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Autor: Osterwalder, K.

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Constructive quantum field theory: goals, methods, results

By K. Osterwalder

Mathematik Eidg. Technische Hochschule, Zürich

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I. Introduction

The successes of quantum electrodynamics and more recently the enormous progress in the attempts to unify all the four fundamental forces in one theory give hope to the belief that a relativistic, non-abelian quantum field theory will eventually lead to a satisfactory description of elementary particle physics. Clearly, the correct Lagrangian has yet to be found, the full symmetry has to be established – maybe it is even super! – and it is conceivable that fields have to be replaced by strings. But no matter what the theory will finally look like, it has become abundantly clear that we have to understand the details of its mathematical structure and that we have to be able to prove its logical consistency. It is well known that we are still far from such a goal. Not even in the case of quantum electrodynamics have we been able to reach it. In fact, nowadays most people believe that QED is inconsistent, that is does not exist as a mathematical model beyond formal perturbation theory.

The main goal of constructive quantum field theory is to provide a sound mathematical foundation for elementary particle theories. The program can be summarized as follows:

1. Starting from a formal Langrangian density construct a relativistic quantum field theory satisfying some system of axioms (e.g. the Wightman axioms or their Euclidean counterpart). The dynamics of the theory thus obtained is determined by the Lagrangian in a well-defined manner.
2. Determine whether the model constructed in 1 describes nontrivial scattering and whether the scattering matrix is unitary (asymptotic completeness).
3. Study the rôle of the formal power series expansion: is it asymptotic, is it Borel summable; for which values of the expansion parameters does it make precise predictions?
4. Investigate the rôle of symmetries: broken symmetries, Goldstone bosons, phase transitions etc.

5. In models containing gauge fields study all the mechanismus predicted by formal arguments: Faddeev–Popov formalism, Gribov ambiguities, Higgs mechanism, confinement etc.
6. Check the validity and reliability of computer calculations.

Up to now all realistic models have resisted the attempts to deal with them according to these ideas. However, many simplified models have been constructed successfully and some of them have been analysed in great details. The simplifications have been

to simplify the interactions
to simplify the symmetries
to reduce the number of space time dimensions,

the latter being the most serious one. Right now the situation looks quite promising: recent progress seems to justify the hope that the construction of a four dimensional quantum field theory model, satisfying all the axioms, is within reach.

The purpose of this talk is to describe the major developments and results of constructive quantum field theory and then to focus on some of the most recently proposed methods.

Quantum field theory was one of the major areas in Ernst Stueckelberg's research and he has made many contributions of lasting value to this fundamental area of theoretical physics. Some of the ideas that will be discussed here have their roots in the deep work of Stueckelberg and I would like to dedicate this talk to his memory.

II. Major developments

1. *Hamiltonian approach*

The origins of constructive quantum field theory can be traced to early, seminal work of A. Jaffe [1] and of O. Lanford [2]. In his 1965 thesis Jaffe studied a self-coupled scalar field with an interaction term $\lambda \int :\varphi(x)^4: d^3x$ except that he introduced a space cutoff function $g \in C_0(\mathbb{R}^3)$ and a high momentum cutoff κ to define a Hamiltonian

$$H_{g,\kappa} = H_0 + \lambda \int g(x) : \varphi_\kappa(x)^4 : d^3x$$

where H_0 is the Hamiltonian of a free field. He showed that $H_{g,\kappa}$ is a self-adjoint operator with unique vacuum state. Similar results were obtained by Lanford for a model coupling bosons and fermions via a Yukawa interaction.

Now the problem was to remove the cutoffs and to prove that the limiting theory defined the correct dynamics and satisfied all the Wightman axioms. This problem could be solved after another simplification had been introduced: a reduction of the number of space-time dimensions d to $d = 2$ and later to $d = 3$.

The model with the least technical difficulties is the $(\lambda\phi^4)_2$ model, described by the Hamiltonian introduced above but with space dimension 1, instead of 3. In perturbation theory this model has no ultra-violet divergences and does not call for any infinite renormalization. By 1972 this model had been shown to exist without any cutoff and to satisfy all the Haag–Kastler axioms and most of the Wightman axioms [3]. These results had also been extended to technically more difficult models, such as

- $P(\varphi)_2$ where $\lambda\varphi^4$ is replaced by an arbitrary polynomial bounded below
- Y_2 where a boson and a fermion interact via a Yukawa interaction (in space time dimension $d = 2$)
- $(\lambda\varphi^4)_3$ the selfinteracting boson model with φ^4 coupling in $d = 3$ dimensions.

