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Autor: Parriaux, O. / Cochet, F.
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Degenerate coupling between dielectric waveguides

By O. Parriaux and F. Cochet

Centre Suisse d'Electronique et de Microtechnique CH-2000 Neuchâtel, Switzerland

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Abstract. The link between the coupled mode formalism and the approach considering the beat of eigenmodes for the description of the degenerate coupling between two dielectric slab waveguides is established via a second order derivation of the dispersion equation of the complete structure.

I. Introduction

The coupling between two identical neighbouring waveguides is a wellknown effect that is widely used in guided wave optics. Distributed couplers are key elements in a number of devices such as power dividers, switches, modulators, etc. If the two waveguides are parallel and uniform along the direction of propagation, the power goes periodically back and forth from one guide to the other. The spatial period L_c of the power transfer along the direction of propagation depends on the distance d between the waveguide boundaries and on the confinement of the modal field.

This phenomenon of periodical power transfer can be understood as a beat of two eigenmodes of the complete structure consisting of the two single waveguides. These two modes, that are both equally excited by the incoming modal field of a single waveguide, have different propagation constants β_1 and β_2, with $\Delta\beta = \beta_2 - \beta_1$ (time t and longitudinal z dependence in the form $\exp(j(\omega t - \beta z))$ is assumed); at distance $L_c/2$ from the coupler input, the phase difference between the two modes is $\Delta\varphi = \Delta\beta L_c/2 = \pi$ and the full power is in the other guide; at distance L_c, $\Delta\varphi = 2\pi$ and the whole power is back into the first guide [1].

As the waveguides used in concrete life are often complicated structures, the direct calculation of β_1 and β_2 requires tedious and costly numerical techniques whose results are not always meaningful. Instead, another approach uses a coupled wave formalism assuming that the modal field of a single waveguide is not significantly perturbed by the presence of a neighbouring waveguide in the coupling section. The coupling strength is described by a coupling coefficient K proportional to the modal field overlap over the cross-section of one of the waveguides, the latter being considered as a dielectric perturbation in the neighbourhood of the other waveguide. In the degenerate case (i.e. when the waveguides are identical) the power flows back and forth from one guide to the other along z according to $\sin(Kz)$, with a spatial period $L_c = \pi/K$; the full power transfer is reached when $z = L_c/2 = \pi/(2K)$.

A comparison between the two approaches shows that $K = \Delta\beta/2$. This was demonstrated in the case of two coupled planar waveguides to be approximately true [2] and also in the case of two circular optical fibers [3, 4], where the coupling coefficient K can be obtained analytically, by solving numerically the complete two-waveguides structure for β_1 and β_2. The coupled wave formalism gives fairly accurate results that depart significantly from exact ones only when the two waveguides touch each other.

Hereafter, an analytical derivation of the expressions for β_1 and β_2 is performed in the case of degenerate coupling between two planar waveguides by calculating the Taylor expansion of the dispersion equation of the complete structure to second order. A comparison with the analytical expression for K will explicitly show the validity of the coupled wave approach.

II. Derivation of $\Delta\beta$

The coupled structure considered (Fig. 1) is lossless and consists of two identical homogeneous dielectric waveguides of width w, relative permittivity ε_2, at a distance d from each other, embedded in an infinite dielectric medium of permittivity ε_1.

The dispersion equation $\mathcal{F}(\beta)=0$ for the TE modes of the 5-layers structure is obtained by matching the tangential field components across the 4 dielectric boundaries. It can be derived straightforwardly by using a transition matrix formalism [5]:

$$\begin{aligned}\mathcal{F}(\beta)=0=\operatorname{ch}(dp_1)\left(2\cos(2wp_2)+\sin(2wp_2)\left(\frac{p_1}{p_2}-\frac{p_2}{p_1}\right)\right)\\ +\operatorname{sh}(dp_1)\left(2+\sin^2(wp_2)\left(\frac{p_1}{p_2}-\frac{p_2}{p_1}\right)^2\right.\\ \left.+\sin(2wp_2)\left(\frac{p_1}{p_2}-\frac{p_2}{p_1}\right)\right)\end{aligned} \tag{1}$$

where $p_1=\sqrt{\beta^2-k_0^2\varepsilon_1}$

$p_2=\sqrt{k_0^2\varepsilon_2-\beta^2}$

$k_0=2\pi/\lambda$ wavenumber in vacuum

λ wavelength

β propagation constant of the TE modes of the complete structure.

