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Unitary scheme model study of ^{4}He with the Gogny, Pires and de Tourreil interaction

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Abstract. The Unitary Scheme Model, with number of quanta of excitation $0 \leq N \leq 6$, has been applied to the nucleus ^{4}He. The Gogny et al. interaction has been used to calculate the energy levels, the mean-square radius and the integral cross section of the γ-quanta photoabsorption by the ^{4}He nucleus. The obtained results are in good agreement with the corresponding experimental findings.

1. Introduction

The methods of expanding the nuclear wave function in terms of a complete set of orthonormal functions, bases, have been used on a large scale especially for the nuclei with $3 \leq A \leq 6$. In principle, the predicted results for the nuclear characteristics should be independent of the particular bases chosen when the number of terms in the expansion is kept large enough.

In this paper, we shall apply specifically the Unitary Scheme Model [1, 10] (USM), which has shown good results for the structure of light nuclei, and present results for calculations of the ^{4}He energy levels, mean-square radius and the integral cross section of the γ-quanta photoabsorption by this nucleus.

For the even-parity states of ^{4}He we consider bases of the USM corresponding to number of quanta of excitation $N = 0, 2, 4$ and 6. For the odd-parity states we consider bases corresponding to $N = 1$, 3 and 5. The realistic potential employed is the Gogny, Pires and De Tourreil [6] (GPT) potential. This potential is a smooth local nucleon-nucleon potential which gives an acceptable fit to two nucleon data up to 300 MeV and reasonable properties for finite nuclei.

In a previous paper [3] the USM has been applied to study the binding energy, the nuclear wave function, the first-excited state and the mean-square radius of ^{4}He. The GPT-potential along with the Hu and Massey [7] (HM) potential are used in this study. Bases corresponding to $N = 0, 2, 4, 6$ and 8 and having irreducible representations of the symmetric group S_4, $[f]$, equal to [4], [22] and [211] (for $N = 2$) have been only considered. The HM-potential has shown better agreement with the corresponding experimental data than the GPT-potential. This may be due to the fact that bases corresponding to $[f] = [31]$ (for $N \geq 4$) have not been considered. In the present work all the irreducible representations of the symmetric group S_4 are included.

2. The GPT-potential

For each state of total spin s and isospin t the potential is written in the form [6]

$$V(r) = V_c(r) + V_T(r)S_{12} + V_{Ls}(r)\vec{l} \cdot \vec{s} + V_{LL}(r)L_{12} \tag{2.1}$$

The central, tensor and spin-orbit terms are standard. The operator L_{12} is defined, with the usual notations, as

$$L_{12} = (\vec{\sigma}_1 \cdot \vec{\sigma}_2) l^2 - \tfrac{1}{2}\{(\vec{\sigma}_1 \cdot \vec{l})(\vec{\sigma}_2 \cdot \vec{l}) + (\vec{\sigma}_2 \cdot \vec{l})(\vec{\sigma}_1 \cdot \vec{l})\} \tag{2.2}$$

Each term of the potential is expressed as a sum of Gaussians:

$$V_i(r) = \sum_\alpha V_\alpha \exp(-r^2/r_\alpha^2), \tag{2.3}$$

$i = C, T, LS, LL.$

3. Construction of the wave functions and the energy matrices

Consider the Hamiltonian $\mathscr{H}$ of A nucleons

$$\mathscr{H} = \frac{1}{2m} \sum_{i=1}^{A} P_i^2 + \tfrac{1}{2} \sum_{i=1}^{A} \sum_{\neq j=1}^{A} V(|\vec{r}_i - \vec{r}_j|) \tag{3.1}$$

The translational invariance of the Hamiltonian permits the separation of the centre of mass motion, and as a consequence the Hamiltonian corresponding to the internal motion becomes

$$H = \mathscr{H} - T_{CM}, \tag{3.2}$$

where T_{CM} denotes the centre of mass kinetic energy

$$T_{CM} = \frac{p^2}{2mA} = \frac{1}{2mA} \left(\sum_{i=1}^{A} \vec{P}_i \right)^2$$