For a more complete description of these early developments and for references see e.g. [3], [4].

2. *The Euclidean formulation*

In 1972/73 several new ideas were introduced which made old results much easier to derive and opened the road to the solution of many problems that had been out of reach before. Most of these 'new' ideas were actually a mathematically rigorous formulation of a program formulated by K. Symanzik in the mid sixties [5]. The first one of these was the connection of boson quantum field theories with probability theory via Markoff processes [6]. The second one was a more general connection between an arbitrary (Wightman-) quantum field theory in Minkowski space and Euclidean Green's functions in Euclidean space [7]. The third discovery was the close connection between Euclidean quantum field theory and classical statistical mechanics [8].

The direct construction of a relativistic quantum field theory through its Hamiltonian was now abandoned. Instead the goal became to prove the existence of Schwinger functions (Euclidean Green's functions) and then to make use of an axiomatic result that guarantees that the relativistic theory can be reconstructed in a unique fashion.

On a formal level the Schwinger function of a single scalar boson particle are given as expectation values of a Euclidean field $\Phi(x)$,

$$S(x_1, \ldots, x_n) = \frac{1}{Z} \langle \Omega, \Phi(x_1) \cdots \Phi(x_n) e^{-\int \mathscr{L}_I(\Phi(x)) d^4x} \Omega \rangle$$

Here Φ is a free field, determined by

$$[\Phi(x), \Phi(y)] = 0, \qquad \text{for all } x, y \text{ in } \mathbb{R}^4,$$

and

$$\begin{aligned}\langle \Omega, \Phi(x)\Phi(y)\Omega \rangle &= (2\pi)^{-2} \int \frac{1}{p^2 + m^2} e^{ip(x-y)}\, d^4p \\ &= (-\Delta + m^2)^{-1}(x, y).\end{aligned}$$

$\mathscr{L}_I(\Phi(x))$ is the interaction Lagrangian such as: $\Phi(x)^4$: and Z is a normalization constant. For models with several fields and with fermions the formulas are similar.

Notice that the Euclidean boson fields, unlike their relativistic counterparts are *abelian.* As a consequence the Schwinger functions can also be defined by functional integrals as the moments of a probability measure μ on function space

$$S(x_1, \ldots, x_n) = \frac{1}{Z} \int \phi(x_1) \cdots \phi(x_n) e^{-\int \mathscr{L}_I(\phi(x)) d^4x} \, d\nu_G(\phi)$$

$$= \int \phi(x_1) \cdots \phi(x_n) \, d\mu(\phi)$$

Still on the formal level $d\mu$ is a Gaussian measure $d\nu_G$ with mean zero and covariance $G = (-\Delta + m^2)^{-1}$ multiplied by the factor $\exp\left(-\int \mathscr{L}_I(\phi(x))\, dx\right)$.

For a rigorous construction of the Schwinger functions one has to start from a regularized theory, just as in the Hamiltonian approach. To get rid of the volume divergences one restricts integration in $\int \mathscr{L}_I(\phi(x))\, d^4x$ to a finite volume Λ or one puts the whole theory on a torus. The high momentum divergences are taken care of by an ultra-violet cutoff κ. Thus the doubly cutoff Schwinger functions

$$S_{\Lambda,\kappa}(x_1, \ldots, x_n)$$

are well defined and the problem is then to prove that the limit

$$S_{\Lambda,\kappa} \xrightarrow[\Lambda \to \mathbb{R}^4]{\kappa \to \infty} S$$

exists and that the limiting quantities S have all the properties which are required for them to define a relativistic quantum field theory. In general the no cutoff limit will not exist or be trivial unless some κ-dependent modification of the coefficients appearing in $\mathscr{L}_I$ have been performed: this is the renormalization problem. We remark that one particular method to cutoff the large momenta is to put the whole theory on a lattice, i.e. to allow the variable x only to take values in a lattice such as $a \cdot \mathbb{Z}^4$, where $a \sim \kappa^{-1}$ is called the lattice spacing. The Laplacian that appears in the formal definition of $S(x_1, \ldots, x_n)$ is then replaced by the finite difference Laplacian. One can easily see that the approximate Schwinger functions thus obtained, are the correlation functions of a classical lattice spin system, with a single spin distribution essentially determined by $\exp[-\mathscr{L}_I(\phi(x))]$. The non local part of the finite difference Laplacian generates a nearest neighbour ferromagnetic interaction. This establishes the mathematical equivalence of Euclidean quantum field theory and classical statistical mechanics.