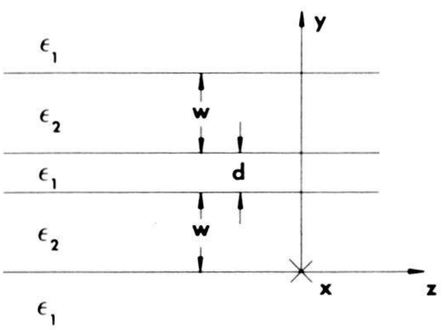

Figure 1
5-layers coupled structure. Propagation takes place along z, y is normal to the interfaces.

When $d \to \infty$, (1) reduces to

$$\operatorname{tg}(wp_2) = \frac{2p_1p_2}{p_2^2 - p_1^2}, \tag{2}$$

which is the wellknown dispersion equation of a single slab waveguide of width w, whose solutions $\{\beta_0\}$ are the propagation constants of the propagating TE modes.

If $d \to 0$ (1) reduces to the corresponding equation for the TE modes of a slab waveguide of width $2w$ with the corresponding set of propagation constants $\{\beta_i\}$:

$$\operatorname{tg}(2wp_2) = \frac{2p_1p_2}{p_2^2 - p_1^2}. \tag{3}$$

The effect of bringing a second waveguide from infinity to a finite distance d from the first one is to split each value of the initial set $\{\beta_0\}$ into two neighbouring values. These pairs of solutions get further apart as the second waveguide comes closer to the first one, and become identical to two consecutive values in the $\{\beta_i\}$ set of the waveguide of width $2w$ when $d = 0$. This mode transformation is sketched in Fig. 2 showing as an example the transverse field distribution of the E_x component of the TE_1 mode belonging to the w-wide waveguide, transforming

a)