By adding and subtracting an oscillator potential referred to the centre of mass, the internal Hamiltonian becomes

$$\begin{aligned} H = {} & \frac{1}{2m} \sum_{i=1}^{A} P_i^2 - \frac{P^2}{2mA} + \tfrac{1}{2} m w^2 \sum_{i=1}^{A} (\vec{r}_i - \vec{R})^2 \\ & + \tfrac{1}{2} \sum_{i=1}^{A} \sum_{\neq j=1}^{A} V(|\vec{r}_i - \vec{r}_j|) - \tfrac{1}{2} m w^2 \sum_{i=1}^{A} (\vec{r}_i - \vec{R})^2, \end{aligned} \tag{3.3}$$

The internal Hamiltonian can be recasted, in terms of the relative coordinates of the nucleons, in the form

$$H = H_0 + V', \tag{3.4}$$

where

$$\begin{aligned} H_0 &= \frac{1}{2A} \sum_{i=1}^{A} \sum_{\neq j=1}^{A} \left[\frac{1}{2m} (\vec{P}_i - \vec{P}_j)^2 + \tfrac{1}{2} m w^2 (\vec{r}_i - \vec{r}_j)^2 \right] \\ &= \frac{1}{A} \sum_{i<j=1}^{A} \left[\frac{(\vec{P}_i - \vec{P}_j)^2}{2m} + \tfrac{1}{2} m w^2 (\vec{r}_i - \vec{r}_j)^2 \right], \end{aligned} \tag{3.5}$$

which is known as the USM-Hamiltonian, and

$$V' = \tfrac{1}{2}\sum_{i=1}^{A}\sum_{j=1\neq}^{A}\left[V(|\vec{r}_i-\vec{r}_j|)-\frac{mw^2}{2A}(\vec{r}_i-\vec{r}_j)^2\right]$$
$$= \sum_{i<j=1}^{A}\left[V(r_{ij})-\frac{mw^2}{2A}r_{ij}^2\right] = \sum_{i<j=1}^{A} v'_{ij}, \tag{3.6}$$

is the residual interaction.

The energy eigenvalues and the corresponding eigenfunctions of the Hamiltonian H_0 are given by [2]:

$$E_N^{(0)} = [N+\tfrac{3}{2}(A-1)]\hbar w, \tag{3.7}$$

$$|A\Gamma M_L;\Gamma_S M_S M_T\rangle \equiv |AN\{\rho\}_{(\nu)[f]};SM_S TM_T\rangle, \tag{3.8}$$

where Γ and Γ_S are the sets of all orbital and spin–isospin quantum numbers characterizing the states, respectively. The number of quanta of excitation N is an irreducible representation (IR) of the unitary group $u_{3(A-1)}$. The representation $\{\rho\}=\{\rho_1,\rho_2,\rho_3\}$ is related to Elliott symbol $(\lambda\mu)$ [4] by the relations $\lambda=\rho_1-\rho_2$, $\mu=\rho_2-\rho_3\cdot\{\rho\}$ is an IR of the unitary group u_{A-1} and the unitary unimodular subgroup of three dimensions $S\mathcal{U}_3$, at the same time. L and M_L stand for the orbital angular momentum and its z-component and are also IR of the rotational groups SO_3 and SO_2, respectively. The representation (ν) is an IR of the orthogonal group O_{A-1} and $[f]$ is an IR of the symmetric group S_A. S, M_S are the spin, its z-component and T, M_T are the isospin, its z-component which are IR of the direct product of the groups $S\mathcal{U}_2\dot{\times}S\mathcal{U}_2\cdot\alpha$ is a repetition quantum number.