3. *Superrenormalizable models: methods and results*

With the Euclidean setup at hand the problem of studying limits of regularized operators $H_{g,\kappa}$ had now been replaced by the problem of constructing limits of measures or of their moments $S_{\Lambda,\kappa}$. Two basic sets of methods proved to be extremely powerful:

a) *Correlation inequalities*: some of them were already known from statistical mechanics [9], many others were found in the seventies and proved to be very helpful tools both in field theory and in statistical mechanics [8], [10], [11]. Existence of limits and some information on the models constructed could be obtained by these methods. The strength of the correlation inequality methods is their elegance and the fact that they usually hold without restrictions on the absolute values of the coupling parameters in $\mathscr{L}_I$. Their weakness is that they put otherwise unnecessary restrictions on the signs of the coupling parameters and – more seriously – that they do not hold for models with fermions.

b) *Expansion methods*: if we replace $\mathscr{L}_I$ by $\lambda\mathscr{L}_I$ and formally expand the Schwinger functions S in powers of λ then we obtain the standard Feynman diagram expansion which in most models is known or believed to be divergent. This difficulty can be overcome if one treats different regions in space separately and uses asymptotic expansions only. This leads to the so called *cluster expansion* (or high temperature expansion, as it is called in statistical mechanics) which converges whenever the theory is far from a critical point, i.e. if the measure $d\mu$ is close to a Gaussian. The expansion method is in general more complicated than the correlation inequality method and its range is limited in the sense just explained. However, as a tool, it is more powerful: in addition to the proof of the existence of the limit of $S_{\Lambda,\kappa}$ it gives detailed information about the spectrum of the model: multiplicity of the ground state, the existence of isolated particle spectrum, the existence or absence of bound states, asymptotic completeness at low energies etc. It also allows for the study of such things as analyticity in the coupling parameters, Borel summability of the formal power series and the problem of phase transitions and multiphase regions.

By the beginning of the eighties these methods had been brought to such a perfection that a large class of models was completely under control: the superrenormalizable models. They are characterized by the fact that the κ-dependent adjustments of the parameters in $\mathscr{L}_I$, which are necessary for the $\kappa\to\infty$ limit to exist (i.e. the renormalization) can be written down explicitly as a *polynomial* in the overall coupling parameter λ. They include the following models:

For d = space-time dimension = 2

$\mathscr{L}_I = P(\Phi(x))$	P: a polynomial, bounded from below
$= \sin \varepsilon\Phi(x)$	Sine–Gordon model
$= e^{\alpha\Phi(x)}$	Höegh–Krohn model
$= \bar{\Psi}(x)\Psi(x)\phi(x)$	Yukawa Model (Y_2)

and Abelian Higgs models.

For $d = 3$:

$\mathscr{L}_I = (\Phi(x))^4$	$(\phi^4)_3$ model
$= \bar{\Psi}(x)\Psi(x)\phi(x)$	Y_3 model

(and QED).

For all these models the following results have been obtained (or could be obtained with a large but finite amount of additional work):

- the no cutoff limits of the Schwinger functions exist and define a relativistic quantum field theory.
- non triviality of the scattering matrix,
- analysis of the particle spectrum,
- equation of motion,
- symmetry breaking, phase transitions, multiple phase diagram,
- the meaning of formal power series expansion: Borel summability.

Unfortunately, no model in 4 space-time dimensions is superrenormalizable and it became clear in the mid-seventies that the methods described so far would not be sufficient for the construction of a nontrivial, four dimensional model.

4. *Beyond superrenormalizability: renormalization group and asymptotic freedom*

To make the difference between superrenormalizable and merely renormalizable models more transparent let us look at a concrete example: the $(\Phi^4)_d$ model. Formally the measure $d\mu(\phi)$ is given by

$$d\mu(\Phi) = N^{-1} \exp\left(-\int \mathscr{L}_I(\Phi(x))\, d^d x\right) d\nu_G(\Phi)$$

where $d\nu_G$ is Gaussian measure with mean 0 and covariance $G = (-\Delta + m^2)^{-1}$ and $\mathscr{L}_I(\Phi(x)) = \lambda\phi^4(x)$.