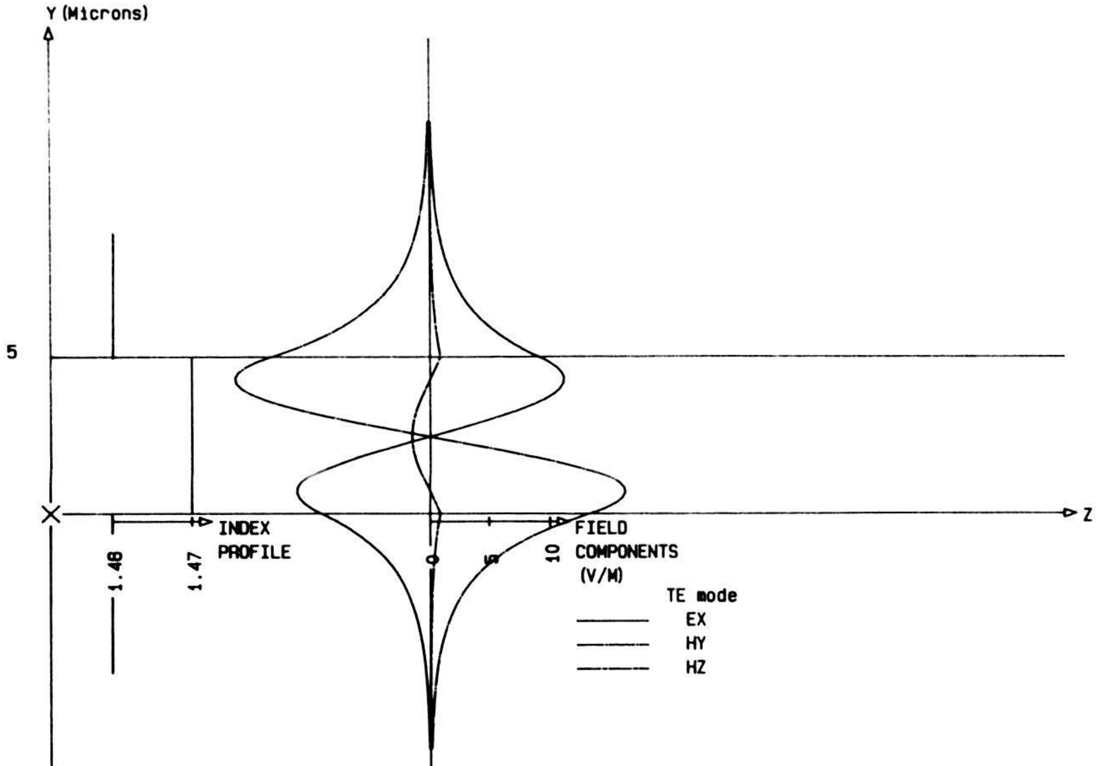

Figure 2
Mode transformation as the waveguide spacing d decreases from infinity to zero. The waveguides have a substrate index of 1.46 and an index increase $\Delta n = 0.01$, $w = 5\ \mu$m, $\lambda = 1\ \mu$m. a) TE_1 mode field components E_x, H_y, H_z in V/m, with H_i expressed in V/m after multiplication by the vacuum impedence, normalized to 1 W of propagated power. b) E_x field component normalized to 1 W power of the coupled structure when $d \to \infty$. c) E_x field component normalized to 1 W power of the even and odd TE_2 and TE_3 modes for $d = 3\ \mu$m. d) E_x field component of the second and third order modes of the $2w$-wide waveguide when $d = 0$.

b)

