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A note on mixing-increasing and mixing-decreasing maps

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Abstract. We collect a number of results on affine maps of the normal state space of a W^*-algebra which either increase or decrease the *chaoticity* or *mixedness* of states in the sense of Uhlmann ([1]).

Introduction

The theory of the *more-mixed-than* partial order relation for states of a C^*-algebra introduced by Uhlmann ([1]), has reached a considerable level of sophistication ([2]). Its physical connotations are discussed in part in [3].

We are here concerned with the particular case of W^*-algebras, and affine endomorphisms of their normal state space which are *mixing-increasing* or *mixing-decreasing*.

Definitions

Consider a W^*-algebra A, We write A^+, A^u, and A^p for the *positive* elements, *unitary* elements, and *projections* of A respectively. The *von Neumann–Murray partial order* for A^p will be written $<$, and the corresponding *equivalence relation* is $\sim$. The *state space* $S(A)$ (or simply S) consists of the positive linear functionals f on A which satisfy $f(1) = 1$, 1 being the identity of A. A state f is *normal* if it is completely additive on families of pairwise orthogonal projections, or alternatively if $f(\sup\{a_i : i \in I\}) = \sup\{f(a_i) : i \in I\}$, for every uniformly bounded, increasing directed set $\{a_i : i \in I\}$ in A^+. The *normal states* of A will be denoted by $S_n(A)$ (or simply S_n). We write *Aff*(S), resp. *Aff*(S_n), for the affine maps of S, resp. S_n, into itself. If $v \in Aff(S_n)$, then there exists (by extension and duality, see e.g. [4]) a unique linear map α from A to A, such that

$$v(f) = f \circ \alpha, \quad \text{for all} \quad f \in S_n. \tag{1}$$

This map α is positive, normal (i.e. w^*xw^*-continuous), and $\alpha(1)=1$. For the general theory of W^*-algebras we refer to the standard texts [4], [5], [6], [7].

Definition 1 (Uhlmann). For f, and g in S, we say f is *more mixed than* g, and write $f \succ g$ if f lies in the w^*-closure of the convex hull of $\{g(u^* \cdot u): u \in A^u\}$.

One of the basic results of the theory is that $f \succ g$ if and only if $F(f) \leq F(g)$ for all unitarily invariant, lower-semicontinuous, convex functions F on S (see 3.2 of [2]). This says that all entropy-like functions (i.e. $-F$'s) are larger for f than for g, and this is the basis for the mathematical physicists interest in $\succ$. Notice that if both states are normal then w^*-closure can be replaced by norm-closure in the above definition.

Definition 2. $v \in \mathit{Aff}(S)$ is said to be *mixing-increasing*, resp. *mixing-decreasing* if $v(f) \succ f$, resp. $f \succ v(f)$, for every $f \in S$.

If $v \in \mathit{Aff}(S_n)$ then two natural definitions for mixing-increase/decrease are available.

Definition 3. Let $v \in \mathit{Aff}(S_n)$, and α be the corresponding dual map given by (1). Then v is said to be *W-mixing-increasing*, resp. *-decreasing*, if $v(f) \succ f$, resp. $f \succ v(f)$, for every $f \in S_n$. v is said to be *S-mixing-increasing*, resp. *-decreasing*, if the map in $\mathit{Aff}(S)$ defined by $f \to f \circ \alpha$, $f \in S$, is mixing-increasing resp. -decreasing.

Clearly, S-mixing-increase/decrease implies W-mixing-increase/decrease. The converse is not a priori evident, and will be shown to apply in a variety of cases.