To regularize the model we restrict the integration in $\int \mathscr{L}_I(\Phi(x))\, d^d x$ to a finite volume $\Lambda \subset \mathbb{R}^d$ and replace the inverse free propagator $(-\Delta + m^2)$ by some cutoff version G_κ^{-1}, e.g. by $G_\kappa^{-1} = (-\Delta + m^2)e^{(-\Delta+m^2)/\kappa^2}$. The crucial point is that in $\tilde{G}_\kappa(p)$ values of p with $p^2 \gtrsim \kappa$ are strongly suppressed and $\tilde{G}_\kappa(p) \to (p^2 + m^2)^{-1}$ for $\kappa \to \infty$. We will say that in G_κ momenta larger than κ have been cut off. Formal power series expansion in powers of λ indicates that before passing to the limit $\kappa \to \infty$ we have to renormalize the regularized measure by setting

$$d\mu_\kappa^{\mathrm{ren}}(\Phi) = N_\kappa^{-1} \exp\left(-\int_\Lambda \mathscr{L}_{I,\kappa}^{\mathrm{ren}}(Z_\kappa^{1/2}\Phi(x))\, dx^d\right) d\nu_{G_\kappa}(Z_\kappa^{1/2}\phi)$$

where

$$\mathscr{L}_{I,\kappa}^{\mathrm{ren}}(Z_\kappa^{1/2}\Phi) = \frac{Z_\kappa}{2}\, \delta m_\kappa^2 \Phi(x)^2 + Z_\kappa^2 \lambda_\kappa \Phi(x)^4$$

and N_κ is an appropriate normalization constant to make the total weight of $d\mu_\kappa^{\mathrm{ren}}$ equal to 1.

We thus have to admit κ-dependent wave function renormalization Z_κ, a mass correction δm_κ^2 and coupling constant renormalization $\delta\lambda_\kappa = \lambda_\kappa - \lambda$ and these quantities may become singular as $\kappa \to \infty$. In two and three space-time dimensions we may choose Z_κ to be 1 and δm_κ^2 and λ_κ to be polynomials in λ, with κ-dependent coefficients that can be calculated explicitly: these models are *superrenormalizable*.

In four space-time dimensions, however, these quantities are only given as *formal* power series in λ which are most likely to be divergent (even at finite values of κ). Such models are *renormalizable* but not superrenormalizable. In order to procede with a nonperturbative construction of the model we have to find alternative methods of determining the renormalization quantities Z_κ, δm_κ^2 and λ_κ. They have to be defined implicitly. The new technique to implement this program was found in the renormalization group approach [12]. In constructive quantum field theory this approach was pioneered in 1978 in a study of the (superrenormalizable!) $(\phi^4)_3$ model in [13] and it has since been applied successfully to various models which are just renormalizable [14]. Even for the construction of gauge theories this method appears to be the key ingredient [15]. On a more pedagogical level it has led to a completely new understanding of perturbative renormalization, reducing the original BPHZ schemes to very transparent and (almost!) utterly simple procedures [16].

The basic idea of this new technique is to carry out the integration over function space with respect to the measure $d\mu_\kappa^{\text{ren}}(\Phi)$ as a *sequence of fixed momentum scale integrals.* More precisely we split the covariance G_κ of $d\nu_{G_\kappa}$ as follows

$$G_\kappa = G_{\kappa'} + \Gamma, \qquad \kappa' < \kappa,$$

where in $G_{\kappa'}$ the momenta are cutoff at κ', while in Γ they are basically localized in the range $\kappa' \leq p^2 \leq \kappa$. This allows us to split the field Φ into high and low momentum parts

$$\Phi = \Phi' + \Psi$$

and to write

$$d\nu_{G_\kappa}(Z_\kappa^{1/2}\Phi) \quad \text{as} \quad d\nu_{G_{\kappa'}}(Z_\kappa^{1/2}\Phi') \times d\nu_\Gamma(Z_\kappa^{1/2}\Psi)$$

Substituting this into $d\mu_\kappa^{\text{ren}}$ and integrating over Ψ we define the effective action $V_{\kappa'}(Z_{\kappa'}^{1/2}\Phi')$ by setting the result equal to

$$\exp\left(-V_{\kappa'}(Z_{\kappa'}^{1/2}\Phi')\right) d\nu_{G_{\kappa'}}(Z_{\kappa'}^{1/2}\Phi')$$

(up to normalization).