The basis functions (3.8) transform according to the following chain of groups [2]:

$$\begin{array}{ccccc}
 & & S\mathcal{U}_4 & \supset & S\mathcal{U}_2\dot{\times}S\mathcal{U}_2 \\
\mathcal{U}_{4A} & \supset \mathcal{U}_{4A} & \supset\ \dot{\times} & & \dot{\times} \\
 & & \mathcal{U}_A & \supset & S_A \\
\mathcal{J}\supset\ \dot{\times} & & \dot{\times} & & \dot{\times} \\
\mathcal{U}_{3(A-1)} & \supset \mathcal{U}_{A-1} & \supset O_{A-1} & \supset & S_A \\
 & \dot{\times} & \dot{\times} & & \dot{\times} \\
 & SU_3 & \supset SO_3 & \supset & SO_2
\end{array} \tag{3.9}$$

By virtue of the basis functions (3.8) one constructs the nuclear wave function with given total momentum J, isotopic spin T and parity π, in the usual manner as follows:

$$|J^\pi TM_J M_T\rangle = \sum_{\Gamma,S} C_{\Gamma S}^{J^\pi T} \sum_{M_L+M_S=M_J} (LM_L, SM_S \mid JM_J)$$
$$\times |A\Gamma M_L;\Gamma_S M_S M_T\rangle, \tag{3.10}$$

where $C_{\Gamma S}^{J^\pi T}$ are the state-expansion coefficients and $(LM_L, SM_S \mid JM_J)$ are the Clebsch-Gordan coefficients [9] of the rotational group SO_3. In the summation (3.10) N is permitted to be either even or odd integer depending on the parity of the state π.

The nuclear wave function (3.10) is an eigenfunction of the operator V' with matrix elements given by [2]

$$\langle J^\pi T M_J M_T | V' | J^\pi T M_J M_T \rangle = \frac{A(A-1)}{2} \sum_{\Gamma S \bar{\Gamma} \bar{S} \bar{J} \Gamma_a \Gamma' S' \Gamma'_a}$$
$$\times C^{J^\pi T}_{\Gamma S} C^{J^\pi T}_{\Gamma' S'} \langle A \Gamma S J T | A-2\bar{\Gamma}\bar{S}\bar{J}\bar{T}; 2\Gamma_a \rangle$$
$$\times \langle A \Gamma' S' J T | A-2\bar{\Gamma}\bar{S}\bar{J}\bar{T}; 2\Gamma'_a \rangle \begin{pmatrix} \bar{L} & \bar{S} & \bar{J} \\ l & s & j \\ L & S & J \end{pmatrix} \begin{pmatrix} \bar{L} & \bar{S} & \bar{J} \\ l' & s & j \\ L' & S' & J \end{pmatrix}$$
$$\times \langle (\varepsilon l s) j t | V'_{A-1,A} | (\varepsilon' l' s) j t \rangle, \tag{3.11}$$

where $\langle A \Gamma S J T | A-2\bar{\Gamma}\bar{S}\bar{J}\bar{T}; 2\Gamma_a \rangle$ stand for the two-particle total fractional parentage coefficients [2, 9, 10] (which are product of orbital and spin–isospin coefficients), the barred indices refer to the set of A-2 nucleons, Γ_a is the set of all orbital and spin–isospin quantum numbers characterizing the two-particle states, and $\varepsilon = 2n + l$, in which n is the radial quantum number of the inter-particle distance joining the last pair. In (3.11) $\begin{pmatrix} j_1 & j_2 & j_{12} \\ j_3 & j_4 & j_{34} \\ j_{13} & j_{24} & J \end{pmatrix}$ are the normalized $9j$-symbols and $\langle (\varepsilon l s) j t | V'_{A-1,A} | (\varepsilon' l' s) j t \rangle$ are the two particle matrix elements of the operator V' which are given by

$$\langle (\varepsilon l s) j t | V'_{A-1,A} | (\varepsilon' l' s) j t \rangle = \langle (\varepsilon l s) j t | V(|\vec{r}_{A-1} - \vec{r}_A|) | (\varepsilon' l' s) j t \rangle$$
$$- \langle \varepsilon l | \frac{m w^2}{2A} (\vec{r}_{A-1} - \vec{r}_A)^2 | \varepsilon' l' \rangle \tag{3.12}$$

