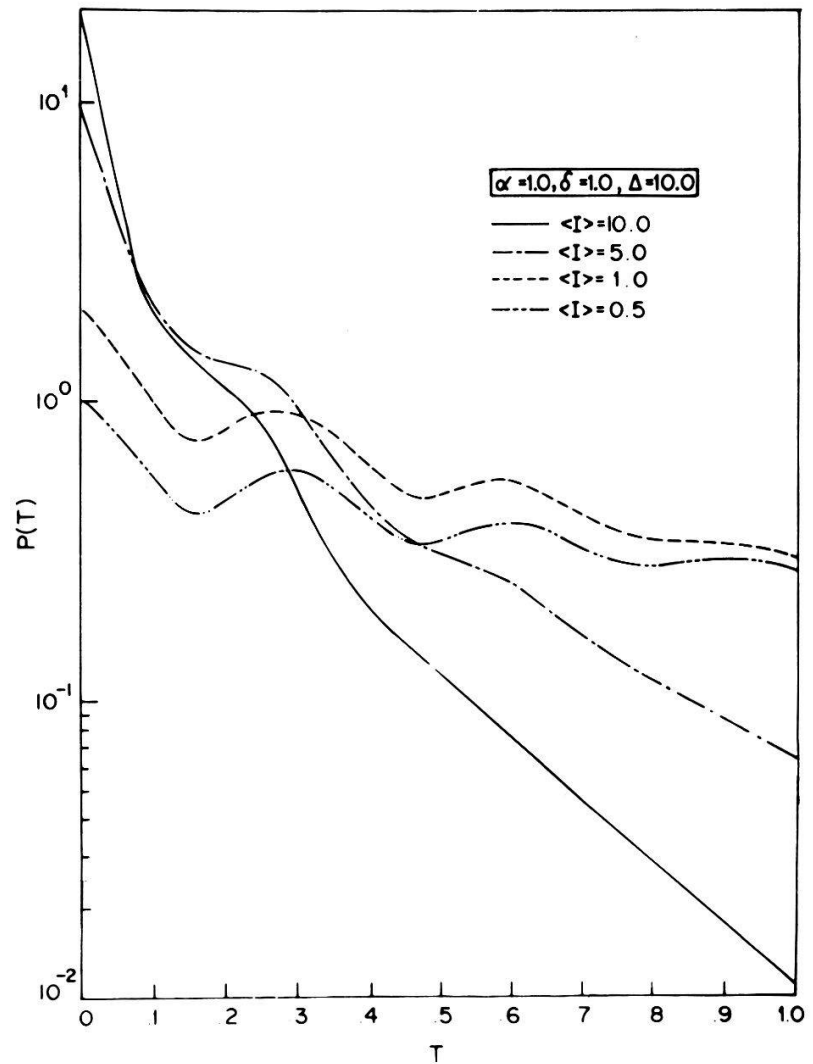


Figure 3
The time-interval-distribution of registering a conditional photo-count at time T .

pronounced 'oscillations' for $P(T)$ for the same $\langle I \rangle$ s and T s. This is to be expected only, as $P(T)$ gives a more sensitive account of the TIS, for its direct dependence on the field-correlation $g(\tau)$ – as evident from the simple equation (28). The general oscillatory behaviour of the TIDs for the present symmetric two-peaked spectrum is due to the 'cos ΔT ' factor in $g(T)$ in equation (6). At low count rates, the heterodyning between the Brillouin components produces fluctuations in the intensity on the time scale of the order of Δ^{-1} , and we clearly see the oscillations. But as the mean count rate $\langle I \rangle$ goes higher, these intensity fluctuations get arrested and as a result, the oscillations due to heterodyning also vanish, as made clear in Figs. (2) and (3).

It is instructive to compare the behaviour of the two TIDs, $V(T)$ and $\dot{P}(T)$, for the same count rates and interval times. This we do in Fig. 4. It is interesting to notice that whereas $P(T)$ oscillates over the $V(T)$ values for a low count rate ($\langle I \rangle = 0.1$), $P(T)$ decays-off faster as compared to $V(T)$ for relatively higher count rate ($\langle I \rangle = 5.0$) (equation (24) or (27)) with $T = 0.08$ as the point of departure-true to the suggestion of equation (27)!

