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RENORMALIZATION GROUP STUDY OF SCALAR FIELD THEORIES

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An approximate RG equation is derived and studied in scalar quantum field theories in d dimensions. The approximation allows an infinite number of different couplings in the potential, but excludes interactions containing derivatives. The resulting non-linear partial differential equation can be studied by simple means. Both the Gaussian and the non-Gaussian fixed points are described qualitatively correctly by the equation. The RG flows in d = 4 and the problem of defining an "effective" field theory are discussed in detail.

The scalar field theory is regularized in momentum space :
The change of the effective potential $V(\varphi)$ under the RG transformation $\Lambda^{cut} \rightarrow e^{-t}\Lambda^{cut}$ is descrived by the approximate RG equation:

$$\left(1+\frac{\partial f}{\partial \varphi}\right)\frac{\partial f}{\partial t}=\frac{A_d}{2}\frac{\partial^2 f}{\partial \varphi^2}+\left(1+\frac{\partial f}{\partial \varphi}\right)\left[\left(1-\frac{d}{2}\right)\varphi\frac{\partial f}{\partial \varphi}+\left(1+\frac{d}{2}\right)f\right], \qquad (1)$$

where

$$f(\varphi,t)=\frac{\partial}{\partial\varphi}V(\varphi,t),$$

d is the dimension, A_d is a constant.[1] The equation has a non-trivial fixed-point solution corresponding to the usual fenomagnetic phase transition for $2<d<4$. In d=3 the leading and subleading critical indices ν and ω are predicted to be 0.687 and 0. 595 respectively, while $\eta=0$ is implied by the approximation. In d=4 only the Gaussian fixed-point exists ($f^*(\varphi)\equiv 0$) corresponding to a free field theory. The flow speed of the "almost marginal" operator ("φ^4" coupling) in d=4 can be determined both in the perturbative and in the non- perturbative regime.

The approximation and other aspects of the analysis are related to those of Wilson's recursion relation[2]. Eq (1) is most easily derived from the exact integro- differential functional equation of Wegner and Houghton[3].

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