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Polarization formalism for elastic scattering of leptons by spin ½ and spin 1 particles in the one photon exchange approximation

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Abstract. A simple and transparent method is used to obtain the polarization parameters (analyzing powers and spin correlation coefficients) for elastic scattering of a polarized lepton beam by a polarized spin $\frac{1}{2}$ or spin 1 target when the polarizations of the final particles are not detected. The expressions are obtained in the one photon exchange approximation, but the lepton mass is retained throughout and the calculation is made entirely in the laboratory frame.

I. Introduction

The elastic scattering of a polarized lepton beam by a polarized target was studied by Dombey [1] for a spin $\frac{1}{2}$ target and by Gourdin [2, 3] for a spin 1 target. In each case a sum was made over the spin states of the final particles, corresponding to an experiment in which the polarization of neither final particle is detected. The calculations were made in the one photon exchange ($1\gamma E$) approximation, and the results are summarized in the review article by Gourdin [4]. However, they are not given in the form of expressions for the standard polarization parameters and, particularly for a spin 1 target, it is slightly uncertain what these parameters should be in terms of the results of Ref. 4. Moreover, the lepton mass is set equal to zero. While this is a good approximation in most experimental situations, it turns out to be almost as easy to retain the lepton mass throughout the calculation. One can then see which polarization parameters vanish identically in the $1\gamma E$ approximation and which are small because of the smallness of the lepton mass compared with the target mass and the lepton beam energy.

In this paper we present a simple and transparent method for obtaining the analyzing powers and spin correlation coefficients for the elastic scattering of a polarized lepton beam by a polarized spin $\frac{1}{2}$ or spin 1 target, when the polarizations of the final particles are not detected. We work in the $1\gamma E$ approximation, but retain the lepton mass throughout and make the calculation entirely in the laboratory frame. There is no need to make use of the Breit frame.

We find that there is an error in one of the results of Ref. 4 for a spin 1 target. In Section II we derive the necessary kinematical results, write the expression for the differential cross section and give the $1\gamma E$ approximation. We deal with the part of the calculation arising from the lepton vertex in Section III and proceed to derive the polarization parameters for spin $\frac{1}{2}$ and spin 1 targets in Sections IV and V respectively. It is useful to have these results collected together in one place in a consistent notation.

II. Kinematics, cross section and the one photon exchange approximation

In the laboratory system the initial 4-momenta are

$$p_l' = (E', \mathbf{l}'), \qquad p_T' = (M, \mathbf{0}), \tag{2.1}$$

and the final 4-momenta are

$$p_l'' = (E'', \mathbf{l}''), \qquad p_T'' = (\sqrt{M^2 + T^2}, \mathbf{T}), \tag{2.2}$$

where

$$T = |\mathbf{T}|, \qquad E'^2 = m_l^2 + l'^2, \qquad l' = |\mathbf{l}'|,$$

and similarly for doubly primed quantities. The lepton and target masses are m_l and M respectively, and l and T refer to beam and target quantities. We define the 4-momentum transfer q as

$$q = p_l' - p_l'' = p_T'' - p_T'. \tag{2.3}$$

The angle of scattering of the lepton in the laboratory frame is denoted by θ, while θ_T denotes the angle between the outgoing target particle and the beam direction. It is convenient to use two frames of reference in the calculation. In the (xyz) frame the 3-momenta of the particles are

$$\mathbf{l}' = (0, 0, l'), \qquad \mathbf{l}'' = (l'' \sin\theta, 0, l'' \cos\theta), \quad \mathbf{T} = (-T \sin\theta_T, 0, \cos\theta_T). \tag{2.4}$$

In the (123) frame on the other hand we have the 3-momenta

$$\mathbf{l}' = (l' \sin\theta_T, 0, l' \cos\theta_T), \qquad \mathbf{T} = (0, 0, T). \tag{2.5}$$

The y-axis and the 2-axis coincide; this axis is perpendicular to the scattering plane, in the direction of $\mathbf{l}' \times \mathbf{l}''$ or $\mathbf{T} \times \mathbf{l}'$. The z-axis is along the beam direction, while the 3-axis is along the direction of the outgoing target particle. The x-axis and the 1-axis are then defined by the requirement that the triads be right handed.

As well as θ it is convenient to use another second variable η to characterize the scattering. It is defined by

$$-q^2 = 4M^2\eta. \tag{2.6}$$

It follows from (2.1)–(2.3) that

$$4M^2\eta = 2(E'E'' - l'l'' \cos\theta - m_l^2)$$
$$= 2M(\sqrt{M^2+T^2} - M) = 2M(E' - E''). \tag{2.7}$$

The equality in (2.7) leads to a quadratic equation for l'', whose solution is

$$\frac{l''}{l'} = \frac{(ME' + m_l^2)\cos\theta + (E'+M)(M^2 - m_l^2 \sin^2\theta)^{1/2}}{(E'+M)^2 - l'^2\cos^2\theta}. \tag{2.8}$$

It also follows from (2.7) that

$$T^2 = 4M^2\eta(1+\eta), \qquad E'' = E' - 2M\eta,$$
$$\mathbf{l}'\cdot\mathbf{l}'' = l'^2 - 2M(E'+M)\eta,$$
$$\mathbf{l}'\cdot\mathbf{T} = l'T\cos\theta_T = \mathbf{l}'\cdot(\mathbf{l}'-\mathbf{l}'') = 2M(E'+M)\eta. \tag{2.9}$$

To obtain an exact expression for η in terms of E' and θ, we use

$$\eta = \frac{l'^2 - l'l''\cos\theta}{2M(E'+M)} = \frac{l'^2(E'\sin^2\theta + M - \cos\theta(M^2 - m_l^2\sin^2\theta)^{1/2})}{2M[(E'+M)^2 - l'^2\cos^2\theta]}. \tag{2.10}$$

Note that η increases monotonically as θ increases from 0 to π, and that

$$\eta(\pi) = l'^2/(2E'M + M^2 + m_l^2).$$

Since $\mathbf{l}'' = \mathbf{l}' - \mathbf{T}$, it follows that

$$\mathbf{T}\times\mathbf{l}' = \mathbf{T}\times\mathbf{l}'' = \mathbf{l}'\times\mathbf{l}''. \tag{2.11}$$

