Zeitschrift: Helvetica Physica Acta
Band: 62 (1989)
Heft: 4

Artikel: Stochastic quantum dynamics and relativity

Autor: Gisin, N.

DOI: https://doi.org/10.5169/seals-116034

Helvetica Physica Acta, Vol. 62 (1989) 363–371
0018–0238/89/040363–09$1.50 + 0.20/0

Stochastic quantum dynamics and relativity

By N. Gisin

Groupe de Physique Appliquée, Université de Genève, 20 rue de l'Ecole de Médecine, 1211 Genève 4, Switzerland

(13. I. 1989)

Abstract. Our aim is to unify the Schrödinger dynamics and the projection postulate. We first prove that the Schrödinger evolution is the only quantum evolution that is deterministic and compatible with relativity. Next we present a non deterministic generalization of the Schrödinger equation compatible with relativity. This time continuous pure state valued stochastic process covers the whole class of quantum dynamical semigroups, as exemplified by the damped harmonic oscillator and spin relaxation. Finally a symmetry preserving generalization of the Ghirardi–Rimini–Weber model is presented.

1. Introduction

The Schrödinger evolution is deterministic and linear:

$$\psi_0 \mapsto \psi_t = e^{-iHt}\psi_0 \tag{1}$$

The 'collapse of the wave packet', on the contrary, is stochastic and non linear:

$$\psi_0 \mapsto \begin{cases} \dfrac{P\psi_0}{\|P\psi_0\|} & \text{with probability } \langle P \rangle \\ \dfrac{(1-P)\psi_0}{\|(1-P)\psi_0\|} & \text{with probability } \langle 1-P \rangle \end{cases} \tag{2}$$

There is no continuous transition from (1) to (2) and this is the root of the quantum measurement problem. In particular it is unclear where the jump between (1) and (2) occurs. This leads to some vague notions like 'measurement', 'macroscopic', 'observer', 'mind' . . . . This problem is however not a 'vital problem' since with some intuition a physicist can always avoid using (2). Actually the projection postulate is only necessary in order to avoid the prediction of macroscopic superpositions. From this point of view it looks like a ad-hoc postulate whose sole purpose is to save quantum theory from blatant contradiction with every day experience. This curious situation is made possible, among others, by the following fact: The density operator which contains at any time the available information about the state of a quantum system evolves in a closed form as well according to (1) than to (2): if $\rho_0 = \sum c_i P_{\psi_i}$ then (1) implies

$$\rho_t = e^{-iHt}\rho_0 e^{iHt} \tag{3}$$

and (2) implies

$$\begin{aligned}\rho_t &= \sum c_i \langle P \rangle_{\psi_i} \frac{P\psi_i}{\|P\psi_i\|}\left(\frac{P\psi_i}{\|P\psi_i\|}\right)^\dagger \\ &\quad + \sum c_i \langle 1-P \rangle_{\psi_i} \frac{(1-P)\psi_i}{\|(1-P)\psi_i\|}\left(\frac{(1-P)\psi_i}{\|(1-P)\psi_i\|}\right)^\dagger \\ &= P\rho_0 P + (1-P)\rho_0(1-P)\end{aligned} \tag{4}$$

We emphasize that the non linear stochastic process (2) leads to a closed form for the density operator: ρ_t depends only on ρ_0, not on a particular decomposition. Mathematically it is quite miraculous that all the non linear terms in (2) cancel when computing ρ_t. Physically it makes it possible from a pragmatic point of view to ignore (2) and, moreover it is at the root of the 'peaceful coexistence' between quantum mechanics and relativity [1]. Indeed in situations considered by Einstein–Podolsky–Rosen it is possible to prepare at a distance different mixtures of pure states corresponding to the same density operator. Now, if the density operator would not evolve in a closed form, it would be possible to distinguish different mixtures and thus to signal faster than light.

