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Recent Developments in Quantum Mechanics *

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Abstract : *It is essentially a review of recent progress in Quantum Mechanics obtained by the "Geneva School", put all together in a synthesis for the first time.*

During these twelve last years Quantum Mechanics has developed deeply in three aspects :

1) The interpretation has been completely clarified ([1] to [10]) but many "senior" physicists delight in the mystery of their school-days Quantum Mechanics and do not want to change their minds .

2) The formalism has been developed and generalized to many (if it is not all) physical situations.

3) Many new rules of calculation have been developed.

In conclusion many paradoxes and/or unsolved problems have been solved and many calculations which usually appear just as tricks can be explained and justified [27]. I want here to give a brief survey of each one of these three points and to end by some examples which show the power and the efficiency of this new theory.

1. The Interpretation

By a physical system we understand a part of reality conceived as existing in space and time and which is isolated from the rest of the universe. We take the usual realistic point of view, the system is and it is what it is; whether it is known or not by the physicist does not change anything of its own reality. A theory is nothing else than a model, a mathematical model, sufficiently precise to describe the reality as it is but idealized to give a powerful intuition on its reaction to the exterior actions of the physicist. In other words we take the point of view of Einstein (for example in E.P.R. [25]). At the very basis is the notion of experimental project (in short test or question), an experiment that you can perform on the system and where you have exactly defined in advance what is for you a positive result. Each experimental project defines a particular property of the system : We say that a property is actual for a given (one) system (or it is an element of reality in E.P.R.) if in the event of the corresponding experiment the positive result would be certainly obtained. Other properties which are not actual are said to be potential. The

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complete set of all actual properties is by definition the state of the system. Of course by itself or by the influence of the exterior the system changes, in other word its state changes : some actual properties becoming potential while others, potential at the start, appear in actuality.

The main problem is to exhibit a mathematical framework capable of describing a system by its complete family of properties, its possible states and the corresponding structures inherent from these notions. To do this we proceed in the following way. We first suppose known the set Q of all experimental projects (or tests) that you can in the event perform on the system. A test $\alpha \in Q$ is said to be true if the corresponding property is actual which means as we have written that if we would actually perform α the positive result would be certain. Let us consider the following structure on Q. A test α is said to be stronger than a test β (i.e. $\alpha < \beta$) if whenever α is true β is true as well. A test α and a test β will be said to be equivalent if (and only if) $\alpha < \beta$ and $\beta < \alpha$. We identify the equivalence class containing α with its corresponding property a. The set $\mathcal{L}$ of all properties (the set of all equivalence class in Q) inherits an ordering relation which makes $\mathcal{L}$ a complete lattice [3].

The set Σ of all possible states of the system has also a particular structure. Two states $\mathcal{E}_1$ and $\mathcal{E}_2$ are said to be orthogonal (i.e. $\mathcal{E}_1 \perp \mathcal{E}_2$) if there is in Q a test α such that the positive result (in the event you would perform α) is certain for $\mathcal{E}_1$ but impossible for $\mathcal{E}_2$. Such an orthogonality relation gives a characterization of classical physics. In classical physics two different states are always orthogonal to each other. This is a direct consequence of the following hidden postulate known as the classical prejudice : Given any property of $\mathcal{L}$, there is in its equivalence class a test, an experimental project, such that no matter the state, the positive result is either certain or impossible. Such a postulate is unwarranted and fails in fact in physics; it does not follow from first principles. In particular from a principle for the deterministic evolution, whatever that may be, we cannot conclude that the result of a hypothetical experiment is certain.

The reader has understood that in usual quantum mechanics described by a Hilbert space, closed subspaces are in one to one correspondence with properties and rays define states. In other words a property is actual if the ray which is the state is in the corresponding closed subspace. Apparently nothing is changed in the interpretation but the measurement corresponding to a closed subspace is here a test i.e. an experimental project which is not ideal and takes time, some times a long time. As a conclusion we want to quote [26] : "Einstein in 'Quantum-Mechanik and Wirklichkeit' begins his essay with the distinction between the two positions one might take on the relation between the Ψ-function and some state of affairs of an individual system. The point of view now labelled 'I_a' holds that the system possesses determinate values of all of its observables even if these values cannot be determined simultaneously by measurment .... The other point of view, 'I_b' holds that determinate values of observables do not really exist until they are brought into being by measurments ...". According to our interpetation given here it is now clear that it is the second point of view 'I_b'which is the correct one and not the first 'I_a'.