In Fig. 5, we compare the approximate values of $P(T)$ as given by Blake and Barakat in [3], against the exact ones obtained here. First, at the outset, we

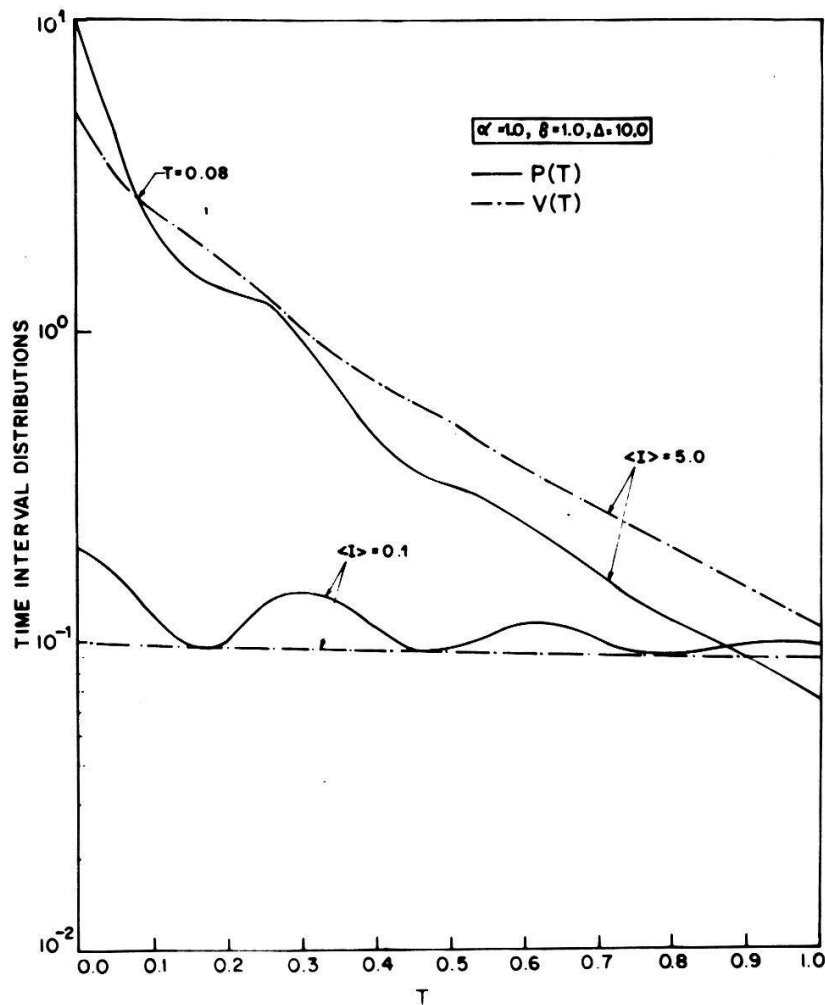


Figure 4

A comparative study of the time-interval-distribution $V(T)$ and $P(T)$, as a function of count rates. For $\langle I \rangle = 5.0$ (moderate count rate $P(T)$ decays off faster than $V(T)$ after $T = 0.08$ but $P(T)$ oscillates above $V(T)$ values throughout, when $\langle I \rangle = 0.1$ (low count rate).