Further,

$$\mathbf{T}\times\mathbf{l}' = l'T\sin\theta_T(0, 1, 0) \tag{2.12}$$

in either frame of reference and, from (2.9),

$$(l'T\sin\theta_T)^2 = 4M^2\eta[(E'^2 - 2ME'\eta - M^2\eta) - m_l^2(1+\eta)]. \tag{2.13}$$

To write the differential cross section we need the expression for phase space in the laboratory system. The differential of Lorentz invariant phase space, expressed in laboratory system variables, is

$$\text{dLips} = \frac{l''^2\, d\Omega_{\text{LAB}}}{4(l''(E'+M) - E''l'\cos\theta)}. \tag{2.14}$$

Using (2.8) and the further result

$$E'' = \frac{(E'+M)(E'M + m_l^2) + l'^2\cos\theta(M^2 - m_l^2\sin^2\theta)^{1/2}}{(E'+M)^2 - l'^2\cos^2\theta},$$

we find that

$$l''(E'+M) - E''l'\cos\theta = l'(M^2 - m_l^2\sin^2\theta)^{1/2}.$$

Thus, from (2.14),

$$\mathrm{dLips} = \left(\frac{l''}{l'}\right)^2 \frac{l'\, d\Omega_{\mathrm{LAB}}}{4(M^2 - m_l^2 \sin^2\theta)^{1/2}}. \tag{2.15}$$

Since the differential of cross section is

$$d\sigma = \frac{\mathrm{dLips}}{[(p_l' \cdot p_T')^2 - m_l^2 M^2]^{1/2}} \cdot \frac{|T_{fi}|^2}{16\pi^2},$$

it follows from (2.15) that

$$\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{LAB}} = \left(\frac{l''}{l'}\right)^2 \frac{|T_{fi}|^2}{64\pi^2 M(M^2 - m_l^2 \sin^2\theta)^{1/2}}. \tag{2.16}$$

The next step is to write T_{fi} in the $1\gamma E$ approximation. The standard result is

$$T_{fi} \approx \frac{4\pi\alpha g^{\mu\nu}}{-q^2}(2\pi)^6 \langle \mathbf{l}''|\, J_\mu^l(0)\, |\mathbf{l}'\rangle \langle \mathbf{T}|\, J_\nu^T(0)\, |\mathbf{0}\rangle, \tag{2.17}$$

and this result is substituted into (2.16). So far we have suppressed the spin indices of all the particles. We are assuming that the polarizations of the final particles are not detected, and therefore sum over their spins. But we consider a polarized beam and a polarized target, described by density matrices ρ^l and ρ^T. We can then no longer simply write $|T_{fi}|^2$ in (2.16). Instead, using the $1\gamma E$ approximation of (2.17), we have

$$\begin{aligned}\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{LAB}} &= \frac{\alpha^2 F}{16 M^2 l'^4 (1-\cos\theta)^2} g^{\nu\sigma} g^{\mu\rho} \\ &\times (2\pi)^6 \sum_{r's';\,r''} \overline{\langle \mathbf{l}''r''|\, J_\sigma^l(0)\, |\mathbf{l}'s'\rangle} \langle \mathbf{l}''r''|\, J_\rho^l(0)\, |\mathbf{l}'r'\rangle \rho^l_{r's'} \\ &\times (2\pi)^6 \sum_{\lambda'\mu';\,\lambda''} \overline{\langle \mathbf{T}\lambda''|\, J_\nu^T(0)\, |\mathbf{0}\mu'\rangle} \langle \mathbf{T}\lambda''|\, J_\mu^T(0)\, |\mathbf{0}\lambda'\rangle \rho^T_{\lambda'\mu'},\end{aligned} \tag{2.18}$$

where

$$F = \left(\frac{l''}{l'\eta}\right)^2 \frac{l'^4(1-\cos\theta)^2}{4M^3(M^2 - m_l^2\sin^2\theta)^{1/2}}.$$

Using (2.8) and (2.10), one finds after some manipulation that

$$F = \frac{M}{(M^2 - m_l^2 \sin^2\theta)^{1/2}}\left[1 - \frac{m_l^2(1-\cos\theta)}{M(M + (M^2 - m_l^2\sin^2\theta)^{1/2})}\right] \approx 1, \tag{2.19}$$

provided that $m_l^2 \ll M^2$. We shall continue to include the factor F (which depends only on θ), though even for a muon beam and a proton target the approximation (2.19) will be very good.

III. The lepton vertex

For a spin $\frac{1}{2}$ point particle the matrix element of the current is

$$(2\pi)^3\langle \mathbf{l}''r''|\, J^l_\mu(0)\,|\mathbf{l}'r'\rangle = 2m_l\bar{u}^{(r'')}(\mathbf{l}'')\gamma_\mu u^{(r')}(\mathbf{l}'), \tag{3.1}$$

where the conventions of Bjorken and Drell [5] are used throughout. Using (3.1) we then evaluate the quantity

$$\begin{aligned} &g^{\nu\sigma}g^{\mu\rho}(2\pi)^6\sum_{r''}\overline{\langle \mathbf{l}''r''|\, J^l_\sigma(0)\,|\mathbf{l}'s'\rangle}\,\langle \mathbf{l}''r''|\, J^l_\rho(0)\,|\mathbf{l}'r'\rangle \\ &\quad= (E'+m_l)^{-1}\chi^{(s')*}[(E'+m_l)\mathbb{1}-\boldsymbol{\gamma}\cdot\mathbf{l}']\gamma^\nu(\gamma_0E''-\boldsymbol{\gamma}\cdot\mathbf{l}''+m_l\mathbb{1}) \\ &\qquad\times\gamma^\mu[(E'+m_l)\mathbb{1}-\boldsymbol{\gamma}\cdot\mathbf{l}']\chi^{(r')}, \end{aligned}$$

where χ denotes a rest spinor satisfying $\gamma_0\chi=\chi$. Using the fact that an odd number of spatial γ-matrices gives zero when placed between rest spinors, we find for this expression the following results:

$$\begin{aligned} &\nu=0, \quad \mu=0 \quad && 2(E'E''+\mathbf{l}'\cdot\mathbf{l}''+m_l^2);\\ &\nu=0, \quad \mu=i && 2(E''l'_i+E'l''_i)+(E'+m_l)i(\boldsymbol{\sigma}\times\mathbf{l}'')_i\\ & && -(E'+m_l)^{-1}(2m_l(E'+m_l)+\mathbf{l}'\cdot\mathbf{l}'')i(\boldsymbol{\sigma}\times\mathbf{l}')_i\\ & && +(E'+m_l)^{-1}(i\boldsymbol{\sigma}\cdot(\mathbf{l}'\times\mathbf{l}'')l'_i+i(\mathbf{l}'\times\mathbf{l}'')_i\boldsymbol{\sigma}\cdot\mathbf{l}');\\ &\nu=j, \quad \mu=0 && 2(E''l'_j+E'l''_j)-(E'+m_l)i(\boldsymbol{\sigma}\times\mathbf{l}'')_j\\ & && +(E'+m_l)^{-1}(2m_l(E'+m_l)+\mathbf{l}'\cdot\mathbf{l}'')i(\boldsymbol{\sigma}\times\mathbf{l}')_j\\ & && -(E'+m_l)^{-1}(i\boldsymbol{\sigma}\cdot(\mathbf{l}'\times\mathbf{l}'')l'_j+i(\mathbf{l}'\times\mathbf{l}'')_j\boldsymbol{\sigma}\cdot\mathbf{l}');\\ &\nu=j, \quad \mu=i && 2M(E'-E'')\delta_{ij}+2(l'_il''_j+l'_jl''_i)+2m_l(E'-E'')i\varepsilon_{ijk}\sigma_k\\ & && +2i\varepsilon_{ijk}(l''_k-(E'+m_l)^{-1}(E''+m_l)l'_k)\boldsymbol{\sigma}\cdot\mathbf{l}'. \end{aligned}$$

It is understood that these expressions stand between the rest spinors. We have put the spatial index below when 3-vectors are involved. The 3-vector $\boldsymbol{\sigma}$ is defined by

$$\sigma_1=\sigma^{23}, \qquad \sigma_2=\sigma^{31}, \qquad \sigma_3=\sigma^{12},$$

where

$$\sigma^{\mu\nu}=\tfrac{1}{2}i(\gamma^\mu\gamma^\nu-\gamma^\nu\gamma^\mu). \tag{3.2}$$

Then

$$-\gamma^i\gamma^j=\sigma_i\sigma_j=\delta_{ij}\mathbb{1}+i\varepsilon_{ijk}\sigma_k,$$

and this relation is used in deriving the above results. Note that the σ_i are 4×4 matrices; there is no need to specialize to the familiar representation involving Pauli matrices.

The lepton density matrix is

$$\rho^l_{r's'} = \chi^{(r')*}\tfrac{1}{4}(\quad + \boldsymbol{\sigma}\cdot\mathbf{p}^l)\chi^{(s')},$$

where the polarization vector $\mathbf{p}^l$ satisfies $0 \le |\mathbf{p}^l| \le 1$. The sum over r', s' means that we multiply the results in the previous paragraph by $\frac{1}{4}(\mathbb{1} + \boldsymbol{\sigma}\cdot\mathbf{p}^l)$, take the trace and use $\operatorname{tr}\mathbb{1} = 4$, $\operatorname{tr}\sigma_i = 0$. To write the final result in its most convenient form, we use equations (2.9) and (2.11)–(2.13) and introduce the vector

$$\tilde{\mathbf{l}}' = \mathbf{l}' - (\mathbf{l}'\cdot\mathbf{T})\mathbf{T}/T^2 = (l'\sin\theta_T, 0, 0) \quad \text{in the (123) frame.}$$

A substantial amount of manipulation then leads to the following results:

$$\nu = 0, \quad \mu = 0 \qquad 4(E'^2 - 2ME'\eta - M^2\eta);$$

$$\nu = 0, \quad \mu = i \qquad 4(E' - M\eta)\tilde{l}'_i + \frac{2(E'^2 - 2ME'\eta - M^2\eta)T_i}{M(1+\eta)} + iV_i;$$

$$\nu = j, \quad \mu = 0 \qquad 4(E' - M\eta)\tilde{l}'_j + \frac{2(E'^2 - 2ME'\eta - M^2\eta)T_j}{M(1+\eta)} - iV_j;$$

$$\nu = j, \quad \mu = i \qquad 4M^2\eta\delta_{ij} + 4\tilde{l}'_i\tilde{l}'_j + \frac{2(E' - M\eta)(\tilde{l}'_iT_j + T_i\tilde{l}'_j)}{M(1+\eta)}$$
$$+ \frac{[(E'^2 - 2ME'\eta - M^2\eta) - M^2(1+\eta)]T_iT_j}{M^2(1+\eta)^2} + i\varepsilon_{ijk}W_k.$$

In the (123) frame the vectors $\mathbf{V}$ and $\mathbf{W}$ are given by

$$\mathbf{V} = 2T(-m_l p^l_y,\ m_l\cos\theta_T p^l_x + E'\sin\theta_T p^l_z,\ 0), \tag{3.3}$$

$$\mathbf{W} = 4M\eta(m_l\cos\theta_T p^l_x + E'\sin\theta_T p^l_z,\ m_l p^l_y,$$
$$-m_l\sin\theta_T p^l_x - 2M\{E'(E' - M\eta) - m_l^2(1+\eta)\}(l'T)^{-1}p^l_z). \tag{3.4}$$

Though the components of $\mathbf{V}$ and $\mathbf{W}$ are conveniently written in the (123) frame, the components of the beam polarization vector $\mathbf{p}^l$ in the (xyz) frame have been used in (3.3) and (3.4), since experimentally one uses a longitudinally or transversely polarized beam.