The peculiar relation between (2) and (4) together with the wish to unify the dynamics (1) and (2) are at the origin of this article. In Section 2 we prove that any generalization of quantum dynamics must share this peculiar relation or would contradict relativity. Hence the Schrodinger evolution (1) is the only deterministic evolution compatible with relativity. In Section 3, following Ph. Pearle [2, 3], we present a class of time continuous non linear stochastic evolution equations for state vectors compatible with relativity. In Section 4 we emphasize that this generalization covers the whole class of quantum dynamical semigroups, and exemplify it with the damped harmonic oscillator and spin relaxation. In the conclusion we point to the connections with the theory of continuous quantum measurements [4, 5] and with the Ghirardi–Rimini–Weber [6, 7] attempt to unify micro and macrodynamics. Finally some milestones for the unification of (1) and (2) are discussed.

2. No instantaneous signal constraint

Let H be a Hilbert space. In this article we adopt the usual quantum kinematics. In particular the projectors $P = P^\dagger = P^2$ are identified with the measurable physical quantities that can only take the two values 0 and 1. One dimensional projectors $P_\psi = |\psi\rangle\langle\psi|$ are identified with the possible states of the system. A deterministic evolution over a fixed period of time in a given environment is described by a map g on the set of pure states:

$$g : P_\psi \to g(P_\psi)$$

Consequently a mixture of states P_{ψ_i} with weights x_i evolves into a mixture of states $g(P_{\psi_i})$ with the same weights x_i (a mixture represents our ignorance of the exact state of a system, it is thus clear that the different states P_{ψ_i} can not interact). The corresponding density operators evolve thus as follows:

$$\sum_i x_i P_{\psi_i} \rightarrow \sum_i x_i g(P_{\psi_i})$$

Now, let $\sum_i x_i P_{\psi_i} = \sum_j y_j P_{\varphi_j}$ be two different mixtures corresponding to the same density operator. Assume that

$$\sum_i x_i g(P_{\psi_i}) \neq \sum_j y_j g(P_{\varphi_j}) \tag{5}$$

then the two mixtures can be distinguished after a finite time. But below we prove that any two mixtures corresponding to the same density operator can be prepared at a distance, thanks to *EPR* like correlations. The assumption (5) thus contradicts relativity and (under the assumption that relativity is correct) the map *g must extend to the space of density operators:*

$$g : \rho_0 \rightarrow g(\rho_0)$$

where $g(\rho_0) = \sum_i x_i g(P_{\psi_i})$ for any mixture satisfying $\rho_0 = \sum_i x_i P_{\psi_i}$. Let us notice that the assumption (5) implies that the density operator formalism would not be appropriated for the description of the evolution of mixtures, but it could remain useful for the computation of mean values of observables. Note also that once the fact that g extends to the space of density operators is established, its linearity is obvious.

The proof that any two mixtures corresponding to the same density operator can be prepared at a distance is based on the following technical lemma.

Lemma. *Let* $\psi_i, \varphi_j \in \mathrm{H}$, $x_i, y_j \in]0, 1]$, $i = 1, \ldots, n$, $j = 1, \ldots, m$.

If $\sum x_i P_{\psi_i} = \sum y_j P_{\varphi_j}$, *then there is a state vector* χ *and* $\{\alpha_i\}$, $\{\beta_j\}$ *two orthonormal basis of an Hilbert space* κ *such that*

$$\chi = \sum_i \sqrt{x_i}\, \psi_i \otimes \alpha_i = \sum_j \sqrt{y_j}\, \varphi_j \otimes \beta_j$$

Proof. Assume that $n > m$. Let $\kappa = \mathbb{C}^n$ and $\{\alpha_i\}$ be an o.n. basis of κ. Let E be the space spanned by the ψ_i. One has

$$E = \{\theta \mid \langle \sum x_i P_{\psi_i} \rangle_\theta = 0\}^\perp$$

E is thus also the space spanned by the φ_j. Consequently for all ψ_i there are complex numbers b_{ij} such that $\psi_i = \sum_j b_{ij} \varphi_j$, hence

$$\sum_i \sqrt{x_i}\, \psi_i \otimes \alpha_i = \sum_{ij} \sqrt{x_i}\, b_{ij} \varphi_j \otimes \alpha_i = \sum_j \varphi_j \otimes \tilde{\beta}_j$$

where

$$\begin{aligned}\langle \tilde{\beta}_j \mid \tilde{\beta}_l \rangle &= \sum_i x_i b_{ij} b_{il}^* \\ &= \langle \varphi_j | \sum_{i,m,n} x_i b_{in} b_{im}^* \varphi_n \varphi_m^\dagger | \varphi_l \rangle \\ &= \langle \varphi_j | \sum_i x_i P_{\psi_i} | \varphi_l \rangle \\ &= y_j \delta_{jl}\end{aligned}$$