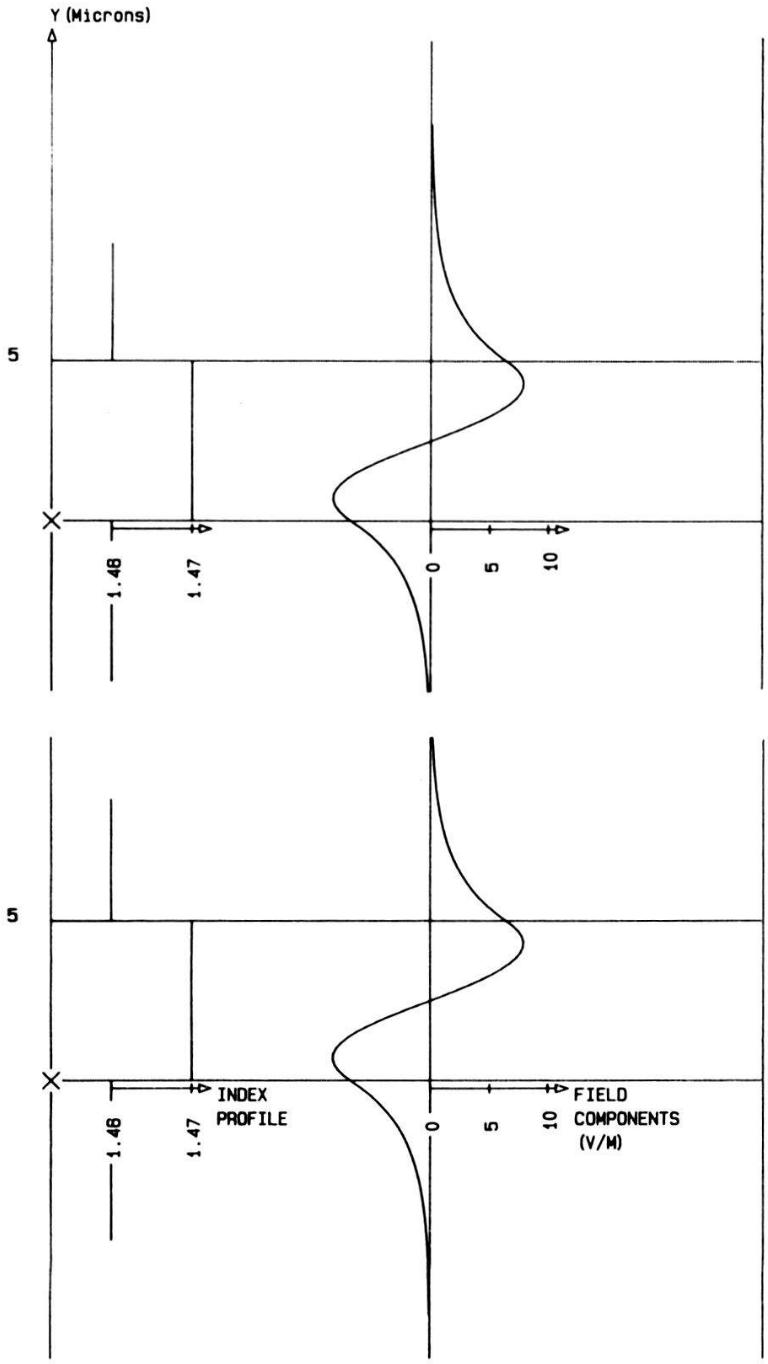

FIGURE 2 (*continued*)

into the TE_2 and TE_3 modes of the $2w$-wide waveguide. Figure 2 has been obtained by solving numerically the dispersion equation of the full 5-layers structure. The modal fields are normalized to unit power [6].

Hereafter the propagation constants belonging to a given pair of modes as described above will be designated by β_1 and β_2. The search for β_1 and β_2 must be performed by solving numerically the transcendental equation (1). In order to obtain an analytical expression for $\Delta\beta = \beta_2 - \beta_1$, a second order Taylor expansion of $\mathscr{F}(\beta)$ will be performed around β_0:

$$\mathscr{F}(\beta) = 0 = F_0 + \frac{\delta\beta}{1!} F_0' + \frac{\delta\beta^2}{2!} F_0'' \tag{4}$$