IV. Results for a spin $\frac{1}{2}$ target

For a spin $\frac{1}{2}$ target there are two form factors F_1, F_2 defined by the equation

$$(2\pi)^3\langle\mathbf{T}\lambda''|\, J^T_\mu(0)\, |\mathbf{0}\lambda'\rangle = \bar{u}^{(\lambda'')}(\mathbf{T})[2MF_1\gamma_\mu + i\kappa F_2\sigma_{\mu\nu}(p''_T - p'_T)^\nu]u^{(\lambda')}(\mathbf{0}). \tag{4.1}$$

Using (3.2) and the Dirac equation for both the initial and the final spinor, we may replace the square bracket in (4.1) by

$$2MG_M\gamma_\mu - (G_M - G_C)(p''_T + p'_T)_\mu/(1+\eta),$$

where

$$G_C = F_1 - \eta\kappa F_2, \qquad G_M = F_1 + \kappa F_2.$$

We have used G_C rather than the customary G_E to denote the *charge* form factor; this then agrees with the usual notation for a spin 1 target. The quantity κ is the anomalous static magnetic moment of the spin $\frac{1}{2}$ target particle in units $(2M)^{-1}$. Hermiticity of the electromagnetic current implies that G_C and G_M are real functions of η in the spacelike region $\eta > 0$, and $G_C(0) = 1$, $G_M(0) = 1 + \kappa$.

There is a further simplification. Since, using the Dirac equation,

$$\bar{u}(\mathbf{l}'')\gamma\cdot(p''_l - p'_l)u(\mathbf{l}') = 0,$$

it follows from (2.3) that in the calculation we can replace p''_T by p'_T. The square bracket in (4.1) can then be replaced by

$$2MG_M\gamma_\mu - 2(G_M - G_C)(p'_T)_\mu/(1+\eta).$$

It follows that

$$\begin{aligned}(2\pi)^6 \sum_{\mu''} &\overline{\langle \mathbf{T}\lambda''|\, J^T_\nu(0)\,|\mathbf{0}\mu'\rangle}\, \langle \mathbf{T}\lambda''|\, J^T_\mu(0)\,|\mathbf{0}\lambda'\rangle \\ &\doteq 2M\chi^{(\mu')*}\left[G_M\gamma_\nu - \frac{(G_M - G_C)(p'_T)_\nu}{M(1+\eta)}\right](\gamma_0\sqrt{M^2+T^2} - \boldsymbol{\gamma}\cdot\mathbf{T} + M\mathbb{1}) \\ &\times\left[G_M\gamma_\mu - \frac{(G_M - G_C)(p'_T)_\mu}{M(1+\eta)}\right]\chi^{(\lambda')},\end{aligned} \tag{4.2}$$

where $\doteq$ implies that equality in (4.2) holds for the purpose of calculating the right side of (2.18). In evaluating the right side of (4.2) we need to take account of the change of sign for covariant indices when ρ, σ take spatial values.

As in Section III the density matrix is

$$\rho^T_{\lambda'\mu'} = \chi^{(\lambda')*}\tfrac{1}{4}(\mathbb{1} + \boldsymbol{\sigma}\cdot\mathbf{p}^T)\chi^{(\mu')},$$

with $0 \le |\mathbf{p}^T| \le 1$. Evaluating the right side of (4.2), then including the density matrix and summing on λ', μ', we arrive at the following results:

$\nu = 0,$	$\mu = 0$	$4M^2(G_C + \eta G_M)^2/(1+\eta);$
$\nu = 0,$	$\mu = i$	$2MG_M(G_C + \eta G_M)(1+\eta)^{-1}[-T_i + i(\mathbf{T}\times\mathbf{p}^T)_i];$
$\nu = j,$	$\mu = 0$	$2MG_M(G_C + \eta G_M)(1+\eta)^{-1}[-T_j - i(\mathbf{T}\times\mathbf{p}^T)_j];$
$\nu = j,$	$\mu = i$	$4M^2G_M^2\eta(\delta_{ij} - i\varepsilon_{ijk}p^T_k).$

Now inserting these results and those given at the end of Section III into the right side of (2.18), we have, using (2.9) and (2.13),

$$\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{LAB}} = \frac{\alpha^2 F}{l'^4(1-\cos\theta)^2}$$
$$\times \left[\frac{(E'^2 - 2ME'\eta - M^2\eta)}{1+\eta}\{(G_C+\eta G_M)^2 - 2G_M(G_C+\eta G_M)\eta\}\right.$$
$$+ G_M^2\eta(3M^2\eta + (E'^2 - 2ME'\eta - M^2\eta) - M^2\eta - m_l^2)$$
$$\left. + \frac{G_C G_M}{4M(1+\eta)}\mathbf{p}^T\cdot(\mathbf{T}\times\mathbf{V}) + G_M^2\eta\mathbf{p}^T\cdot\left(\frac{\mathbf{T}\times\mathbf{V}}{4M(1+\eta)} + \tfrac{1}{2}\mathbf{W}\right)\right]. \quad (4.3)$$

From (3.3) and (3.4) it follows that, in the (123) frame,

$$\frac{\mathbf{T}\times\mathbf{V}}{4M(1+\eta)} = 2M\eta(-m_l\cos\theta_T p_x^l - E'\sin\theta_T p_z^l, -m_l p_y^l, 0), \quad (4.4)$$

$$\frac{\mathbf{T}\times\mathbf{V}}{4M(1+\eta)} + \tfrac{1}{2}\mathbf{W} = 2M\eta(0, 0, -m_l\sin\theta_T p_x^l$$
$$- 2M\{E'(E'-M\eta) - m_l^2(1+\eta)\}(l'T)^{-1}p_z^l). \quad (4.5)$$

The expression for $(d\sigma/d\Omega)_{\mathrm{LAB}}$ in terms of the polarization parameters is (see, for example, Ohlsen [6])

$$(d\sigma/d\Omega)_{\mathrm{LAB}} = I_0(1 + A_y^l p_y^l + A_2^T p_2^T + C_{x1}p_x^l p_1^T + C_{x3}p_x^l p_3^T$$
$$+ C_{y2}p_y^l p_2^T + C_{z1}p_z^l p_1^T + C_{z3}p_z^l p_3^T). \quad (4.6)$$