Put $\beta_j = (1/\sqrt{y_j})\tilde{\beta}_j$, this concludes the proof of the lemma.

In order to achieve the proof that any two mixtures corresponding to the same density operator can be prepared at a distance let A and B be two self adjoint operators with non degenerated eigenvectors α_i and β_j. By measuring A or B on the system represented by the Hilbert space κ, one forces the system represented by H into the mixture $\sum x_i P_{\psi_i}$ or $\sum y_j P_{\varphi_j}$ respectively. The Hilbert space κ of the lemma can obviously be chosen larger and include spatial degrees of freedom, thus the eigenvectors α_i and β_j can have supports in any region of space far away from the system represented by H.

A similar result has been presented by Ph. Pearle in Ref. 8.

We like to emphasize that the linearity of the Schrodinger equation is usually associated to the linearity of the Hilbert space. Here we give a completely different argument based on a theory which, together with quantum mechanics, is at the basis of today's understanding of the physical world.

The fact that a deterministic evolution compatible with relativity must be linear puts heavy doubts on the possibility to solve the measurement problem or to describe quantum friction by adding non linear terms to the Schrodinger equation (9–13).

However, if the evolution g is not deterministic: $g : P_\psi \rightarrow \sum x_i P_{\psi_i}$ with x_i the probabilities for the occurrence of P_{ψ_i}, then the projection postulate (2) shows that the result no longer holds. In the next section we present examples of such stochastic evolutions continuous in time.

3. State vector valued quantum stochastic processes

The existence of time continuous state vector valued stochastic processes satisfying the no instantaneous signal constraint has been first proven in Ref. 14 where an explicit example is given. Here we generalize that example. The generalization is rich enough to cover the whole class of quantum dynamical semigroups.

Consider the following Itô stochastic process:

$$\begin{aligned}d\psi_t &= (B - \langle B \rangle_t)\psi_t \, d\xi_t \\ &\quad - D(B^\dagger B - 2\langle B^\dagger \rangle_t B + \langle B^\dagger \rangle_t \langle B \rangle_t)\psi_t \, dt \qquad (6)\end{aligned}$$

where the Wiener processes ξ_t satisfies $(d\xi_t)^2 = 2D\,dt$, and $\langle B\rangle_t = \langle\psi_t| B |\psi_t\rangle$ is the quantum mean value of the operator B. The motivations for (6) are given below. By using the Stratonovich product $\circ$ one can rewirte the equation (6):

$$d\psi_t = (B - \langle B\rangle_t)\psi_t \circ d\xi_t - D(B^2 - \langle B^2\rangle_t)\psi_t\,dt$$
$$+ 2D\langle B^\dagger + B\rangle_t(B - \langle B\rangle_t)\psi_t\,dt - D(B^\dagger B - \langle B^\dagger B\rangle_t)\psi_t\,dt \qquad (7)$$

Recall that the Stratonovich and Itô products are related as follows [15]: $X\circ dY = X\,dY + \frac{1}{2}\,dX\,dY$, hence $d(XY) = X\circ dY + dY\circ Y = X\,dY + dX\,Y + dX\,dY$. Thus, using the form (7) of our stochastic process it is immediate that the norm of ψ_t is preserved: $d\langle\psi_t \mid \psi_t\rangle = 0$. This is the motivation for the term multiplying $d\xi_t$ in (6). Notice that if $B^\dagger = -B$ then equation (7) reduces to the Schrodinger equation with a fluctuating Hamiltonian.