notice that for both the count rates $\langle I \rangle = 0.1$ and $\langle I \rangle = 1.0$, the $P(T)$ values of Blake and Barakat [3] violate $P(0) = 2\langle I \rangle$, as suggested by equations (26) and/or equation (28). For $\langle I \rangle = 0.1$ at $T = 0$, equation (28) suggests $P(0) = 0.2$ but the reported value due to Blake and Barakat [3] is 1.0, thereby suggesting an error of $\sim 400\%$ for $\langle I \rangle = 0.1$. For $\langle I \rangle = 1.0$, equation (28) gives $P(0) = 2$ at $T = 0$, but Blake and Barakat [3] gives this value as 1.25 which now suggests an error of $\sim 38\%$ for $\langle I \rangle = 1.0$. However, the exact and overall estimate of the error in the reported values for $P(T)$ by Blake and Barakat [3], can be determined only from our exact analysis (for arbitrary $\langle I \rangle$ and T) (Fig. 5), as equation (28) is constrained by the condition $\langle I \rangle T \ll 1$. From Fig. 5, it is evident that as T grows the error also grows for $\langle I \rangle = 1.0$, but the reverse trend is observed for $\langle I \rangle = 0.1$ and an average error of $\sim 77\%$ for $\langle I \rangle = 1.0$ and $\sim 209\%$ for $\langle I \rangle = 0.1$ is found in the numerical-scheme due to Blake and Barakat [3]. However, interestingly the qualitative features are found to be in some agreement with ours, as can be seen from the oscillations in Fig. 5.

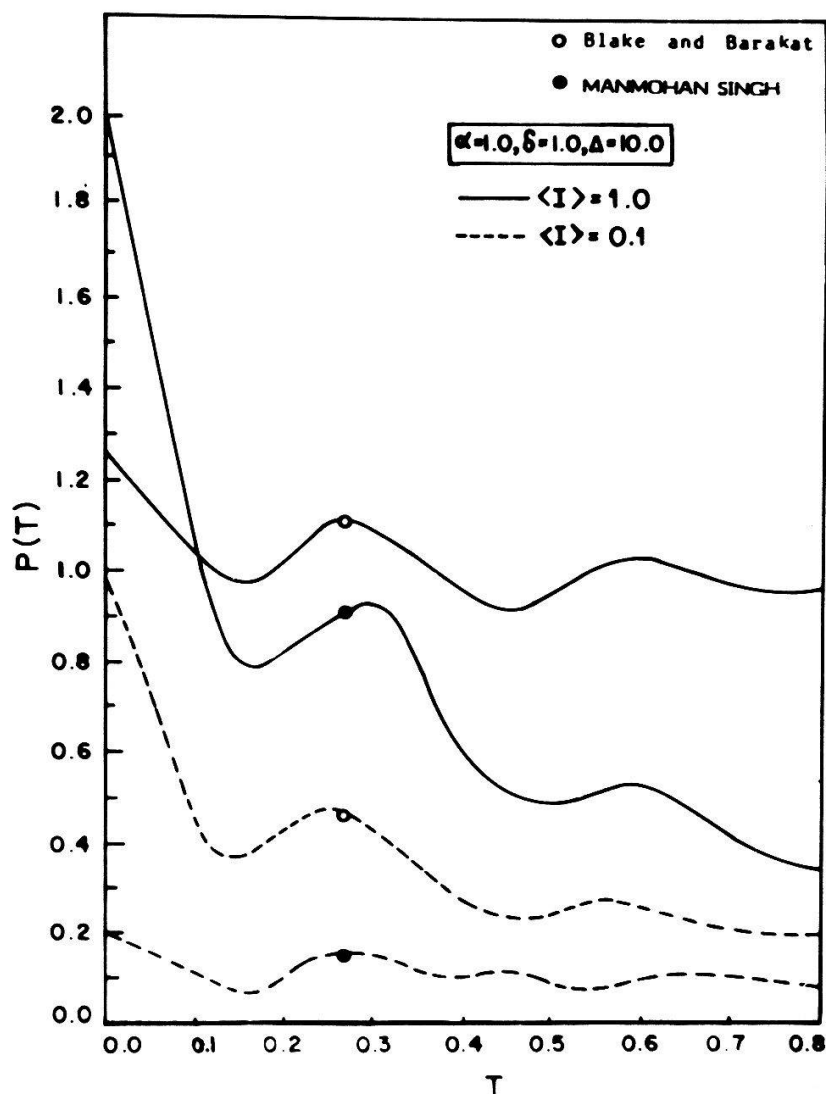


Figure 5

The approximate values of the time-interval-distribution for the conditional photocount reported by Blake and Barakat [3], compared against the exact ones obtained here.

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