From equations (4.3)–(4.6) we can read out the required results. They are

$$I_0 = \frac{\alpha^2 F}{l'^2(1-\cos\theta)^2}\left[\frac{(E'^2 - 2ME'\eta - M^2\eta)}{l'^2(1+\eta)}(G_C^2 + \eta G_M^2) + \frac{(2M^2\eta - m_l^2)}{l'^2}\eta G_M^2\right],$$

$$I_0 A_y^l = I_0 A_2^T = 0,$$

$$I_0 C_{x1} = -\frac{\alpha^2 F}{l'^2(1-\cos\theta)^2}\cdot\frac{2G_C G_M(E'+M)Mm_l\eta^{3/2}}{l'^3(1+\eta)^{1/2}},$$

$$I_0 C_{x3} = -\frac{\alpha^2 F}{l'^2(1-\cos\theta)^2}\cdot\frac{2G_M^2[(E'^2 - 2ME'\eta - M^2\eta) - m_l^2(1+\eta)]^{1/2}Mm_l\eta^2}{l'^3(1+\eta)^{1/2}},$$

$$I_0 C_{y2} = -\frac{\alpha^2 F}{l'^2(1-\cos\theta)^2}\cdot\frac{2G_C G_M Mm_l\eta}{l'^2},$$

$$I_0 C_{z1} = -\frac{\alpha^2 F}{l'^2(1-\cos\theta)^2}\cdot\frac{2G_C G_M[(E'^2 - 2ME'\eta - M^2\eta) - m_l^2(1+\eta)]^{1/2}ME'\eta}{l'^3(1+\eta)^{1/2}},$$

$$I_0 C_{z3} = -\frac{\alpha^2 F}{l'^2(1-\cos\theta)^2}\cdot\frac{2G_M^2[E'(E'-M\eta) - m_l^2(1+\eta)]M\eta^{3/2}}{l'^3(1+\eta)^{1/2}}.$$

The expression for I_0 is the exact form of the familiar Rosenbluth formula.

The two analyzing powers are identically zero in the $1\gamma E$ approximation and there are three spin correlation coefficients which are proportional to the lepton mass and will normally be small. The expressions for C_{z1} and C_{z3} are the exact forms of the approximate expressions given by Gourdin [4]. Our results have been written in terms of the components of $\mathbf{p}^T$ in the (123) frame. However, experimentally it is easiest to polarize the target in the beam direction. One therefore wants to write spin correlation coefficients in which the components of $\mathbf{p}^T$ in the (xyz) frame are used. The results are

$$\begin{aligned} C_{xx} &= C_{x1}\cos\theta_T - C_{x3}\sin\theta_T, \qquad & C_{xz} &= C_{x1}\sin\theta_T + C_{x3}\cos\theta_T, \\ C_{zx} &= C_{z1}\cos\theta_T - C_{z3}\sin\theta_T, \qquad & C_{zz} &= C_{z1}\sin\theta_T + C_{z3}\sin\theta_T, \end{aligned} \tag{4.7}$$

where, from (2.9) and (2.13),

$$\cos\theta_T = \frac{(E'+M)\eta^{1/2}}{l'(1+\eta)^{1/2}}, \qquad \sin\theta_T = \frac{[(E'^2 - 2ME'\eta - M^2\eta) - m_l^2(1+\eta)]^{1/2}}{l'(1+\eta)^{1/2}}. \tag{4.8}$$

Besides I_0, the easiest quantities to measure experimentally are C_{xz} and C_{zz}. C_{xz} would be too small to measure in the case of an electron beam, but for a muon beam of energy up to a few hundred MeV, C_{xz} and C_{zz} would be of comparable magnitude.

V. Results for a spin 1 target

There are now three form factors; in the notation of Arnold, Carlson and Gross [7],

$$\begin{aligned} &(2\pi)^3\langle \mathbf{T}\lambda''|\, J_\mu^T(0)\, |\mathbf{0}\lambda'\rangle \\ &\quad = -[G_1\overline{\varepsilon(\lambda'')}\cdot\varepsilon(\lambda') - G_3\overline{\varepsilon(\lambda'')}\cdot q\varepsilon(\lambda')\cdot q/2M^2](p_T''+p_T')_\mu \\ &\qquad + G_2(\overline{\varepsilon(\lambda'')}_\mu\varepsilon(\lambda')\cdot q - \varepsilon(\lambda')_\mu\overline{\varepsilon(\lambda'')}\cdot q), \end{aligned} \tag{5.1}$$

where ε is the polarization 4-vector of the spin 1 target, so that

$$\varepsilon(\lambda')\cdot p_T' = \varepsilon(\lambda'')\cdot p_T'' = 0.$$

Hermiticity of the current ensures that the form factors G_1, G_2, G_3 are real functions of η in the spacelike region $\eta > 0$. The charge, electric quadrupole and magnetic dipole form factors are introduced via the equations

$$\begin{aligned} G_C &= (1+\tfrac{2}{3}\eta)G_1 - \tfrac{2}{3}\eta G_2 + \tfrac{2}{3}\eta(1+\eta)G_3, \\ G_Q &= G_1 - G_2 + (1+\eta)G_3, \qquad G_M = G_2. \end{aligned} \tag{5.2}$$

Then $G_C(0) = 1$, $G_Q(0) = Q$ (the static quadrupole moment of the target particle in units M^{-2}) and $G_M(0) = \mu$ (the static magnetic dipole moment in units $(2M)^{-1}$).