Let $P_\psi = |\psi\rangle\langle\psi|$. From (6) one gets

$$dP_\psi = ((B - \langle B\rangle_t)P_\psi + P_\psi(B - \langle B\rangle_t))\,d\xi_t$$
$$- D\{B^\dagger B, P_\psi\}\,dt + 2DBP_\psi B^\dagger\,dt$$

where $\{\ ,\ \}$ denotes the anticommutator. Now let ρ_t denote the corresponding density operator, i.e. the average of P_ψ over the Wiener process ξ_t: $\rho_t = \langle\langle P_\psi\rangle\rangle_{\xi_t}$ one has

$$\dot\rho_t = -D\{B^\dagger B, \rho_t\} + 2DB\rho_t B^\dagger \qquad (8)$$

We emphasize that $\dot\rho_t$ depends only on ρ_t and not on a particular decomposition of ρ_t, the stochastic process (6) satisfies thus the constraint discussed in the previous section. This is the motivation for the terms multiplying dt in (6).

The generator of an arbitrary quantum dynamical semigroup is a sum of terms like the right-hand side of equation (8) [16], consequently by introducing several independent Wiener processes $d\xi_i$ ($d\xi_i\,d\xi_j = 0\ \forall i \neq j$) with different operators B_i, one recovers the whole class of quantum dynamical semigroups (QDSQ). Let us emphasize that good physical arguments allow one to think of QDSG as the most general possible evolutions of density operators [17, 18]. This will be exemplified in the next section.

Let us now concentrate on the case $B = A = A^\dagger$. Equations (6), (7) and (8) become

$$d\psi_t = (A - \langle A\rangle_t)\psi_t\,d\xi_t - D(A - \langle A\rangle_t)^2\psi_t\,dt$$
$$d\psi_t = (A - \langle A\rangle_t)\psi_t\circ(d\xi_t + 4D\langle A\rangle_t\,dt) - 2D(A^2 - \langle A^2\rangle_t)\psi_t\,dt \qquad (9)$$
$$\dot\rho_t = -D[A, [A, \rho_t]]$$

Moreover one has

$$d\langle A\rangle_t = 2(\langle A^2\rangle_t - \langle A\rangle_t^2)\,d\xi_t$$

Hence the average of $\langle A\rangle_t$ over the Wiener process ξ_t is constant: $\langle\langle\langle A\rangle_t\rangle\rangle_{\xi_t} = \langle A\rangle_0 =$ constant. But on the average the standard deviation decreases with time:

$$\frac{d}{dt}\langle\langle\Delta A_t^2\rangle\rangle = -8D\langle\langle\Delta A_t^4\rangle\rangle \leq 0$$

with $\Delta A_t^2 = \langle A^2 \rangle_t - \langle A \rangle_t^2$. Accordingly the state vectors concentrate on the eigenspaces of A, but the proportions are such that the mean value $\langle\langle\langle A \rangle\rangle\rangle$ is constant. *The quantum probabilities are thus turned into a classical probability distribution:*

$$\psi_0 = \sum_i c_i \alpha_i \rightarrow \rho_\infty = \sum_i |c_i|^2 P_{\alpha_i}$$

To conclude this section let us emphasize that the equations (6) and (9) generalize to the case of several operators with several correlated Wiener processes. For instance equation (9) generalizes to:

$$d\psi_t = \sum_i (A_i - \langle A_i \rangle_t)\psi_t \, d\xi_i - \sum_{ij} D_{ij}(A_i - \langle A_i \rangle_t)(A_j - \langle A_j \rangle_t)\psi_t \, dt$$

with $d\xi_i \, d\xi_j = 2D_{ij} \, dt$.

4. Damped harmonic oscillator and spin relaxation

In this section we exemplify the connection between the stochastic process (6) and quantum dynamical semigroups with the two standard examples of dissipative quantum systems.