Much of the beauty of the $\succ$-theory in W^*-algebras comes from the fact that the $\succ$-relation is dual to the von Neumann–Murray relation on A^p. In their full generality the following results are due to Alberti (see [2]). Define $K: SxA^+ \to \mathbb{R}^+$, by

$$K(f, a) := \sup \{f(u^*au): u \in A^u\}; \tag{2}$$

then K takes values in the interval $[f(a), \|a\|]$, is subadditive and lower-semicontinuous in each variable, and is order-determining in the following sense:

$$f \succ g \quad \text{if and only if} \quad K(f, p) \leq K(g, p) \quad \text{for all} \quad p \in A^p; \tag{3}$$

$$p \prec q \quad \text{if and only if} \quad K(f, p) \leq K(f, q) \quad \text{for all} \quad f \in S. \tag{4}$$

Before proceeding to the discussion of mixing-increase/decrease, we consider the question of the replacement of S by S_n in (4). This is possible for finite W^*-algebras (see [2]); we show that it is also possible for countably decomposable (i.e. σ-finite) algebras.

Lemma 1. *If A is countably decomposable, $p, q \in A^p$, and $K(f, p) \leq K(f, q)$ for every $f \in S_n$, then $p \prec q$.*

Proof. Take any faithful normal trace t on A^+. There exists (e.g. V.2 of [5], and Theorem 2.7.11 of [4]) an increasing net $\{f_j : j \in J\}$ of positive normal linear functionals of A with $t(a) = \sup\{f_j(a) : j \in J\}$, for every $a \in A^+$. Let z be a central projection of A, then

$$K(f_j, pz) = \sup\{f_j(u^*pzu) = f_j(u^*puz) : u \in A^u\}$$
$$\leq \sup\{f_j(u^*quz) = f_j(u^*qzu) : u \in A^u\} = K(f_j, qz)$$

where the inequality follows from the assumption by renormalizing the positive normal linear functional $f_j(\cdot z)$ to a state. We now prove that

$$t(a) = \sup\{K(f_j, a) : j \in J\}, \quad \text{for every} \quad a \in A^+.$$

Indeed, let $K(a)$ be the l.u.b of the increasing net $\{K(f_j, a) : j \in J\}$. From (2), $f_j(a) \leq K(f_j, a)$, thus $t(a) \leq K(a)$. On the other hand, $f_j(u^*au) \leq t(u^*au) = t(a)$, and thus $K(a) \leq t(a)$. From the above relations, we conclude that:

$$t(pz) \leq t(qz), \quad \text{for every central projection} \quad z \in A. \tag{*}$$

But in a countably decomposable W^*-algebra, (*) is equivalent to $p \prec q$ (see Exercise 7.13 of [6]). q.e.d.

Affine Bijections

The affine bijections of S_n are in one-to-one correspondence (see e.g. Theorem 3.2.8 of [4]) with the Jordan-$*$-automorphisms of A, via duality. Recall that a Jordan-$*$-automorphism α of A is a linear bijection of A sending selfadjoints into selfadjoints elements, and preserving the Jordan-product, i.e. $\alpha(ab + ba) = \alpha(a)\alpha(b) + \alpha(b)\alpha(a)$, for all $a, b \in A$. α is then $*$-preserving, positive, isometric, normal, and $\alpha(1) = 1$. Furthermore, $\alpha(ax) = \alpha(a)\alpha(x)$, for all $a \in A$ when x is in the center of A, and $\alpha(aba) = \alpha(a)\alpha(b)\alpha(a)$, for all $a, b \in A$. α also maps A^u and A^p bijectively onto themselves and preserves all the lattice operations of the latter. Using these properties it is easy to prove the following result:

Lemma 2. *For a Jordan-$*$-automorphism α of A, $f \in S$, and $a \in A^+$, $K(f \circ \alpha, a) = K(f, \alpha(a))$.*

Proposition 1. *Let $v \in Aff(S_n)$ be bijective and let α be the corresponding Jordan-$*$-automorphisms of A such that $v(f) = f \circ \alpha$, for all $f \in S_n$. The following pair of three conditions are mutually equivalent when A is countably decomposable:*

(i) *v is W-mixing-increasing [resp. W-mixing-decreasing];*
(ii) *v is S-mixing-increasing [resp. S-mixing-decreasing];*
(iii) *$\alpha(p) \prec p$ [resp. $p \prec \alpha(p)$] for every $p \in A^p$.*

It follows that α restricted to the center of A is the identity.