In the calculation we may use the same trick as for the spin $\frac{1}{2}$ case and replace p_T'' by p_T'. To perform the sum over λ'' in (2.18), one uses

$$\sum_{\lambda''} \varepsilon(\lambda'')_\rho \overline{\varepsilon(\lambda'')}_\sigma = -g_{\rho\sigma} + (p_T'')_\rho (p_T'')_\sigma / M^2.$$

Further, since $\varepsilon(\lambda')$ is the polarization vector for a particle at rest,

$$\varepsilon(\lambda') \cdot q = \varepsilon(\lambda') \cdot p_T'' = -\boldsymbol{\varepsilon}(\lambda') \cdot \mathbf{T}.$$

One also needs the relation

$$q \cdot p_T'' = \tfrac{1}{2} q^2 = -2M^2 \eta.$$

A tedious but straightforward calculation using (5.1) and (5.2) shows that

$$\begin{aligned}
&(2\pi)^6 \sum_{\lambda''} \overline{\langle \mathbf{T}\lambda'' | J_\nu^T(0) | \mathbf{0}\mu' \rangle} \langle \mathbf{T}\lambda'' | J_\mu^T(0) | \mathbf{0}\lambda' \rangle \\
&\doteq 4(p_T')_\nu (p_T')_\mu [(G_C - \tfrac{2}{3}\eta G_Q)^2 \boldsymbol{\varepsilon}(\lambda') \cdot \overline{\boldsymbol{\varepsilon}(\mu')} \\
&\quad + \{(1+\eta)^{-1} G_Q (G_C + \tfrac{1}{3}\eta G_Q + \eta G_M) + \tfrac{1}{4} G_M^2\} \boldsymbol{\varepsilon}(\lambda') \cdot \mathbf{T} \overline{\boldsymbol{\varepsilon}(\mu')} \cdot \mathbf{T} / M^2] \\
&\quad + 4M^2 \eta (1+\eta) G_M^2 \overline{\varepsilon(\mu')}_\nu \varepsilon(\lambda')_\mu - G_M^2 \overline{\boldsymbol{\varepsilon}(\mu')} \cdot \mathbf{T} \boldsymbol{\varepsilon}(\lambda') \cdot \mathbf{T} g_{\nu\mu} \\
&\quad - 2G_M (G_C - \tfrac{2}{3}\eta G_Q) [\boldsymbol{\varepsilon}(\lambda') \cdot \mathbf{T} (p_T')_\nu \overline{\varepsilon(\mu')}_\mu + \overline{\boldsymbol{\varepsilon}(\mu')} \cdot \mathbf{T} \varepsilon(\lambda')_\nu (p_T')_\mu] \\
&\quad + 2G_M (G_C + \tfrac{4}{3}\eta G_Q + \eta G_M) [\overline{\boldsymbol{\varepsilon}(\mu')} \cdot \mathbf{T} (p_T')_\nu \varepsilon(\lambda')_\mu + \boldsymbol{\varepsilon}(\lambda') \cdot \mathbf{T} \overline{\varepsilon(\mu')}_\nu (p_T')_\mu].
\end{aligned} \tag{5.3}$$

From (5.3) one can read out immediately the expressions for the four choices of (ν, μ).

Now combining (5.3) with the results for the lepton vertex given at the end of Section III, we obtain

$$\begin{aligned}
&(2\pi)^6 \sum_{r's';\, r''} \overline{\langle \mathbf{l}''r'' | J_\sigma^l(0) | \mathbf{l}'s' \rangle} \langle \mathbf{l}''r'' | J_\rho^l(0) | \mathbf{l}'r' \rangle \rho_{r's'}^l g^{\nu\sigma} g^{\mu\rho} \\
&\quad \times (2\pi)^6 \sum_{\lambda''} \overline{\langle \mathbf{T}\lambda'' | J_\nu^T(0) | \mathbf{0}\mu' \rangle} \langle \mathbf{T}\lambda'' | J_\mu^T(0) | \mathbf{0}\lambda' \rangle \\
&= 16M^2 \overline{\boldsymbol{\varepsilon}(\mu')} \cdot \boldsymbol{\varepsilon}(\lambda') [(E'^2 - 2ME'\eta - M^2\eta)(G_C - \tfrac{2}{3}\eta G_Q)^2 + M^2 \eta^2 (1+\eta) G_M^2] \\
&\quad + 4\overline{\boldsymbol{\varepsilon}(\mu')} \cdot \mathbf{T} \boldsymbol{\varepsilon}(\lambda') \cdot \mathbf{T} [(1+\eta)^{-1} (E'^2 - 2ME'\eta - M^2\eta) \\
&\quad \times \{4G_Q(G_C + \tfrac{1}{3}\eta G_Q) + G_M^2\} + (M^2\eta - m_l^2) G_M^2] \\
&\quad - 16M(E' - M\eta)\eta G_Q G_M (\overline{\boldsymbol{\varepsilon}(\mu')} \cdot \tilde{\mathbf{l}}' \boldsymbol{\varepsilon}(\lambda') \cdot \mathbf{T} + \overline{\boldsymbol{\varepsilon}(\mu')} \cdot \mathbf{T} \boldsymbol{\varepsilon}(\lambda') \cdot \tilde{\mathbf{l}}') \\
&\quad + 4M(G_C + \tfrac{1}{3}\eta G_Q + \tfrac{1}{2}\eta G_M) G_M i (\boldsymbol{\varepsilon}(\lambda') \cdot \mathbf{T} \overline{\boldsymbol{\varepsilon}(\mu')} \cdot \mathbf{V} - \boldsymbol{\varepsilon}(\lambda') \cdot \mathbf{V} \overline{\boldsymbol{\varepsilon}(\mu')} \cdot \mathbf{T}) \\
&\quad + 16M^2 \eta (1+\eta) G_M^2 \overline{\boldsymbol{\varepsilon}(\mu')} \cdot \tilde{\mathbf{l}}' \boldsymbol{\varepsilon}(\lambda') \cdot \tilde{\mathbf{l}}' + 4M^2 \eta (1+\eta) G_M^2 i \varepsilon_{ijk} \varepsilon(\lambda')_i \overline{\varepsilon(\mu')}_j W_k,
\end{aligned} \tag{5.4}$$

where $\mathbf{V}$, $\mathbf{W}$ are given by (3.3), (3.4).