Let $H = a^\dagger a$ be the Hamiltonian of the harmonic oscillator, with a and $a^\dagger$ the usual annihilation and creation operators. The standard description of the damped quantum oscillator in terms of density operators is [19]

$$\dot{\rho}_t = -i[a^\dagger a, \rho_t] + k(2a\rho_t a^\dagger - \{a^\dagger a, \rho_t\})$$

The associated stochastic process is

$$d\psi_t = (a - \langle a \rangle_t)\psi_t \, d\xi_t - k(a^\dagger a - 2\langle a^\dagger \rangle_t a + \langle a^\dagger \rangle_t \langle a \rangle_t)\psi_t \, dt \tag{10}$$

with $(d\xi_t)^2 = 2k \, dt$. Note that for a coherent state $|\alpha\rangle$ (i.e. $a\,|\alpha\rangle = \alpha\,|\alpha\rangle$ with $\alpha \in \mathbb{C}$) the first term in (10) cancels. Accordingly the coherent states remain coherent and follow the path of a classical damped harmonic oscillator:

$$d((a - \alpha)\,|\alpha\rangle) = 0 \quad \text{with} \quad d\alpha = -k\alpha \, dt$$

Spin relaxation is described phenomenologically by the Bloch equations [20]

$$\dot{M}_x = -wM_y - M_x/T_2 \qquad \dot{M}_y = wM_x - M_y/T_z$$
$$\dot{M}_z = (M_0 - M_z)/T_1$$

or equivalently in terms of density operators [19]

$$\begin{aligned}\dot{\rho}_t = &-i\frac{w}{2}[\sigma_z, \rho_t] - D_\perp(2\rho_t - \sigma_x\rho_t\sigma_x - \sigma_y\rho_t\sigma_y) \\ &- D_\parallel(\rho_t - \sigma_z\rho_t\sigma_z) \\ &+ k(2\sigma_+\rho_t\sigma_- - \{\sigma_-\sigma_+, \rho_t\})\end{aligned}$$

where $\vec{M} = \mathrm{Tr}\,(\vec{\sigma}\rho)$, $T_2^{-1} = k + 2D_\perp + 2D_\parallel$, $T_1^{-1} = 2k + 4D_\perp$, $M_0 = 2kT_i$. The fluctuations (i.e. the $D_\perp$ and $D_\parallel$ terms) can be described in terms of a fluctuating magnetic field, whereas the dissipative term must be described in terms of the stochastic processes presented in the previous section:

$$d\psi_t = -iB\sigma_z\psi_t\,dt - i\vec{\sigma}\psi_t \circ d\vec{b} + (\sigma_+ - \langle\sigma_+\rangle_t)\psi_t\,d\xi_t$$
$$- k(\sigma_-\sigma_+ - 2\langle\sigma_-\rangle_t\sigma_+ + \langle\sigma_-\rangle_t\langle\sigma_+\rangle_t)\psi_t\,dt \tag{11}$$

where $d\vec{b}$ are 3 independent Wiener processes, $(db_i)^2 = D_i\,dt$, $db_i\,db_j = 0$ for all $i \neq j$, $db_i\,d\xi = 0$ for all i, $(d\xi)^2 = 2k\,dt$. For symmetry reasons we put $D_\perp = D_x = D_y$, $D_\parallel = D_z$. From (11) one deduces for $\vec{m} = \langle\psi|\,\vec{\sigma}\,|\psi\rangle$:

$$d\vec{m} = 2B\vec{m} \wedge \vec{e}_z\,dt + 2\vec{m} \curlywedge d\vec{b} - \vec{m} \wedge (\vec{m} \wedge \vec{e}_x)\,d\xi + \vec{m} \wedge \vec{e}_y\,d\xi$$
$$- k\vec{m}\,dt + k(2 - m_z)\vec{e}_z\,dt$$

where $\vec{e}_x = (1, 0, 0)$, $\vec{e}_y = (0, 1, 0)$, $\vec{e}_z = (0, 0, 1)$. Hence $d(\vec{m}^2) = 0$, but the average over the stochastic processes $d\vec{b}$ and $d\xi$, $\vec{M} = \langle\langle\vec{m}\rangle\rangle$ follows the Bloch equations.