Proof. (ii)$\Rightarrow$(i) is trivial. (i)$\Rightarrow$(iii) follows from (3) and Lemmas 2 and 1. (iii)$\Rightarrow$(ii) follows from (4), Lemma 2 and (3). Let β be the restriction of α to the center Z of A, then β is a $*$-automorphism of Z. Define $\xi \in Aff(S(Z))$ by $\xi(f) = f \circ \beta$. It follows that ξ is S-mixing-increasing [resp. decreasing]. In either case, ξ (and β) must be the identity mappings for in an abelian algebra $f \succ g$ if and only if $f = g$. q.e.d.

We complement and illustrate the proposition by considering countably decomposable factors.

Proposition 2. *If* v *is an affine bijection of the normal state space of a countably decomposable factor of type I, or* II_1, *or III, then* v *is S-mixing-preserving, i.e. S-mixing-increasing and S-mixing-decreasing.*

Proof. We have to show for any Jordan-$*$-automorphism α of the factors in question, and any projection p therein, $\alpha(p) \sim p$. Since A is a factor, α is either a $*$-automorphism or a $*$-antiautomorphism (see e.g. Proposition 3.2.2 of [4]). Let t be a faithful normal trace on A^+. Then $t \circ \alpha$ is again a faithful normal trace on A^+, so that $t \circ \alpha = ct$, for some $c > 0$. In the finite case, we infer from $\alpha(1) = 1$, and normalizing $t(1) = 1$, that $c = 1$. In the semifinite type I case, we normalize t by $t(p) = 1$ for every atomic projection $p \in A^p$; since α maps atoms into atoms, we again conclude that $c = 1$. In the type III case $t = t \circ \alpha$ is trivially true since $t(a) = \infty$, for all $a \neq 0$. Thus, in all the factors in question, $t\alpha(p) = t(p)$ or $\alpha(p) \sim p$. q.e.d.

The above fails to be true for factors of type II_∞. By the work of Connes [8] (see [9] for a review) on the structure of factors of type III_λ $(0 < \lambda < 1)$, there exists for each such λ a type II_∞ factor $N(\lambda)$ which admits a $*$-automorphism with the scaling property $t \circ \alpha = \lambda t$, for any faithful normal semifinite trace t on $N(\lambda)^+$. It follows that $t \circ \alpha^{-1} = \lambda^{-1} t$. For a non-finite, non-zero $p \in N(\lambda)^p$, we have $\alpha(p) \prec p$, and $p \prec \alpha^{-1}(p)$ with $\alpha(p) \not\sim p \not\sim \alpha^{-1}(p)$. The map $f \to f \circ \alpha$ (resp. $f \to f \circ \alpha^{-1}$) provides us with an affine bijections of S or S_n which is mixing-increasing (resp. -decreasing) but not mixing-decreasing (resp. -increasing). The proof of Proposition 2 shows that any affine bijection of the normal state space of a factor of type II_∞ is either mixing-increasing or mixing-decreasing.

The type III case

This is the simplest case since $\succ$ trivializes.

Proposition 3. *Let* A *be a countably decomposable* W^**-algebra of type III. Then*

(i) $K(f, p) = f(c[p])$, *where* $c[p]$ *is the central support of* p.

(ii) *$f > g$ if and only if $g > f$ if and only if f and g coincide on the center of A.*
(iii) *For every $v \in Aff(S_n)$ with corresponding dual map $\alpha: A \to A$, the following five conditions are equivalent:*

(a) *v is W-mixing-increasing;*
(b) *v is W-mixing-decreasing;*
(c) *v is S-mixing-increasing;*
(d) *v is S-mixing-decreasing;*
(e) *α restricted to the center of A is the identity.*

Proof. (ii) and (iii) follow immediately from (i) using Lemma 1, and (3). Due to the countable decomposability assumption, $p \sim c[p]$ (Proposition 2.2.14 of [7]) for every $p \in A^p$. By (4), $K(f, p) = K(f, c[p])$. But $K(f, z) = f(z)$ when z is in the center of A. q.e.d.

