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The problem of using energy-dependent nucleon–nucleon potentials in nuclear physics

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Abstract. The problem of using energy-dependent nucleon–nucleon potentials in nuclear physics lies in the embedding of such potentials into multinucleon Hilbert spaces. A certain way of embedding, which has been proposed and used in the literature, is simple and parameter-free, but not realistic.

1. Introduction

In a series of papers [1–8] a new nucleon–nucleon potential model has been proposed and used. The new potential model is a phenomenological model with energy-dependent and partial wave dependent nucleon–nucleon potentials. A parameter-free prescription of embedding energy-dependent two-nucleon potentials into multinucleon Hilbert spaces is part of the model.

The new potential model has been used to calculate the triton binding energy [1] and it has been used to investigate the sensitivity of the trinucleon binding energy to the repulsive core range of the nucleon–nucleon interaction [3]. On the basis of the model it has been shown [6] that on-shell contributions of the nucleon–nucleon potential account for -1.18 MeV of the triton binding energy. It has also been shown [8] that a change of ~ 1.25 MeV of the triton binding energy can be obtained from a 0.2 fm variation of the effective range r_{nn} (2.74 to 2.94 fm).

A closer inspection of the new potential model raises objections.

2. The two-nucleon system

The new potential model [1–8] is based on two model assumptions. The first model assumption is concerned with the definition of a class of nucleon–nucleon potentials. It is assumed that two nucleons interact by a potential of the form

$$V(E) = \lambda(E)\bar{V}, \tag{1}$$

where $\bar{V}$ is a common hermitian potential, like a Yamaguchi potential or a Reid potential. The function $\lambda(E)$ is a strength factor which depends on the asymptotic energy of the nucleon–nucleon relative motion. The two-nucleon Schrödinger equation reads

$$(T + V(E) - E)\psi = 0. \tag{2}$$

The authors of Refs. 1–8 justify the energy-dependence of the phenomenological nucleon–nucleon potential by recalling that nucleons are composite particles and by saying that microscopic theories like Feshbach theory, resonating group theory and N-body integral equations yield energy-dependent effective interactions; also relativistic effects are quoted.

As ansatz for a phenomenological potential, $V(E)$ has more flexibility than $\bar{V}$. In some respects this may be an advantage, in other respects it is a disadvantage. It is a disadvantage that the interpretation of the effective range r_0 as 'range of the nuclear force' gets lost. The effective range formula,

$$k \cot \delta = -a^{-1} + \tfrac{1}{2} r_0 k^2, \tag{3}$$

with given nonzero a and r_0, can be fulfilled with a potential $V(E)$ of zero range [6] as well as with a potential of long range.

3. The three-nucleon system

With energy-dependent two-nucleon potentials the three-nucleon Schrödinger equation reads

$$(T + V_{12}(E_{12}) + V_{23}(E_{23}) + V_{31}(E_{31}) - E)\psi = 0. \tag{4}$$

The subsystem energies E_{ij} are operators,

$$E_{ij} = E - \Delta_{ij}. \tag{5}$$

The energy shift operator Δ_{ij} relates the energy of the full system to the energy of the subsystem. Here, the second model assumption is made. It is assumed [2] that Δ_{ij} does not contain any potential and that

$$E_{ij} = E - q_2^2(ij)/(2\mu_2(ij)) \tag{6}$$

is valid. Here, the operator $q_2(ij)$ denotes the second momentum coordinate in a Jacobi coordinate system which has the relative momentum of particles i and j as first coordinate; $\mu_2(ij)$ is the corresponding reduced mass. The following critical remarks are made.

(a) The second model assumption is not in agreement with models which derive effective interactions of composite particles from a microscopic hamiltonian. In Feshbach theory [9], the operator Δ_{ij} is the hamiltonian of the relative motion of the spectator versa the subsystem projected into Q-space. A similar definition of Δ_{ij} arises when channels are formally eliminated from a coupled reaction channels equation [10] or from an orthogonalized coupled channels resonating group equation [11]. In all such microscopic theories the spectator feels an interaction when entering into the volume where all three particles are close together. In fact, the kinetic and potential energies tend to be large and of opposite sign, in the interaction region. Omitting one of them while keeping the other does not seem to be justified.

(b) The new potential model is not compatible with the old (hermitian) potential model. When $\lambda(E)$ is chosen to be equal to unity at all scattering energies, as well as in the vicinity of all bound state energies (see Ref. 2), one expects that three-body observables calculated with V and $\bar{V}$ are equal. It is easy to see why they have to be equal: In the two-nucleon system, V and $\bar{V}$ yield identical spectra. The bound and scattering states form a complete set. From this complete set one can construct a complete Hilbert space like, for instance, the space of harmonic oscillator functions. In this Hilbert space, V and $\bar{V}$ have identical representations. Only these representations are needed to write down the three-nucleon Schrödinger equation in Hilbert space representation. The second model assumption (6) is not needed in this special case. Hence the two Schrödinger equations are identical and have identical solutions.

If one invokes the second model assumption, however, the two Schrödinger equations become different and yield different three-body observables. The difference is *created* by the second model assumption.

4. The multinucleon system

In Ref. 6 it is stated that the new potential model can be applied to four and more-nucleon systems. It is not stated how the second model assumption is extended, in this case. It is not probable that the author intends to include a potential energy part in Δ_{ij} when going over from three to more nucleons. Without a potential energy part in Δ_{ij} the second model assumption for A nucleons reads

$$E_{ij} = E - \sum_{k=2}^{A-1} \vec{q}_k^{\,2}(ij)/(2\mu_2(ij)). \tag{7}$$

Again, $\vec{q}_k(ij)$ and $\mu_k(ij)$ refer to Jacobi coordinate systems which have particles i and j as the first two particles and the remaining particles in arbitrary order.

The Schrödinger equation for A nucleons reads

$$\left(T + \sum_{i>j=1}^{A} V_{ij}(E_{ij}) - E\right)\psi = 0, \tag{8}$$

and now the weak point of the model is seen through a magnifying glass. When going over to ^{208}Pb, for instance, the ground state energy E becomes around -1.600 MeV and the kinetic energy of 206 spectator nucleons becomes around 4.000 MeV, which means that all interactions take place at subsystem energies of minus several GeV. Freezing the energy-dependence of the nucleon–nucleon potential at $E_{ij} = 0$ MeV would certainly be more realistic.

5. Conclusion

The trouble with using energy-dependent nucleon–nucleon potentials in nuclear physics lies in the embedding of such interactions into systems with more then two nucleons. Microscopic theories tell us that for the embedding we need to know interaction potentials of channels which are not present in the two-nucleon system. Despite new work [1–8] the conclusion written by B. H. J. McKellar and C. M. McKay [12] "... we note that a consistent three-body (or many body) theory involving energy-dependent interactions has not yet been achieved" is still valid.

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