5. Conclusion

So far we have proven that a deterministic generalization of the Schrodinger equation is necessarily incompatible with relativity, and that a stochastic generalization is possible and even rich enough to cover all the physically possible evolutions of density operators.

But let us return to our first motivation, namely the unification of the evolutions (1) and (2). A step in this direction would be the choice of a particular operator A in equation (9) together with the postulate that the evolution of a physical system is given by (9) at all times. A first possibility is $A = q$, the position operator. This choice is closely related to the Ghirardi–Rimini–Weber [6, 7] attempt to unify micro and macrodynamics as has been shown by L. Diosi [21] in the case of some appropriate limit. It is also closely related to the theory of continuous position measurement [4, 5]. An other possibility is to consider the set of position projectors $P^n_{\vec{x}} = \mathbb{1} \otimes \cdots \otimes |\vec{x}\rangle\,\langle\vec{x}| \otimes \cdots \otimes \mathbb{1}$ of the nth particle, with Wiener processes $d\xi_t(\vec{x})$ and correlation

$$D(\vec{x}, \vec{y}) = \frac{\lambda}{2}\exp\left\{-\frac{\alpha}{4}(\vec{x} - \vec{y})^2\right\}$$

The corresponding stochastic process reads:

$$d\psi_t(\vec{z}_1 \cdots \vec{z}_N) = \sum_n \psi_t(\vec{z}_1 \cdots \vec{z}_N)\left(d\xi_t(\vec{z}_n) - \int d\vec{x}\,\langle P^n_{\vec{x}}\rangle_t\,d\xi_t(\vec{x})\right)$$
$$- \sum_{n,m}\Big(D(\vec{z}_n, \vec{z}_m) - 2\int d\vec{x}D(\vec{x}, \vec{z}_m)\langle P^n_{\vec{x}}\rangle_t$$
$$+ \iint d\vec{x}\,d\vec{y}D(\vec{x}, \vec{y})\langle P^n_{\vec{x}}\rangle_t\langle P^m_{\vec{y}}\rangle_t\Big)\psi_t(\vec{z}_1 \cdots \vec{z}_N)\,dt$$

With this choice the density operator follow exactly the evolution equation of the Ghirardi–Rimini–Weber [6, 7] model:

$$\rho_t(\vec{x}_1 \cdots \vec{x}_N, \vec{y}_1 \cdots \vec{y}_N) = \sum_{n,m} (2D(\vec{x}_n, \vec{y}_m) - D(\vec{x}_n, \vec{x}_m) - D(\vec{y}_n, \vec{y}_m)) \times \rho_t(\vec{x}_1 \cdots \vec{x}_N, \vec{y}_1 \cdots \vec{y}_N)$$

All the nice features of the GRW model translate to the above process, which replaces jumps by continuous wave packet reduction. Moreover all the Hilbert space machinery is now available; in particular it is immediate to see that a symmetric or antisymmetric state will *preserve its symmetry* for all realizations $\xi_t(\vec{x})$. Notice that in the limit $\lambda \to \infty$, $\alpha\lambda =$ constant, one recovers the equation (9) with $A = q$.

A further possibility, more natural in the Hilbert space context is $A = H$, the energy operator:

$$d\psi_t = -iH\psi_t\, dt + (H - \langle H \rangle_t)\psi_t\, d\xi_t - D(H - \langle H \rangle_t)^2 \psi_t\, dt \tag{12}$$

From (12) follows $\langle\langle\langle H\rangle_t\rangle\rangle_{\xi_t} =$ constant, that is the energy is conserved on the average, but not in individual quantum processes. According to (12) a measurement like situation could be described as follows. Let A be the operator representing the physical quantiy to be measured with eigenvalues $+1$ and -1, $\psi_0 = c_+\alpha_+ + c_-\alpha_-(A\alpha_\pm = \pm\alpha_\pm)$ the initial state of the system and φ_0 the initial state of the apparatus. First a very strong interaction correlates the system and the apparatus, as in the well known Von Neumann scheme:

$$\psi_0 \mapsto c_+\alpha_+ \otimes \varphi_+ + c_-\alpha_- \otimes \varphi_-$$

where $\varphi_\pm$ are two eigenvectors of the apparatus Hamiltonian Ha, i.e. $\varphi_\pm$ represent two possible states of the apparatus. Next, from (12) one deduces

$$d\langle A\rangle_t = (\langle \mathrm{Ha}\rangle_{\varphi_+} - \langle \mathrm{Ha}\rangle_{\varphi_-})(1 - \langle A\rangle_t^2)\, d\xi_t \tag{13}$$