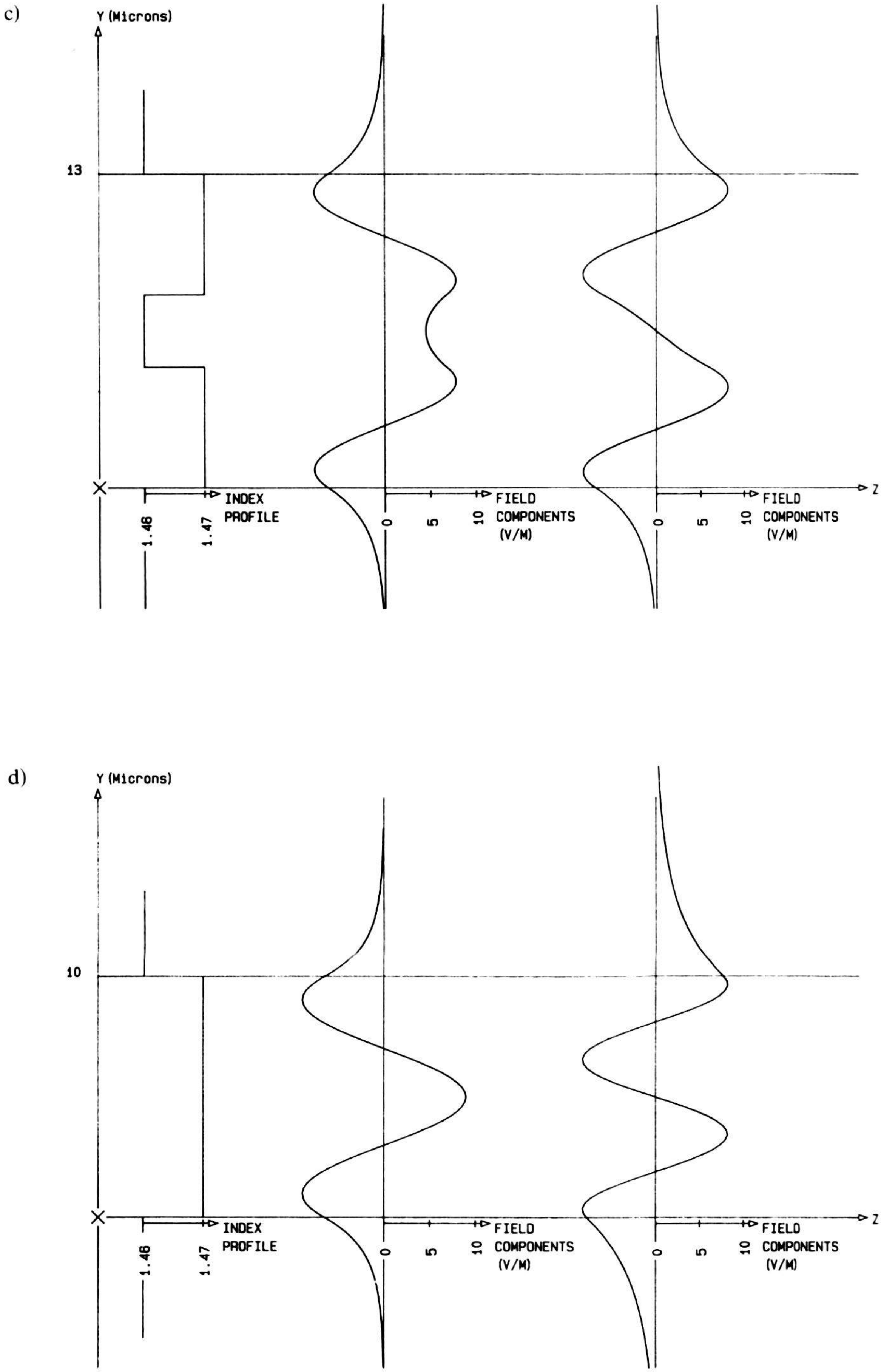

FIGURE 2 (*continued*)

where F_0, F_0' and F_0'' stand for $\mathscr{F}(\beta_0)$, $(\partial\mathscr{F}/\partial\beta)(\beta_0)$ and $(\partial^2\mathscr{F}/\partial\beta^2)(\beta_0)$ resp. After defining the auxiliary variable $\xi = 1/\delta\beta$, one gets:

$$\xi = \frac{1}{2}\left(-\frac{F_0'}{F_0} \pm \sqrt{\left(\frac{F_0'}{F_0}\right)^2 - 2\frac{F_0''}{F_0}}\right). \tag{5}$$

Lengthy calculations yield analytical expressions for the first and second derivatives of the dispersion equation. Then F_0, F_0' and F_0'' are calculated by taking advantage of the fact that β_0 satisfies the dispersion equation (2) for the w-wide waveguide.

One eventually gets:

$$\xi = Ad + B \pm \sqrt{E + C^2 \exp(2p_1 d)} \tag{6}$$

with

$$A = \frac{\beta}{2p_1}$$

$$B = \frac{\beta}{2}\frac{p_2^2 - p_1^2}{p_1^2 p_2^2}(wp_1 + 2)$$

$$C = \frac{\beta}{2}\frac{p_1^2 + p_2^2}{p_1^2 p_2^2}(wp_1 + 2)$$

E is a term of second order in d that can be neglected with respect to the exponential term in (6). One gets the simplified expression:

$$\xi_i = Ad + B \pm Ce^{p_1 d}; \tag{7}$$

From the definition of ξ_i, one gets:

$$\Delta\beta = \delta\beta_1 - \delta\beta_2 = \frac{1}{\xi_1} - \frac{1}{\xi_2} = \frac{-2Ce^{p_1 d}}{(Ad + B)^2 - C^2 e^{2p_1 d}} \tag{8}$$

$$\cong \frac{2}{C}\exp(-p_1 d) \text{ as } |B| \ll C \text{ and } C^2 \exp(2p_1 d) \text{ dominates } (Ad)^2$$

$$= 4\frac{p_1^2 p_2^2}{\beta(p_1^2 + p_2^2)(wp_1 + 2)} e^{-p_1 d}. \tag{9}$$

(9) is exactly twice the expression for the coupling coefficient K calculated analytically in the present structure using the coupled wave formalism [7].

III. Comparisons

As pointed out in the introduction, it is the phase difference $\Delta\beta$, given by (9), between the even and odd modes of the coupled structure, that is responsible for the power transfer from one guide to the other. Figure 3 shows how the effective index of the modes of the structure considered in Fig. 2 evolve versus d. They are degenerate for $d \to \infty$ and reach the effective indices of the second and third modes of the double-width guide. Curve a) resuls from exact calculations, curve b)

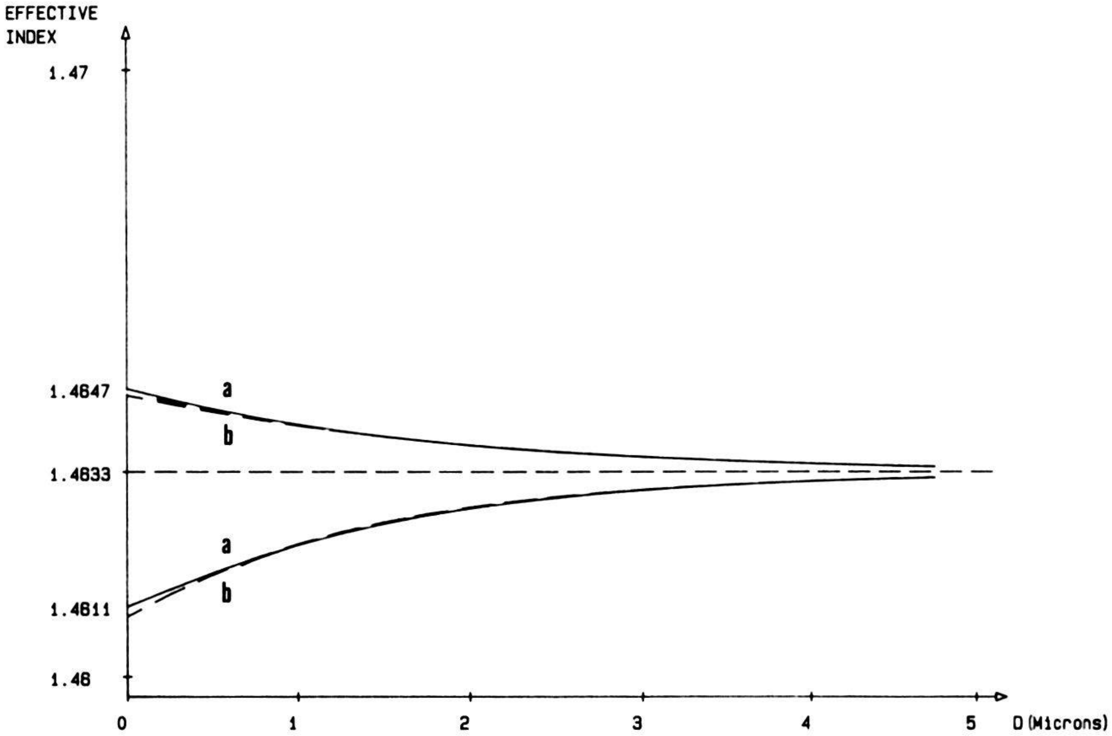

Figure 3
Effective index transformation of the TE_2 and TE_3 modes of the same structure as in Fig. 2 versus waveguide spacing d. When $d \to \infty$ both indices merge with the effective index of the TE_1 mode of a single waveguide. When $d \to 0$, the indices tend to those of the TE_2 and TE_3 of the 2w-wide guide. a) Solid line: exact calculation, b) Dashed line: analytical approximate expression.

