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Limiting particle density of a relativistic Fermi gas in a magnetic field

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Abstract. We investigate the total particle density in the grand canonical ensemble of a relativistic Fermi gas consisting of spin-1/2 particles in the presence of a constant homogeneous external magnetic field. For the special case of equal numbers of particles and antiparticles we compute the behavior of the total particle density in the limit of ultrahigh magnetic fields. Then we calculate the form of the low magnetic field limit, which we compare with the ideal relativistic Fermi gas.

In a previous work [1] we have studied the charged Fermi gas in a strong external homogeneous magnetic field B at sufficiently high temperatures T so that it is necessary to take into account the presence of the antiparticles. However, we computed numerically the special case of a vanishing chemical potential which corresponds to the situation of equal numbers of particles and antiparticles, that is to say that the total charge of the system is zero. An interesting feature of this system was found to be the steady rise of the magnetization for large external fields up to the point of saturation in a manner analogous to the expected behavior in ordinary magnetism. In the present work we have extended our previous investigation further in order to gain a more precise physical understanding of certain limiting cases relating to the total particle number density ρ.

First let us recall that ρ can be written as

$$\rho = \frac{2|e|B}{(2\pi)^2\hbar c}\left[\int_0^\infty dk\left(\frac{1}{e^{\beta(E_0(K)-\mu)}+1}+\frac{1}{e^{\beta(E_0(K)+\mu)}+1}\right)\right.$$
$$\left.+2\int_0^\infty dk\sum_{n=1}^{\infty}\left(\frac{1}{e^{\beta(E_n(K)-\mu)}+1}+\frac{1}{e^{\beta(E_n(K)+\mu)}+1}\right)\right], \tag{1}$$

for which we have maintained our previous notations (this is the equation (9) in Ref. 1 where $|e|^2$ was incorrectly printed instead of $|e|$). $E_n(K)$ represents the energy spectrum of the Fermi particles in an external magnetic field. One can express the integrals appearing here in a compact form in terms of the modified Bessel functions [2] in the integral representation

$$K_\nu(x) = \int_0^\infty e^{-x\cosh t}\cosh(\nu t)\,dt. \tag{2}$$

Thus we may write the total particle number density of both the particles and anti-particles as follows:

$$\rho = \frac{|e|\,Bm}{\pi^2\hbar^2}\sum_{l=0}^{\infty}(-1)^l \cosh((l+1)\beta\mu)[K_1((l+1)\beta mc^2)$$
$$+2\sum_{n=1}^{\infty}\sqrt{(1+2nB/B_c)}K_1((l+1)\beta mc^2\sqrt{(1+2nB/B_c)})] \tag{3}$$

where, as previously, $B_c = m^2c^3/|e|\hbar$.

We now return to the special case of $\mu = 0$ in the high field limit of the particle density ρ. From equation (3) we can see for $B \gg B_c$ that $\rho(B)$ undergoes a steady growth which corresponds to the creation of many particle-antiparticle pairs. We have plotted the numerical evaluation of $\rho(B)$ in Fig. 1, which shows an approximate linear growth in terms of B/B_c. This behavior can be analytically understood by the fact that $\rho(B)$ contains a direct proportionality to B. The sum of the integrals at high fields is roughly of the same type as that of the magnetization M/V which we have shown in Ref. 1 to saturate at large values of B. Thus our numerical calculations substantiate the increasing number of particle-antiparticle pairs under these extreme conditions.

In the opposite case of a low magnetic field ($B \ll B_c$) we find that $\rho(B)$ approaches a limiting value ρ_L which depends upon the temperature. This fact we illustrate for three different temperatures in Fig. 1. Furthermore, we give the estimated numerical values of $\rho_L(T)$ for various temperatures in terms of the electron restmass energy in Table 1. These values of $\rho_L(T)$ can be directly compared to the particle number density of the ideal relativistic Fermi gas $\rho_G(T)$ at the same temperatures. Thus we can see that the values of $\rho_L(T)$ and $\rho_G(T)$ are numerically very close to each other.