The solution of (13) is well known [14, 22]. The distribution of $\langle A\rangle_t$ concentrates asymptotically in time on $\langle A\rangle_t = \pm 1$. The speed of this concentration however, depends on the energy difference $\langle \mathrm{Ha}\rangle_{\varphi_+} - \langle \mathrm{Ha}\rangle_{\varphi_-}$. The constant D of the model can thus be chosen such that the equation (12) is practically identical to the Schrodinger equation whenever the energies are small, but the same equation (12) gets close to the evolution (2) whenever superposition of states with large energy differences occur. Generalization to measurements with more than two outcomes is straightforward.

One could of course object that the possible states of an apparatus can have the same energy. But, in order to contradict the model, one would have to show that the energy of the two states are almost the same during the whole amplification process, from the microscopic level until the macroscopic level.

We acknowledge that the model is highly speculative, but we hope to have convinced the reader that a unification of the deterministic linear Schrodinger evolution and the stochastic non linear projection postulate is not a priory impossible.

REFERENCES

[1] A. SHIMONY, in *Foundations of Quantum Mechanics in the Light of New Technology*, ed. S. Kamefuchi, Phys. Soc. Japan, Tokyo (1983).
[2] PH. PEARLE, *Phys. Rev. 13D*, 857 (1976).
[3] PH. PEARLE, in *New Techniques and Ideas in Quantum Measurement Theory*, ed. D. M. Greenberger, Ann. N.Y. Acad. Sc. (1986).
[4] A. BARCHIELLI, L. LANZ and G. M. PROSPERI, *Nuovo Cim. 72B*, 79 (1982).
[5] A. BARCHIELLI and G. LUPIERTE, *J. Math. Phys. 26*, 2222 (1985).
[6] G. C. GHIRARDI, A. RIMINI and T. WEBER, *Phys. Rev. 34D*, 470 (1986).
[7] J. S. BELL, in *Schrodinger Centenary of a Polymath*, Cambridge University Press (1987).
[8] PH. PEARLE, *Phys. Rev. 33D*, 2240 (1986).
[9] I. BIALYNICKI-BIRULA and J. MYCIELSKI, *Ann. Phys. 100*, 62 (1976).
[10] M. D. KOSTIN, *J. Stat. Phys. 12*, 145 (1975).
[11] N. GISIN, *J. Phys. 14A*, 2259 (1981).
[12] R. W. HASSE, *J. Math. Phys. 16*, 2005 (1975).
[13] K. ALBRECHT, *Phys. Lett. 56B*, 127 (1975).
[14] N. GISIN, *Phys. Rev. Lett. 52*, 1657 (1984).
[15] L. ARNOLD, *Stochastic Differential Equations: Theory and Applications*, Wiley, New-York (1974).
[16] G. LINBLAD, *Commun. Math. Phys. 48*, 119 (1976).
[17] V. GORINI, A. KOSSAKOWSKI and E. C. G. SUDARSHAN, *J. Math. Phys. 17*, 821 (1976).
[18] E. B. DAVIES, *Quantum Theory of Open Systems*, Academic Press, London (1976).
[19] F. HAAKE, *Statistical Treatment of Open Systems by Generalized Master Equations*, Vol. 66 of Springer Tracts in Modern Physics, ed. G. Hohler, Springer, Berlin (1973).
[20] A. ABRAGAM, *Les Principes du Magnetisme Nucleaire*, Presse Universitaire de France (1961).
[21] L. DIOSI, *Physics Letters 129A*, 419 (1988).
[22] M. O. HONGLER, *Phys. Lett. 75A*, 3 (1979).