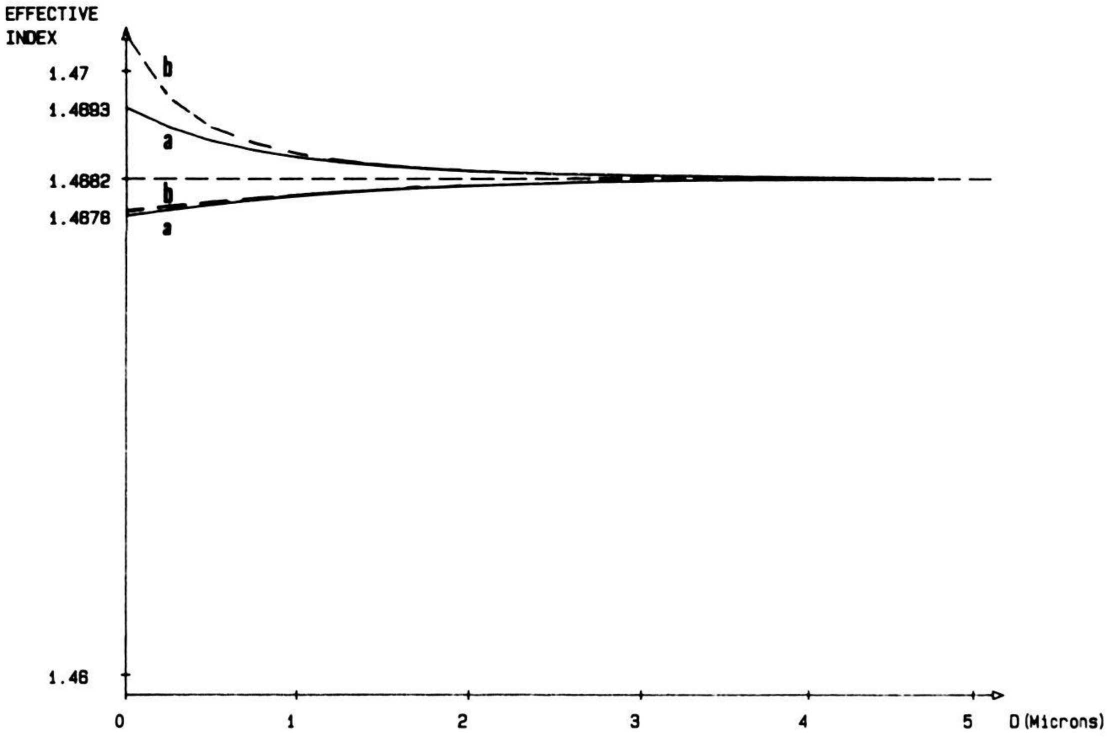

Figure 4
As in Fig. 3, but for modes TE_0 and TE_1.

from the analytical formulation given in (7). In most coupling problems it is generally assumed that the only quantity of interest is $\Delta\beta$. However Fig. 3 reveals that the phase delay $(\beta_1+\beta_2)/2-\beta_0$ affects the field of the coupled structure. This delay is of no consequence as far as the coupled power is concerned, but it can have a polarization effect if the coupling involves TE together with TM modes. Expression (7) gives correctly the sign of the phase delay as illustrated in Fig. 4 with the same structure for the TE_0 and TE_1 modes. From (7) it can be concluded that the phase delay sign changes close to the situation where $\beta_0 \cong k_0(n_1+\Delta n/2)$.