2. The Formalism

We have defined first the properties and the lattice $\mathcal{L}$ and, from that, the states as particular subsets of $\mathcal{L}$. From the mathematical point of view it is very convenient to define first the states and then the properties as a secondary notion. Let us suppose given the set Σ of all possible states of our system. Each property $a \in \mathcal{L}$ will be now represented by a subset A of Σ. By definition a state $\mathcal{E}$ will be in A if and only if the property a is actual for $\mathcal{E}$. With very simple axioms (see [1], [2] or [3]) it is possible to characterize the subset of Σ which represents properties. Define for any subset A of Σ, $A^{\perp}$ the subset of all states orthogonal to each one of A and also $A^{\perp\perp}$ the subset of all states orthogonal to each one of $A^{\perp}$. Evidently $A \subset A^{\perp\perp}$ since orthogonality is a symmetric relation, but A represents a property if and only if $A = A^{\perp\perp}$. Moreover a singleton, a subset containing only one state, also represents a property, in other words we have $\{\mathcal{E}\}^{\perp\perp} = \{\mathcal{E}\}$ for each $\mathcal{E} \in \Sigma$. In the following we will identify $\mathcal{L}$ with the lattice of all biorthogonal subsets of Σ. To give more insight on the structure of $\mathcal{L}$ let us define two new concepts :

Two properties A and B are said to be orthogonal to each other if each state in A is orthogonal to each state in B (i.e. if $A \subset B^{\perp}$ or also if $B \subset A^{\perp}$). In particular $A^{\perp} = A^{\perp\perp\perp}$ is a property, orthogonal to A, called the orthocomplement and the map $A \mapsto A^{\perp}$ defined on $\mathcal{L}$ is the orthocomplementation of $\mathcal{L}$.

A property A is said to be classical if for each state $\mathcal{E} \in \Sigma$ either $\mathcal{E} \in A$ or $\mathcal{E} \in A^{\perp}$. If A is classical $A^{\perp}$ is classical and for any collection of classical properties their greatest lower bound, their intersection, is also a classical property. This means that the classical properties form a lattice, a sublattice of $\mathcal{L}$, called the classical property lattice. Two states $\mathcal{E}_1$ and $\mathcal{E}_2$ are said to be classicaly equivalent if for any A classical, $\mathcal{E}_1 \in A$ is equivalent to $\mathcal{E}_2 \in A$. Such an equivalence class is called a macrostate and the classical property lattice is just the set of subsets of macrostates. The set of macrostates is also called the set of superselection rules [12]. From this analysis we see that in first approximation any physical system appears as a classical one !

When the physical system is one entity making a whole we can impose more axioms and thus justify the Hilbert space structure [11]. But when the system is composed of two or more separated entities such axioms fail and cannot be maintained without contradictions [13]. In fact two separated entities taken together must be described in the following way. Given Σ_1 and Σ_2 the sets of states of each entity and $\perp_1$ and $\perp_2$ the corresponding relations of orthogonality, the whole system has a set of states given by the direct product $\Sigma_1 \times \Sigma_2$. Two global states $(\mathcal{E}_1, \mathcal{E}_2)$ and $(\mathcal{E}_1', \mathcal{E}_2')$ are orthogonal iff $\mathcal{E}_1 \perp_1 \mathcal{E}_1'$ or $\mathcal{E}_2 \perp_2 \mathcal{E}_2'$. As in the general case the corresponding property lattice is then also the lattice of biorthogonal subset of $\Sigma_1 \times \Sigma_2$. The important point is that if Σ_1 is the set of rays of a Hilbert space and Σ_2 the set of rays of an other one the property lattice of the global system cannot be embeded in the linear closed subspace lattice of the tensor product of these two Hilbert spaces. It is then not possible to describe two separated entities by the closed subpaces of the tensor product even if you restrict the states to the ones in $\Sigma_1 \times \Sigma_2$.