Mixing-increase and mixing-decrease

After our discussion of mixing for bijections and type III algebras, we turn to the general situation. Most of the results are direct applications of the results put forward in [2], and are often implicit there. Mixing-increase has been considered previously (see [2], where the terminology is *c-process,* and [10]), but mixing-decrease has apparently not been considered, although it is interesting for the very same reasons.[1])

This section contains four subsections A)–D).

A) *Some notation, and generalities*

If $v \in Aff(S_n)$, we let α be the corresponding dual map and write $\bar{v}(\in Aff(S))$ for the map $\bar{v}(f) = f \circ \alpha$, $f \in S$.

Recall that a state $f \in S$ is said to be singular if given $p \in A^p$ with $p \neq 0$ there exists $q \in A^p$ with $p \geqslant q \neq 0$ and $f(q) = 0$. We write S_s for the singular states of A. Every $f \in S$ admits a unique decomposition as $f = x f_n + (1-x) f_s$, with $0 \leqslant x \leqslant 1$, $f_n \in S_n \cup \{0\}$, and $f_s \in S_s \cup \{0\}$.

Lemma 3. *If $v \in Aff(S_n)$ is W-mixing-increasing or decreasing then α restricted to the center of A is the identity.*

Proof. Let Z be the center of A. For positive $z \in Z$ we have $f(\alpha(z)) = v(f)(z) = f(z)$ for every $f \in S_n$, by (15) of 3.2 of [2]. The assertion follows. q.e.d.

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1) One has an increase of "information", and this is reminiscent of the "measurement process", "reduction of the wave packet", and such situations.

One cannot conclude in general that for W-mixing-increasing or -decreasing $v \in Aff(S_n)$, $\bar{v}(S_s) \subseteq S_s$; simple examples hereto are given in B) & C) below.

Proposition 4. *Let A be an infinite countably decomposable factor and $f \in S$ with decomposition $f = xf_n + (1-x)f_s$ into normal and singular parts. For every $p \in A^p$,*

$$K(f, p) = \begin{cases} xK(f_n, p), & \textit{if } p \textit{ is finite,} \\ 1, & \textit{otherwise.} \end{cases}$$

Proof. If p is not finite, then $p \sim 1$, and $K(f, p) = 1$ by (4). If p is finite and $f \in S_s$, then we show that

$$f(p) = 0. \tag{*}$$

Granted this, we conclude that $K(f, p) = 0$ if p is finite and f is singular, since u^*pu is finite for each $u \in A^u$. But then for general $f \in S$, $K(f, p) = xK(f_n, p)$ for finite p by definition of K. To prove (*), we remark that the linear hull of the finite projections of an infinite factor generate the smallest non-trivial (except in the case of type III) two-sided ideal of this factor. This ideal is annihilated by any singular state since such a state cannot be faithful as is seen directly from the definition. q.e.d.

Corollary 1. *Let A be an infinite countably decomposable factor, and $v \in Aff(S_n)$. If v is W-mixing-increasing then $\bar{v}(S_s) \subseteq S_s$, and v is S-mixing-increasing. If v is W-mixing-decreasing and $\bar{v}(S_s) \subseteq S_s$ then v is S-mixing-decreasing.*

Proof. If v is W-mixing-increasing (resp. -decreasing) and $\bar{v}(S_s) \subseteq S_s$, then using the affinity and the formula for K, we deduce that v is S-mixing-increasing (resp. -decreasing). Assume v is W-mixing-increasing. If $f \in S_s$, and $p \in A^p$ is finite, then $\bar{v}(f)(p) \leqslant K(\bar{v}(f), p) \leqslant K(f, p) = 0$, and this implies that $\bar{v}(f)$ is singular. q.e.d.

B) *Mixing-increase in the finite case*

Let A be finite, with center Z, and unique center-valued trace $T: A \to Z$. Let

$$e_z(f) = \sup \{f(x); 0 \leqslant x \leqslant 1, T(x) \leqslant z\}, \qquad f \in S, \qquad z \in Z^+. \tag{5}$$

Theorem 6–21 of [2] gives a characterization of $>$ in terms of the e-functionals, namely:

$$f > g \quad \text{if and only if} \quad e_z(f) \leqslant e_z(g) \quad \text{for all} \quad z \in Z^+. \tag{6}$$

With this characterization it is simple to prove the following result.