We now introduce the density matrix for a spin 1 particle using Cartesian tensor operators for the tensor polarization (again see Ohlsen [6]). Then

$$\rho_{\lambda'\mu'}^T = \varepsilon(\lambda')^* \rho^T \varepsilon(\mu'),$$

where ε is now treated as a 3×1 matrix and the 3×3 matrix ρ^T is

$$\rho^T = \tfrac{1}{3}\mathbb{1}_3 + \tfrac{1}{2}(p_1^T s_1 + p_2^T s_2 + p_3^T s_3) + \tfrac{2}{9}(p_{12}^T s_{12} + p_{23}^T s_{23} + p_{31}^T s_{31}) + \tfrac{1}{18}(p_{11}^T - p_{22}^T)(s_{11} - s_{22}) + \tfrac{1}{6}p_{33}^T s_{33},$$

with

$$s_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{bmatrix}, \qquad s_2 = \begin{bmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & 0 \end{bmatrix}, \qquad s_3 = \begin{bmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$s_{ij} = \tfrac{3}{2}(s_i s_j + s_j s_i) - 2\mathbb{1}_3,$$

so that

$$s_{12} = -\frac{3}{2}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad s_{23} = -\frac{3}{2}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, \qquad s_{31} = -\frac{3}{2}\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix},$$

$$s_{11} - s_{22} = 3\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad s_{33} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix}.$$

The restrictions arising from the requirement that all the eigenvalues of ρ^T be nonnegative are complicated, and need not be discussed here. We have now to take the expression in (5.4), which may be put in the matrix form $\varepsilon(\mu')^* M \varepsilon(\lambda')$, multiply by $\rho^T_{\lambda'\mu'}$ and sum over λ', μ'; the result is $\mathrm{tr}(M\rho^T)$. The terms on the right side of (5.4), when combined with the various matrices appearing in ρ^T, then give the following contributions:

$\overline{\boldsymbol{\varepsilon}(\mu')} \cdot \boldsymbol{\varepsilon}(\lambda')$ $\quad \mathbb{1}_3 \to 3, \quad s_i \to 0, \quad s_{ij} \to 0;$

$\boldsymbol{\varepsilon}(\lambda') \cdot \mathbf{v}\overline{\boldsymbol{\varepsilon}(\mu')} \cdot \mathbf{w}$ $\quad \mathbb{1}_3 \to \mathbf{v}\cdot\mathbf{w}, \quad \mathbf{s} \to -i\mathbf{v}\times\mathbf{w},$

$s_{12} \to -\tfrac{3}{2}(v_1 w_2 + v_2 w_1)$ and similarly for s_{23}, s_{31},

$s_{11} - s_{22} \to 3(-v_1 w_1 + v_2 w_2),$

$s_{33} \to (v_1 w_1 + v_2 w_2 - 2 v_3 w_3);$

$i\varepsilon_{ijk}\varepsilon(\lambda')_i \overline{\varepsilon(\mu')_j} W_k$ $\quad \mathbb{1}_3 \to 0, \quad \mathbf{s} \to 2\mathbf{W}, \quad s_{ij} \to 0.$

Referring again to Ohlsen [6], $(d\sigma/d\Omega)_{\mathrm{LAB}}$ is written in terms of the polarization parameters as

$$\begin{aligned}(d\sigma/d\Omega)_{\mathrm{LAB}} = I_0[1 &+ p_y^l A_y^l + \tfrac{3}{2}p_2^T A_2^T + \tfrac{2}{3}p_{31}^T A_{31}^T + \tfrac{1}{6}(p_{11}^T - p_{22}^T)(A_{11}^T - A_{22}^T) + \tfrac{1}{2}p_{33}^T A_{33}^T \\ &+ \tfrac{3}{2}p_x^l p_1^T C_{x1} + \tfrac{3}{2}p_x^l p_3^T C_{x3} + \tfrac{2}{3}p_x^l p_{12}^T C_{x,12} + \tfrac{2}{3}p_x^l p_{23}^T C_{x,23} \\ &+ \tfrac{3}{2}p_y^l p_2^T C_{y2} + \tfrac{2}{3}p_y^l p_{31}^T C_{y,31} + \tfrac{1}{6}p_y^l (p_{11}^T - p_{22}^T)(C_{y,11} - C_{y,22}) + \tfrac{1}{2}p_y^l p_{33}^T C_{y,33} \\ &+ \tfrac{3}{2}p_z^l p_1^T C_{z1} + \tfrac{3}{2}p_z^l p_3^T C_{z3} + \tfrac{2}{3}p_z^l p_{12}^T C_{z,12} + \tfrac{2}{3}p_z^l p_{23}^T C_{z,23}].\end{aligned} \tag{5.5}$$

Since the polarization parameters of the lepton beam are contained entirely in the vectors $\mathbf{V}$ and $\mathbf{W}$ we see that the vector analyzing powers A_y^T, A_2^T and the seven spin correlation coefficients involving the tensor polarization of the target all vanish identically in the $1\gamma E$ approximation. There remain three tensor analyzing powers and five spin correlation coefficients involving the vector polarization of the target (just as in the spin $\frac{1}{2}$ case).

The results for these eight quantities can be read off from (5.4). For the spin correlation coefficients, the right side of (5.4) leads to the expression

$$8MG_M(G_C + \tfrac{1}{3}\eta G_Q)\mathbf{p}^T \cdot (\mathbf{T} \times \mathbf{V}) + 4M\eta G_M^2 \mathbf{p}^T \cdot (\mathbf{T} \times \mathbf{V} + 2M(1+\eta)\mathbf{W}),$$

from which one reads off the spin correlation coefficients using (4.4) and (4.5). Using (2.18), (5.4) and (5.5) and the results just above (5.5), one comes to the following results:

$$I_0 = \frac{\alpha^2 F}{l'^2(1-\cos\theta)^2}\left[\frac{(E'^2 - 2ME'\eta - M^2\eta)}{l'^2}(G_C^2 + \tfrac{8}{9}\eta^2 G_Q^2 + \tfrac{2}{3}\eta G_M^2) + \frac{2}{3}\frac{(2M^2\eta - m_l^2)}{l'^2}\eta(1+\eta)G_M^2\right],$$

$$I_0 A_{31}^T = \frac{\alpha^2 F}{l'^4(1-\cos\theta)^2} 2(E' - M\eta)[(E'^2 - 2ME'\eta - M^2\eta) - m_l^2(1+\eta)]^{1/2}\eta^{3/2} G_Q G_M,$$