Expression (9) for $\Delta\beta$ given in the literature can now be compared with the exact solution and the second order analytical approach. Figure 5 shows, for the same structure as in Fig. 3, that all three solutions agree very well even for $d=0$. This has been checked to be true in many cases including those where the second guide causes a strong perturbation as in the case of silicon nitride films on silica [8] with $\Delta n=0.5$. Expressions for A, B, C in (6) show that the neglected term A in (9) can become significant with respect to C when w and Δn are small, i.e., in the weak guidance case. However the effect on $\Delta\beta$ results in a 3% inaccuracy even in this case.

In conclusion, the coupling between two planar waveguides has been described from both point of view of the beat of two normal modes of the complete structure, and of the coupled waves formalism, by means of an analytical second order approximation for $\Delta\beta$. The coupled waves expression for the coupling coefficient has been shown to be sufficiently accurate in most cases of interest.

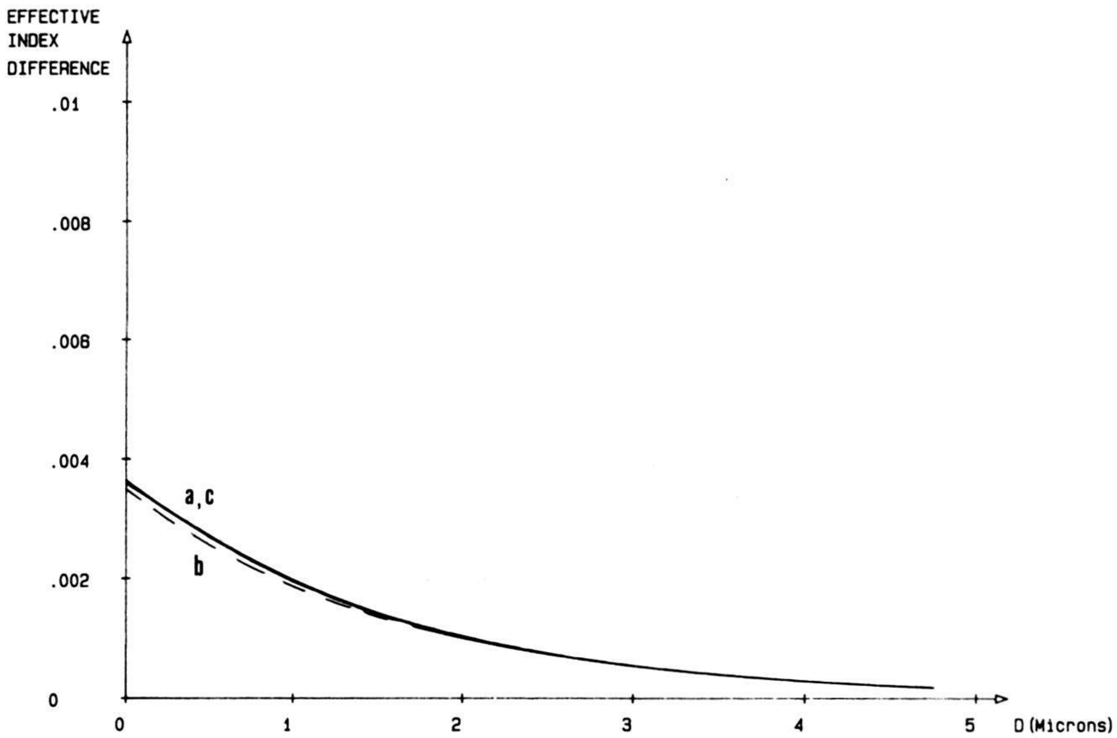

Figure 5
Comparison of the phase difference between TE_2 and TE_3 $\Delta\beta$ versus d, responsible for power coupling, given by a) The exact numerical solution: solid line, b) The analytical approximate expression: dashed line, c) The double of the analytical expression for K, given by the coupled wave theory. Lines a) and c) overlap.

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