Proposition 5. *Let A be a finite W^*-algebra. The following conditions on*

$v \in Aff(S_n)$ are equivalent:

(i) *v is W-mixing-increasing.*
(ii) *v is S-mixing-increasing.*
(iii) *$v(f) = f$ for every tracial normal state f of A.*

Proof. (ii)$\Rightarrow$(i) is trivial. (i)$\Rightarrow$(iii): if f is tracial then $f(u^* . u) = f(.)$, and W-mixing-increase implies that $f = v(f)$. (iii)$\Rightarrow$(ii): We first show that $T \circ \alpha = T$. We refer to the proof of Theorem 7.11 of [6]. For any positive, normal, linear functional g of Z, $f_g = g \circ T$ is the unique extension of g to a positive linear functional of A, and this extension is tracial and normal. By renormalizing, we conclude from

$$g(T(\alpha(x)) = f_g(\alpha(x)), \qquad x \in A, \tag{*}$$

and (iii) that the L.H.S. of (*) equals $f_g(x) = g(T(x))$. Since g was an arbitrary normal positive linear functional on Z, we conclude that $T \circ \alpha = T$. Then,

$$\begin{aligned} e_z(\bar{v}(f)) &= \sup \{\bar{v}(f)(x) : 0 \leq x \leq 1,\ T(x) \leq z\} \\ &= \sup \{f(\alpha(x)); 0 \leq x \leq 1,\ T(x) \leq z\} \\ &\leq \sup \{f(x) : 0 \leq x \leq 1,\ T(x) \leq z\}, \end{aligned}$$

where the inequality applies because from $0 \leq x \leq 1$, and $T(x) \leq z$ we deduce that $0 \leq \alpha(x) \leq 1$, and $T(\alpha(x)) = T(x) \leq z$, since α is positive. (ii) follows from (6). q.e.d.

Remark. Consider a factor of type II_1, and let t be its finite trace normalized by $t(1) = 1$. Set $v(f) = t$, $f \in S_n$. Then $\alpha(a) = t(a)1$, $\bar{v}(S_s) = \{t\} \nsubseteq S_s$, and v is S-mixing-increasing.

C) *Mixing-decrease in type I factors*

We give here a complete characterization of W-mixing-decrease in factors of type I which appears to be new. The result is obtained by combining the observation ([3]) that pure normal states are minimal w.r.t. the $>$ partial order, with deep results of Størmer ([11]) on purity-preserving positive maps of $B(H)$, the algebra of bounded, linear operators on a complex Hilbert space H.

If f is a pure, normal state of a W^*-algebra A then the support of f is an atom of A^p (and conversely). In a factor of type I, any non-zero projection has an atom below it, and atoms are mutually unitarily equivalent; we infer from (4) that $K(f, p) = 1$, if $p \neq 0$. This implies that $g > f$, for all $g \in S$.

Proposition 6. *Let A be a factor of type* I *identified with $B(H)$ for a complex Hilbert space H of suitable dimension. $v \in Aff(S_n)$ is S-mixing-decreasing if and only if one of the following conditions (which are mutually exclusive unless $A = I_1$)*

holds true:

(i) *There exists an isometry* $v \in A$ *such that*

$$\nu(f) = f(v^* . v), \quad \textit{for all} \quad f \in S_n.$$

(ii) *There exists a conjugate-linear isometry* $v : H \to H$ *such that*

$$\nu(f)(a) = f(v^+ a^* v), \quad \textit{for all} \quad f \in S_n, \quad \textit{all} \quad a \in A,$$

where v^+ *is the conjugate-linear adjoint of* v *given by* $\langle v^+ \psi, \phi \rangle = \langle v\phi, \psi \rangle$ *for all* $\phi, \psi \in H$, $\langle . , . \rangle$ *being the scalar product of* H.

