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RENORMALIZATION GROUP STUDY OF SCALAR FIELD THEORIES

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An approximate RG equation is derived and studied in scalar quantum field theories in d dimensions. The approximation allows an infinite number of different couplings in the potential, but excludes interactions containing derivatives. The resulting non-linear partial differential equation can be studied by simple means. Both the Gaussian and the non-Gaussian fixed points are described qualitatively correctly by the equation. The RG flows in $d = 4$ and the problem of defining an "effective" field theory are discussed in detail.

The scalar field theory is regularized in momentum space :
The change of the effective potential $V(\varphi)$ under the RG transformation $\Lambda^{\text{cut}} \rightarrow e^{-t} \Lambda^{\text{cut}}$ is described by the approximate RG equation:

$$\left(1 + \frac{\partial f}{\partial \varphi}\right) \frac{\partial f}{\partial t} = \frac{A_d}{2} \frac{\partial^2 f}{\partial \varphi^2} + \left(1 + \frac{\partial f}{\partial \varphi}\right) \left[\left(1 - \frac{d}{2}\right) \varphi \frac{\partial f}{\partial \varphi} + \left(1 + \frac{d}{2}\right) f \right], \quad (1)$$

where

$$f(\varphi, t) = \frac{\partial}{\partial \varphi} V(\varphi, t),$$

d is the dimension, A_d is a constant. [1] The equation has a non-trivial

fixed-point solution corresponding to the usual ferromagnetic phase transition for $2 < d < 4$. In $d=3$ the leading and subleading critical indices ν and ω are predicted to be 0.687 and 0.595 respectively, while $\eta = 0$ is implied by the approximation. In $d=4$ only the Gaussian fixed-point exists ($f^*(\varphi) \equiv 0$) corresponding to a free field theory. The flow speed of the "almost marginal" operator (" φ^4 " coupling) in $d=4$ can be determined both in the perturbative and in the non-perturbative regime.

The approximation and other aspects of the analysis are related to those of Wilson's recursion relation^[2]. Eq (1) is most easily derived from the exact integro-differential functional equation of Wegner and Houghton^[3].

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