4. The mean-square radius and the total cross section for photoabsorption

The mean-square radius $\mathcal{R}$ is defined as

$$\mathcal{R} = \sqrt{r_p^2 + \langle R^2_{Nuc} \rangle} \tag{4.1}$$

where $r_p = 0.85$ fm is the proton radius and the second term is the mean value of the operator

$$R^2_{Nuc} = \frac{1}{A^2} \sum_{i<j=1}^{A} r_{ij}^2 \tag{4.2}$$

Since the operator R^2_{Nuc} does not depend on the spin–isospin variables of the nuclear wave function (3.10) and from the orthonormality conditions of the spin–isospin fractional parentage coefficients one can obtain

$$\mathcal{R} = \left[r_p^2 + \frac{A-1}{A\left[\frac{Mw}{\hbar}\right]} \sum_{\Gamma \Gamma' \bar{\Gamma} \Gamma_{0a} \Gamma'_{0a}} C^{J^\pi T}_{\Gamma S} C^{J^\pi}_{\Gamma' S} \right.$$
$$\times \langle A \Gamma | A-2\bar{\Gamma}, 2\Gamma_{0a} \rangle \langle A \Gamma' | A-2\bar{\Gamma}, 2\Gamma'_{0a} \rangle \left\{ \left(\varepsilon + \frac{3}{2}\right) \delta^{\varepsilon'}_{\varepsilon} \right.$$
$$\left. \left. - \tfrac{1}{2}\sqrt{(\varepsilon - l + 2)(\varepsilon + l + 3)}\, \delta^{\varepsilon'}_{\varepsilon+2} - \tfrac{1}{2}\sqrt{(\varepsilon - l)(\varepsilon + l + 1)} \delta^{\varepsilon'}_{\varepsilon-2} \right\} \delta^{(\nu')[f']}_{S(\nu)[f]} \right]^{1/2}, \tag{4.3}$$

where $[Mw/\hbar]$, Γ_{0a} and $\langle A\Gamma \mid A-2\bar{\Gamma}, 2\Gamma_{0a}\rangle$ denote the value of $Mw/\hbar$, the set of all orbital quantum numbers of the two-particle states and the two-particle orbital fractional parentage coefficients, respectively.

The partial integral cross-section of the γ-quanta photoabsorption is calculated according to the well-known formula [5]

$$\int \sigma \, dE_f = (2\pi)^3 \frac{e^2}{\hbar c} \frac{(\hbar c)^2}{E_\gamma} \frac{1}{2J_i+1} \times \sum_{\lambda \geq 1} [|\langle f \| T^{el}_\lambda \| i\rangle|^2 + |\langle f \| T^{mag}_\lambda \| i\rangle|^2], \tag{4.4}$$

where $|i\rangle$ and $|f\rangle$ are the wave functions of the initial and final states, respectively, T^{el}_λ and T^{mag}_λ are the one-particle multipole operators of the interaction of the γ-quanta with the nucleus. The right-hand side of (4.4) is calculated according to the dipole approximation.

5. Results and conclusions

The nuclear energy matrices are functions of the oscillator parameter $\hbar w$ and the number of quanta of excitation N. For the even-parity states of ^{4}He, $|J^\pi, T\rangle = |0^+, 0\rangle, |1^+, 0\rangle$ and $|2^+, 0\rangle$, we consider bases of the USM corresponding to $N = 0$, 2, 4 and 6. For the odd-parity states, $|J^\pi, T\rangle = |0^-, 0\rangle$, $|1^-, 0\rangle$, $|2^-, 0\rangle$, $|0^-, 1\rangle$, $|1^-, 1\rangle$ and $|2^-, 1\rangle$, we consider bases corresponding to $N = 1$, 3 and 5. The oscillator parameter $\hbar w$ is allowed to vary in the range $10 \leqslant \hbar w \leqslant 22$ Mev in order to obtain the best fit to the spectrum of ^{4}He.