3. Applications

Two important ingredients of the theory are symmetries and observables. A symmetry is an automorphism i.e. a permutation of the states of Σ which conserves the orthogonality relation in the two directions. Any symmetry generates an automorphism of the orthocomplemented lattice structure of $\mathcal{L}$. Any symmetry generates a permutation of the macrostates and an automorphism of the boolean lattice structure of the classical properties. If Σ is the set of rays of a Hilbert space any symmetry on Σ is generated by a unique either unitary or an antiunitary transformation, but defined up to a phase i.e. up to a complex number of module one [11]. Finally let us remark that many symmetries of Σ are just mathematical objects which have no direct interpretation.

An observable is a map from some complete boolean lattice $\mathcal{B}$ in the property lattice $\mathcal{L}$ which conserves the lattice structure and the orthocomplementation. More precisely a map

$$\mu : \mathcal{B} \to \mathcal{L}$$

is an observable iff $\mathcal{B}$ is a complete boolean lattice (an orthocomplemented complete lattice which is also distributive) and

$$\mu(\vee b_i) = \vee(\mu b_i)$$
$$\mu(b^{\perp}) = (\mu b)^{\perp}$$

Without any loss of generality we can impose also to the observable μ to be injective i.e. such that the minimal element of $\mathcal{B}$ is the only one which is mapped on the minimal element of $\mathcal{L}$.

In the particular case where $\mathcal{L}$ is a classical property lattice, i.e. the set of subsets of a set Σ, it turns out that $\mathcal{B}$ must also be the set of subset of some set (say E) and the observable μ itself can be realized in a unique way as the inverse image of a map from Σ to E. On the opposite case when Σ is the set of rays of some separable Hilbert space any observable μ can be interpreted as the spectral family of some self-adjoint operator [11]. As for the symmetries many observables are just mathematical objects without direct physical interpretation. However each experimental project which has a complete set of well-defined outcomes defines an observable. In fact any subset of such possible outcomes can be interpreted as the positive result of some test and then put in correspondence with a particular property in $\mathcal{L}$. Such correspondence is an observable since it conserves the least upper bound and the orthocomplementation. In practice, it is in general not too difficult to define and select one particular subset of outcomes from a given experiment, but it is in general impossible to define the complete set of possible outcomes.

From these notions of symmetry and observable we have been able to build different particle models. We define a particle as an entity for which the observables position, time and momentum can be defined and we write the conditions which must be imposed a priori on such observables. The classical and the non-classical particle models appear in our theory as two different solutions of the same problem without any appeal to some correspondence or quantization principle [15] (see also [14] Chapter 5). Also the Schrödinger equation can be justified and for a non-relativistic spin $\frac{1}{2}$ particle like an electron the corresponding dynamics compatible with the galilean principle turns out to be the one given by the minimal coupling Schrödinger operator defined for non-abelian gauge based on $SU(2)$ ([14] Chapter V).

If for a given state $\mathcal{E}$ neither the property A nor its orthocomplement $A^{\perp}$ is actual the positive result can not be predicted with certainty in the event you would perform one of the equivalent experimental projects defining A. But in the case of an ideal measurement i.e. an experiment which perturbs the system as little as possible and such that when the positive result is obtained the property A is actual at the end, the a priori probability law for the positive result can be predicted following the usual law of probability. For a system described by some Hilbert space such a priori law is unique and is the one which has been postulated in quantum mechanics ([14] Chapter 4 see also [3]). But here such formula is only valid if the experiment is ideal in our sense. For a real experiment you have in general to add corrections. Such corrections, which can be computed in our formalism, have simply no meaning in the usual theory. From this a priori law of probability we can develop a theory of probability, of random observable and of statistics much more general and powerful that the one derived from density matrix or from W- or C^*-algebra formalism [14].