$$I_0(A_{11}^T - A_{22}^T) = -\frac{\alpha^2 F}{l'^4(1-\cos\theta)^2}[(E'^2 - 2ME'\eta - M^2\eta) - m_l^2(1+\eta)]\eta G_M^2,$$

$$I_0 A_{33}^T = -\frac{\alpha^2 F}{l'^4(1-\cos\theta)^2} \times [(E'^2 - 2ME'\eta - M^2\eta)\eta(\tfrac{8}{3}G_Q(G_C + \tfrac{1}{3}\eta G_Q) + \tfrac{1}{3}G_M^2) + \tfrac{1}{3}(2M^2\eta - m_l^2)\eta(1+\eta)G_M^2],$$

$$I_0 C_{x1} = -\frac{\alpha^2 F}{l'^5(1-\cos\theta)^2}\tfrac{4}{3}(G_C + \tfrac{1}{3}\eta G_Q)G_M(E' + M)Mm_l\eta^{3/2}(1+\eta)^{1/2},$$

$$I_0 C_{x3} = -\frac{\alpha^2 F}{l'^5(1-\cos\theta)^2}\tfrac{2}{3}G_M^2[(E'^2 - 2ME'\eta - M^2\eta) - m_l^2(1+\eta)]^{1/2} Mm_l\eta^2(1+\eta)^{1/2},$$

$$I_0 C_{y2} = -\frac{\alpha^2 F}{l'^4(1-\cos\theta)^2}\tfrac{4}{3}(G_C + \tfrac{1}{3}\eta G_Q)G_M Mm_l\eta(1+\eta),$$

$$I_0 C_{z1} = -\frac{\alpha^2 F}{l'^5(1-\cos\theta)^2}\tfrac{4}{3}(G_C + \tfrac{1}{3}\eta G_Q)G_M[(E'^2 - 2ME'\eta - M^2\eta) - m_l^2(1+\eta)]^{1/2} ME'\eta(1+\eta)^{1/2},$$

$$I_0 C_{z3} = -\frac{\alpha^2 F}{l'^5(1-\cos\theta)^2}\tfrac{2}{3}G_M^2[E'(E' - M\eta) - m_l^2(1+\eta)]M\eta^{3/2}(1+\eta)^{1/2}.$$

As far as one can understand the way in which the results are presented on page 71 of Ref. 4, our exact expressions for the three tensor analyzing powers seem to agree with the approximate expressions given there. However, one of the results for A_1 or A_3 given in Ref. 4 is incorrect by a factor 2.

As for a spin $\frac{1}{2}$ target, one wants to write expressions for the polarization parameters which refer to components of the vector and tensor polarizations of the target in the (xyz) frame. For the spin correlation coefficients, (4.7) holds as before. For the tensor analyzing powers we have the relations

$$A_{zx}^T = \cos 2\theta_T A_{31}^T + \tfrac{1}{4}\sin 2\theta_T (A_{11}^T - A_{22}^T) - \tfrac{3}{4}\sin 2\theta_T A_{33}^T,$$

$$A_{xx}^T - A_{yy}^T = -\sin 2\theta_T A_{31}^T + \tfrac{1}{4}(3+\cos 2\theta_T)(A_{11}^T - A_{22}^T) + \tfrac{3}{4}(1-\cos 2\theta_T)A_{33}^T,$$

$$A_{zz}^T = \sin 2\theta_T A_{31}^T + \tfrac{1}{4}(1-\cos 2\theta_T)(A_{11}^T - A_{22}^T) + \tfrac{1}{4}(1+3\cos 2\theta_T)A_{33}^T,$$

where $\cos\theta_T$, $\sin\theta_T$ are given by (4.8). Experimentally it is possible to have the polarization direction of a deuteron target along or transverse to the beam direction. The technique for making such a polarized target produces tensor and vector polarization together, so that eventually it may be possible to measure some of the spin correlation coefficients (with a polarized lepton beam) or perhaps the analyzing powers $A_{xx}^T - A_{yy}^T$ and A_{zz}^T (though the currently attainable degree of tensor polarization is small).

Measurements of I_0 at various energies and angles suffice to separate $G_C^2 + \frac{8}{9}\eta^2 G_Q^2$ from G_M^2. However, the measurement of a suitable polarization parameter is needed to distinguish G_C and G_Q. It is hard to see which type of experiment is most likely in time to provide this information. If both the beam and the target are unpolarized, the recoiling leptons are unpolarized if the spin state of the outgoing spin 1 particle is undetected. When, however, the spin state of the outgoing lepton is undetected, the recoiling spin 1 particles have a tensor polarization characterized by parameters p_{31}, $(p_{11} - p_{22})$ and p_{33}, where from reciprocity $p_{31} = -A_{31}^T$, $p_{11} - p_{22} = A_{11}^T - A_{22}^T$, $p_{33} = A_{33}^T$. If one could determine, by measuring the asymmetry in a suitable second scattering, the quantity $p_{33} = A_{33}^T$, one could thereby distinguish G_C and G_Q. We have seen that experiments with a polarized target could also do this.

With extra work, the methods of this paper could be extended to calculate polarization transfer coefficients. For example, when a polarized lepton beam is incident on an unpolarized spin 1 target, the recoiling spin 1 particles have, in addition to the tensor polarization given in the previous paragraph, a vector polarization whose parameters are given by

$$p_1 = C_{1x} p_x^l + C_{1z} p_z^l,$$

$$p_2 = C_{2y} p_y^l,$$

$$p_3 = -C_{3x} p_x^l - C_{3z} p_z^l.$$

For a spin $\frac{1}{2}$ target, the same relations also hold for the polarization of the recoiling target particles. More elaborate calculations could be made to obtain other polarization transfer coefficients. For example, we have calculated the coefficients which give the polarization of the outgoing leptons in terms of the polarization of the lepton beam, when the target (spin $\frac{1}{2}$ or spin 1) is unpolarized.

However, experiments to measure these and other polarization transfer coefficients seem a remote possibility.

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