The best fit occurred at $\hbar w = 17$ Mev. The calculated value of the binding energy of ^{4}He is equal to 25.48 Mev. The corresponding nuclear wave function is expressed in terms of the USM bases $|N\{\rho\}(\nu)[f]LS\rangle$ as follows:

$$\begin{aligned}
|0^+0\rangle_1 = {} & -0.9172\,|0\{0\}(0)[4]00\rangle - 0.1267\,|2\{2\}(0)[4]00\rangle \\
& +0.0129\,|2\{2\}(2)[22]00\rangle - 0.2198\,|2\{2\}(2)[22]22\rangle \\
& +0.0093\,|2\{11\}(1)^*[211]11\rangle - 0.0748\,|4\{4\}(0)[4]00\rangle \\
& +0.0066\,|4\{4\}(2)[22]00\rangle - 0.1314\,|4\{4\}(2)[22]22\rangle \\
& +0.1918\,|4\{31\}(2)[31]11\rangle - 0.0632\,|4\{31\}(2)[22]22\rangle \\
& -0.0261\,|4\{22\}(0)[4]00\rangle - 0.0843\,|4\{22\}(2)[22]22\rangle \\
& -0.0492\,|6\{6\}(0)[4]00\rangle + 0.0026\,|6\{6\}(2)[22]00\rangle \\
& -0.1176\,|6\{6\}(2)[22]22\rangle + 0.0798\,|6\{51\}(2)[31]11\rangle
\end{aligned} \tag{5.1}$$

The results of calculating the excited-state energy levels (E^*_{Th}) for ^{4}He, subtracting from them the ground-state energy, are presented, together with the corresponding experimental values (E^*_{Exp}), in Table 1.

A new level for ^{4}He is obtained near the threshold energy value. This is the level $1^+, 0$, with an excited energy $E^*_{Th} = 33.0$ Mev. The main contributions in this level are due to bases having $[f] = [31]$ and little contributions are due to bases having $[f] = [22]$.

It is seen from Table 1 that the order of the levels is correct and the obtained

Table 1
Energy levels for ^{4}He

J^π, T	$(0^+,0)_2$	$0^-,0$	$2^-,0$	$2^-,1$	$(1^-,1)_1$	$0^-,1$	$(1^-,1)_2$	$2^+,0$	$1^-,0$	$1^+,0$
E^*_{Th} (in Mev)	20.7	21.8	23.1	25.2	26.2	26.9	28.7	29.1	29.5	33.0
E^*_{Exp}[8] (in Mev)	20.2	21.4	22.4	24.3	25.1	26.5	27.8	28.0	28.3	—

energy values are in good agreement with the corresponding experimental [8] data.

All the other characteristics of ^{4}He are calculated using the value of $\hbar w = 17$ Mev. The calculated value of the mean-square radius of ^{4}He, obtained by using the wave function (5.1), is $\mathcal{R} = 1.70$ fm.

Among all the levels, given in Table 1, the dipole transition is allowed only for levels [9] having $J = 1$ and $T = 1$. These are the levels $(1^-, 1)_1$ and $(1^-, 1)_2$. The calculated value of the total integral cross section of the γ-quanta photoabsorption by the nucleus ^{4}He is equal to 69 Mev.mb.

The obtained results of the binding energy and the mean square radius of ^{4}He are in good agreement with the corresponding experimental values 28 Mev and 1.63 ± 0.04 fm, respectively. Furthermore when the bases $|8\{8\}(0)[4]00\rangle$, $|8\{8\}(2)[22]22\rangle$ and $|8\{8\}(2)[22]00\rangle$ are added to those appearing in equation (5.1) values of the binding energy and the mean-square radius of ^{4}He equal to 25.62 Mev and 1.69 fm, respectively, are obtained. These bases (corresponding to $N = 8$) are not included in the ground-state wave function for two reasons:

(i) the improvements in the values of $\mathcal{R}$ and the binding energy of ^{4}He are too poor (ii) and the inclusion of bases with $N = 8$ to the positive-parity states must be accompanied by inclusion of bases with $N = 7$ to the negative-parity states, which are not available at hand.

It is to be noticed that bases having IR of the symmetric group S_4, equals to [31] (for $N \geq 4$) are very important and must be considered in order to obtain good agreement between the calculated and the experimental ground-state characteristics of ^{4}He.

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