We can explain the low field limit analytically by means of the following considerations. For the case of vanishing magnetic field strength ($B \to 0$) the sums

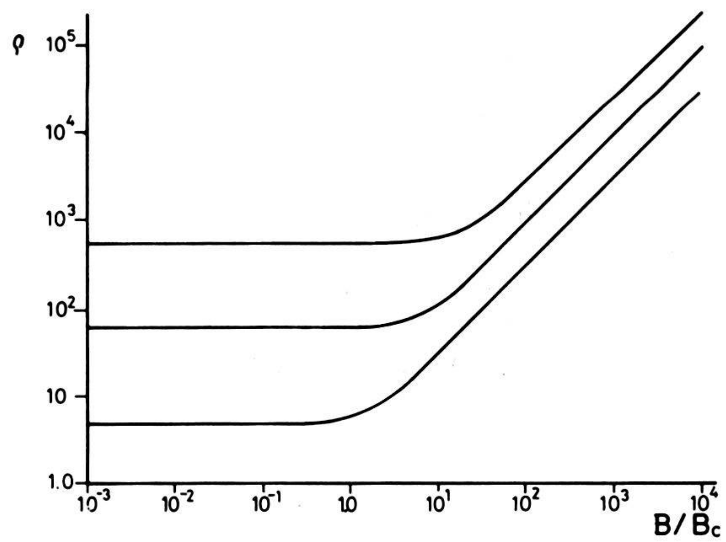

Figure 1
The particle number density ρ (in units of $10^{29}\,\mathrm{cm}^{-3}$) is displayed as a function of B/B_c. The lower curve corresponds to the temperature value $T = 0.5$, the middle one to $T = 1$ and the upper one to $T = 2$, respectively, where T is measured in units of electron restmass energy mc^2 (511 KeV).

Table 1
A comparison of the numerical values of ρ_L and ρ_G in particles per cm^3 for the three temperatures given in Fig. 1. The last column gives the saturation magnetization density in magnetons per cm^3.

T	$\rho_L(T)$	$\rho_G(T)$	M/V
0.5	4.29×10^{29}	4.29×10^{29}	5.80×10^{-4}
1.0	5.33×10^{30}	5.33×10^{30}	4.70×10^{-3}
2.0	4.84×10^{31}	4.84×10^{31}	2.46×10^{-2}

in (3) can be coverted into integrals as follows:

$$\rho_L(\beta)=\lim_{B\to0}\rho(\beta,B)$$
$$=\frac{2}{\pi^2}\left(\frac{mc}{\hbar}\right)^3\sum_{n=0}^{\infty}(-1)^n\int_0^{\infty}\sqrt{1+2x}\,K_1((n+1)\beta mc^2\sqrt{1+2x})\,dx$$
$$=\frac{2}{\pi^2}\left(\frac{mc}{\hbar}\right)^3\sum_{n=0}^{\infty}(-1)^n\int_1^{\infty}y^2K_1((n+1)\beta mc^2y)\,dy. \tag{4}$$

For the case of the three dimensional ideal gas taking into account the spin for the presence of equal numbers of particles and antiparticles the particle number density $\rho_G(\beta)$ is readily calculated to be

$$\rho_G(\beta)=\frac{2}{\pi^2}\left(\frac{mc}{\hbar}\right)^3\sum_{n=0}^{\infty}(-1)^n\frac{K_2((n+1)\beta mc^2)}{(n+1)\beta mc^2}. \tag{5}$$

In order to establish the identity of (4) and (5), we use the following two properties [3] of the functions $K_\nu(x)$:

$$\int_0^{\infty}x^{\mu}K_\nu(ax)\,dx=2^{\mu-1}a^{-\mu-1}\Gamma\left(\frac{1+\mu+\nu}{2}\right)\Gamma\left(\frac{1+\mu-\nu}{2}\right) \tag{6}$$

for Re $(\mu+1\pm\nu)>0$ and Re $a>0$;

$$\int_0^{1}x^{\nu+1}K_\nu(ax)\,dx=2^{\nu}a^{-\nu-2}\Gamma(\nu+1)-a^{-1}K_{\nu+1}(a). \tag{7}$$

These relationships immediately establish $\rho_L(\beta)=\rho_G(\beta)$. This means that the three dimensional rotational symmetry $O(3)$, which is broken in the presence of the external magnetic field B reducing it to $O(2)$, is restored in the limit of vanishing B. Further calculations show that this result holds true for arbitrary spatial dimensions, so that there is no appearance of spontaneous symmetry breaking for the system.

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[2] G. N. WATSON, *A Treatise on the Theory of Bessel Functions* (Cambridge University Press, Cambridge, 1966), see especially Sec. 3.7.
[3] I. S. GRADSHTEYN and I. M. RYZHIK, *Table of Integrals, Series and Products* (English translation, Academic Press, New York, 1965), see p. 648 (Eq. (16)) and p. 683 (Eq. (8)).