(iii) *There exists a pure normal state* f_0 *of* A *such that*

$$\nu(f) = f_0, \quad \textit{for all} \quad f \in S_n.$$

In cases (i) *and* (ii) ν *is S-mixing-increasing, thus S-mixing-preserving.*

Proof. Denote by E the pure normal states of A. Given $f \in S_n$, there exists (spectral theorem for density operators) a family $\{f_j : j \in J\}$ of pairwise orthogonal elements of E and a family $\{x_j : j \in J\}$ in the half-open interval $(0, 1]$ indexed by a countable set J whose cardinality does not exceed the dimension of H, such that

$$f = \sum_{j \in J} x_j f_j, \qquad \sum_{j \in J} x_j = 1,$$

the first sum being $\|.\|$-convergent. Let d denote the von Neumann-Murray dimension function on A^p normalized by $d(p) = 1$ for all atoms $p \in A^p$. By results of Uhlmann and Wehrl (see [2]),

$$K(f, p) = \sum_{j=1}^{d(p)} x_j^{\neq} \tag{*}$$

where $\{x_j^{\neq} : j = 1, 2, \ldots \text{card}(J)\}$ is the decreasing rearrangement (i.e. $x_{j+1}^{\neq} \leq x_j^{\neq}$) of $\{x_j : j \in J\}$, and it is understood that in the sum in (*), $x_j^{\neq} = 0$ if $j > \text{card}(J)$. If $f \in E$, then by minimality or directly from (*), $\nu(f) \in E$. In the terminology of Størmer ([11]), α is then a positive, normal, linear map of class 1. Translating Lemma 5.4 of [11] applied to α back to ν, we get the three alternatives. In the converse direction, clearly (iii) is S-mixing-decreasing by the minimality of the pure normal states. In case (i), we have

$$\nu(f) = \sum_{j \in J} x_j f_j(v^* . v),$$

and the states $f_j(v^* . v)$ are pure, normal, and pairwise orthogonal since v is an isometry. Thus – the eigenvalues do not change – by (*) $K(\nu(f), p) = K(f, p)$ for all $p \in A^p$, so that from (3) $\nu(f) \succ f$, and $f \succ \nu(f)$. The same conclusion obtains in case (ii), by the same argument applied to the conjugate-linear isometry v. This proves that in cases (i) and (ii) ν is W-mixing-preserving. But in these two cases, α maps compacts into compacts, so that $\bar{\nu}(S_s) \subseteq S_s$ follows since a state is singular if and only if it annihilates the compacts. Applying Corollary 1, when A is infinite we have completed the proof, since in the finite case $S = S_n$. q.e.d.

Remark. On an infinite factor of type I, let $\nu(f) = f_0$, $f \in S_n$, where f_0 is a pure normal state. Then $\alpha(a) = f_0(a)1$, $\bar{\nu}(S_s) = \{f_0\} \nsubseteq S_s$, and ν is S-mixing-decreasing.

We have the perhaps unexpected result: The only strictly mixing-decreasing affine maps of the normal state space of a factor of type I are the projections onto a pure normal state. These are precisely the reductions of the wave-packet postulated in the theory of measurements of the first kind ([12], [13]) of an observable with simple discrete spectrum in quantum mechanics.

D) *Conclusions*

We have obtained some information on mixing-increase/decrease, particularly in countably decomposable factors. Manageable conditions for mixing-increase in the semifinite infinite case, and for mixing-decrease in the type II case are still missing. We comment on other approaches.

When A is a semifinite factor with faithful normal semifinite trace t, the predual of A can be identified with $L^1(A, t)$ (see [5]), and the partial order $>$ on $S_n(A)$ can be studied in this setting (see [14], and references therein). There is a connection ([15], see Theorem 2 of [16]) between the K-functional $K(f, .)$ and the generalized s-numbers μ_s (see [17] for the most recent and general account) of the element x_f of $L^1(A, t)$ associated to f:

$$K(f, p) = \int_0^{t(p)} \mu_s(x_f)\, ds, \qquad p \in A^p.$$

Using this formula, Hiai ([15]) has given an example of a strictly W-mixing-decreasing affine map on the normal state space of the hyperfine factor of type II_1.

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