4. Conclusions

Many old unsolved problems of usual quantum mechanics can be solved and explained in the new theory. Let us briefly give some examples :

The irreversible evolution

As we have written before, the equation for a reversible evolution can be proved to be of the Schrödinger type, but the same theory give also the most general equation for the deterministic irreversible evolution [20] and [21]. With such an equation the rest states (usually called stationary) are "of course" the same.

The measuring process

Without any ad hoc hypothesis the measuring process is a puzzling unsolved problem in the usual quantum mechanics. In [16] with our formalism we produce a model for a measuring process which fulfills all the conditions that you can physically impose (such conditions was imposed by Dr. J. Bub, and send to us as a challenge for our theory in a private conversation in Köln in October 1978): no collapse at the end, a law of motion completely deterministic, but an explanation of the probability in terms of some unknown initial correlation between the system and the apparatus.

The decay process

In such process a final state is not a superposition of decayed and undecayed states. The instant of the decay is a random variable. Nevertheless such an instant is well defined for each individual event. In our formalism with continuous (and also non compact) superselection rules we have build a complete model of such process [12].

The nuclear magnetic resonance
We have a unique equation for spin relaxation and spin echo processes [21].

The quantization of the photon field
There are difficulties in the usual theory for the quantization of the electromagnetic field even in the Gupta-Bleuler formalism. This is due to the existence of a continuous superselection rule. This problem is solved in [19] with our formalism.For such field a localisability concept has been developed [17].

The Mössbauer effect
The recoilless emission can not be really explained if you quantize the center of gravity of the crystal. In our formalism with the center of gravity of the crystal treated a as superselection rule, the Lipkin sum rule and the Mössbauer effect can be derived. The perturbation due to a global motion of the crystal can also be predicted.

The two body Hydrogen atom
The usual two-body Schrödinger operator for the Hydrogen atom having a completely continuous spectrum, no rest state exists. Such a paradox is solved by our two-body model in [18].

The two photon decay correlation
As a last example we want to analyze the recent experiment of A. Aspect on the two photon decay correlation more precisely [24].

At the beginning one given exited calcium atom decays in one two-photon composite globule entity. Such an entity can be predicted according to our model (see above : The decay process) taking into account all selection rules and boson spin statistics. After the lens systems and the two frequency filters, the two-photon composite globule entity is localized at the left and at the right of the apparatus, in a non-symmetric state (the boson spin statistics rule fails now) given by :

$$\frac{1}{\sqrt{2}}\Big(f_{\nu_1}(\vec{x}_1)|R\rangle_1 \otimes g_{\nu_2}(\vec{x}_2)|R\rangle_2 + f_{\nu_1}(\vec{x}_1)|L\rangle_1 \otimes g_{\nu_2}(\vec{x}_2)|L\rangle_2\Big)$$
$$= f_{\nu_1}(\vec{x}_1)g_{\nu_2}(\vec{x}_2)\frac{1}{\sqrt{2}}\Big(|R\rangle_1 \otimes |R\rangle_2 + |L\rangle_1 \otimes |L\rangle_2\Big)$$

where $f_{\nu_1}(\vec{x})|R\rangle$ represents one-photon state, localized on the left, going on the left, right circularly polarized and having essentially the frequency ν_1; where $g_{\nu_2}(\vec{x})|R\rangle$ represents one-photon state, localized on the right, going on the right, right circularly polarized and having essentially the frequency ν_2; and so on ...

Such a complicated two-photon state does not correspond to one photon ν_1 on the left and one photon ν_2 on the right but to one entity globally localized on the left and on the right.

After the two polarizers, in each of the four channels the corresponding counter will register photons in a one-photon state, with a given frequency and a given polarization. As it is also explained in [23] the Bell's inequality is violated since the measuring apparatus breaks the one two-photon composite globule entity in two entities, two separated photons.

Since his measuring apparatus (his polarizers) are not perfect (ideal in our sense) according to our theory A. Aspect must correct his experimental data before comparing to the a priori probability calculated with the usual formula of quantum mechanics. This is exactly what he does when he introduces transmission and reflection coefficients for his polarizers.

In conclusion such beautiful experiment proves the reality of the initial state describing the two-photon composite globule entity but puts also in evidence the intrinsic limitations of the old usual formalism.

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