It is an easy calculation to determine the low order (in λ_κ) contributions to $V_{\kappa'}$. With an appropriate choice of $Z_{\kappa'}$ we find that $V_{\kappa'}$ is of the same form as $\mathcal{L}_{I,\kappa}^{\text{ren}}$:

$$V_{\kappa'}(Z_{\kappa'}^{1/2}, \Phi'_{(x)}) = \frac{Z_{\kappa'}}{2}\,\delta m_{\kappa'}^2 \Phi'(x)^2 + Z_{\kappa'}^2 \lambda_{\kappa'} \Phi'(x)^4$$
$$+ \text{‘small terms’}$$

and we calculate

$$\lambda_{\kappa'},\ Z_{\kappa'},\ \delta m_{\kappa'} \qquad \text{as functions of} \qquad \lambda_\kappa,\ Z_\kappa,\ \delta m_\kappa.$$

These are the so called flow equations of the renormalization group transformation. In order for the 'small terms' to be really small we have to choose κ' sufficiently close to κ and λ_κ has to be small. Thus, in order to end up with a small value of κ', starting from a large value of κ, we have to proceed in many small steps; i.e. we write $\kappa' = \kappa_0$ and introduce intermediate values $\kappa_0 < \kappa_1 < \kappa_2 < \cdots < \kappa_N = \kappa$. Then we proceed from V_{κ_N} to $V_{\kappa_{N-1}}$ all the way to V_{κ_0}.

The crucial problem is the choice of λ_κ. The flow equation gives us λ_{κ_0} as a function of $\lambda_{\kappa_N} = \lambda_\kappa$ (disregarding the other parameters). Assume this relation is invertible. Then we may fix λ_{κ_0} and choose λ_{κ_N} accordingly for every value of N. If, for $N \to \infty$, λ_{κ_N} tends to zero, then our procedure should work, because for small λ_{κ_0} *all* λ_{κ_i} are small and we should be able to control the 'small terms' above. If this happens we say the theory is *asymptotically free* in the ultraviolet. On the other hand, if for $N \to \infty$, λ_{κ_N} tends to infinity then, no matter how small we choose the low momentum coupling constant λ_{κ_0}, the λ_{κ_i} will eventually grow large and the nonperturbative control of the renormalization group transformation steps get out of hand. The only way to keep λ_{κ_N} finite would be to choose λ_{κ_0} 'arbitrarily small', i.e. equal to zero. We would thus end up with a free theory. We conclude that the renormalization group method can work *only* if the theory is asymptotically free.

The example discussed so far, the $(\lambda\phi^4)_4$ model is not asymptotically free. In fact, many people believe that no matter how one regularizes and renormalizes this model, in the limit $\kappa \to \infty$ it will always end up being trivial [17].

The flow equation for $\lambda_n \equiv \lambda_{\kappa_n}$, with $\kappa_n = 2^n$, looks like this

$$\lambda_{n-1} \approx \lambda_n - \beta_2\lambda_n^2 - \beta_3\lambda_n^3, \quad \text{with} \quad \beta_2 > 0.$$

This shows that for λ_n small, λ_{n-1} is *smaller* than λ_n, contrary to what we need for asymptotic freedom.

However, this flow equation suggests another interesting alternative: if λ_n is small and *negative* then λ_{n-1} is still negative and $|\lambda_{n-1}| > |\lambda_n|$.

A $(\phi^4)_4$ theory with negative coupling *is* asymptotically free. This fact had been discovered in 1973 already [18]. In 1984 the renormalization group methods were finally used to give a rigorous construction of a $(\phi^4)_4$ theory with negative coupling [19].

Though the formal power series expansion in powers of λ of this model agrees with the standard Feynman diagram expansion, it is unlikely to define a *physical* model: it seems to lack the most crucial property which is needed to go back from the Euclidean description to a Minkowski theory (physical positivity, [7]).

The renormalization group method has been used successfully to study several other renormalizable models:

- in $d = 2$ dimensions the model with quartic fermion interaction and more than one flavour: $\mathcal{L}_I = (\sum_\alpha \bar{\Psi}^\alpha \Psi^\alpha)^2$, (the Gross–Neveu Model), [20].
- the infrared limits of several massless models which are asymptotically free in the infrared, see e.g. [21].
- gauge theories in $d = 3$ dimensions [15], see also [22] for results in $d = 4$.

The developments along these lines are far from being complete at this point but many people are confident that within a reasonable amount of time they will allow for a rigorous construction of a $d = 4$ gauge theory with (not too many) fermions.

Maybe by the time this goal has been reached all of high energy physics has been explained by string theories and field theories are of no interest anymore.

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