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The quasipotential formalism for pion exchange effects in the two-nucleon system

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Abstract. A systematic exposition is given of the covariant quasipotential formalism for the two-nucleon system, showing how the potential itself and the electromagnetic 4-current can be consistently obtained within the same formalism. We calculate the impulse approximation charge density and current density with relativistic corrections and the one-pion exchange and two-pion exchange charge densities which it is correct to use in conjunction with c.m. wavefunctions derived from an energy independent potential. The results for the one-pion exchange charge density with pseudoscalar and pseudovector πNN coupling are related via the equivalence theorem and it is shown how one may obtain a physically reasonable result which is independent of the coupling. The difference between our results and those obtained using the transformation method is traced to the difference between the prescriptions for the retardation in the one-pion exchange potential in the two methods.

I. Introduction

It is well known that the calculation of relativistic and meson exchange corrections to the usual nonrelativistic treatment of nuclei presents many difficulties. In particular, the problem of providing a consistent treatment of both the scattering and bound state properties of the two-nucleon system and its electromagnetic properties has received considerable attention in recent years. Such a treatment is necessary if the electromagnetic properties and reactions of the deuteron are to be properly understood.

In our work [1] on the effect of meson exchange on the theoretical calculation of the forward cross section for deuteron photodisintegration (hereafter referred to as JW) we used the quasipotential approach, but did not attempt to develop the formalism properly. As far as we know, there does not exist in the literature a complete and systematic development of the quasipotential formalism for the two-nucleon system. Our aim in this paper is to provide such a development, and to do it in a covariant way in an arbitrary frame of reference. In later papers we shall present applications of the formalism to a variety of problems involving the deuteron, including the forward cross section for deuteron photodistintegration.

In Section II we explain the basic quasipotential formalism. Starting from the

Bethe–Salpeter equation, we use a *covariant* reduction procedure based upon the prescription due originally to Blankenbecler and Sugar [2] (BBS). The BBS prescription is simple only in the two-nucleon rest frame, but we show how it can be given covariantly in an arbitrary frame and how it leads to a relativistic equation of motion for the moving two-nucleon system. This equation of motion can by means of a Lorentz boost be transformed to the rest frame, where it is just the nonrelativistic Schrödinger equation. We carry through this procedure both for a two-nucleon bound state and for a continuum state. The quasipotential which appears in the Schrödinger equation can be expressed in terms of Feynman diagrams. The relationship between the two-nucleon wavefunction in the moving frame and that in the rest frame is very simple, and relativistic effects can be discussed easily. The wavefunction undergoes a Wigner rotation and the nucleon–nucleon relative momentum is shifted. No expansion in powers of c^{-2} is made.

We then give a general expression for the matrix element of the electromagnetic four-current density, taken between wavefunctions derived from the quasipotential. We do this when the matrix element is taken between bound states and when it is taken between a bound state and a continuum state. We are able to put the two-nucleon current density into a compact form which involves only the contributions of certain Feynman diagrams for the process $\gamma NN \to NN$, which are irreducible in the same sense as one uses in referring to $NN \to NN$ diagrams. Moreover, one can take the matrix elements between rest frame wavefunctions, even though the initial and final states cannot simultaneously be in the rest frame, because the transformation properties of the wavefunction are known. With a single equation as the starting point, one can derive in a unified way expressions for the relativistic impulse approximation and meson exchange contributions to the charge and current densities.

The lowest order diagram (no mesons exchanged) gives the relativistic expressions for the impulse approximation charge and current densities, which we derive in Section III. For the charge density we find the well known corrections of order m^{-2} to the nonrelativistic impulse approximation, namely the spin-orbit and Darwin–Foldy terms, whose importance for deuteron photodisintegration was first demonstrated by Cambi, Mosconi and Ricci [3]. They used the result in Appendix D of De Forest and Walecka [4] and we generalize that result to an arbitrary frame of reference. From the relation between the two-nucleon wavefunction in a moving frame and that in the rest frame, we derive the relativistic corrections of order m^{-2} to the wavefunction, and thence obtain additional corrections of the same order to the impulse approximation charge density which arise from the transformation of the initial and final wavefunctions to their individual rest frames. This result also generalizes to an arbitrary frame a result already obtained by other approaches. At the same time, we are able within our formalism to obtain the expression for the impulse approximation current density in a general frame, which we evaluate up to order m^{-2} and to first order in the external momenta.

The remainder of the paper (Sections IV–VIII) is devoted to a thorough study of the one-pion exchange ($1\pi E$) and two-point exchange ($2\pi E$) charge densities within the quasipotential framework. For $\rho(1\pi E)$ we consider both pseudoscalar (PS) and pseudovector (PV) πNN coupling. The leading contributions to $\rho(2\pi E)$ do not depend on the form of coupling. Since the Hamiltonians of the $\pi\gamma N$ system, for PS and PV πNN coupling, can be derived from a single Hamiltonian,

each together with its appropriate contact terms, it should be possible, if all calculations are carried out consistently, to locate any difference between the results for PS and PV coupling in a suitable contact term [5]. The contact term which should account for the difference has been derived for example by Friar [6] in his study of the equivalence theorem, which relates the Hamiltonians for the $\pi\gamma N$ system with PS and PV coupling by means of a unitary Dyson–Foldy transformation. Now H(PV) contains not only the $\gamma\pi NN$ contact interaction which is generated by the usual (minimal) γNN charge coupling, but also a gauge invariant magnetic moment $\gamma\pi NN$ contact interaction which is generated by the Pauli term in the γNN interaction. By carefully considering all the possible contributions to $\rho(1\pi E)$ we have been able to show that it is this second contact interaction which accounts for the difference between the results for PS and PV coupling.

In Section IV we consider a very subtle contribution to $\rho(1\pi E)$ which depends on the coupling and whose origin lies in the $1\pi E$ NN potential. When this potential is calculated to order m^{-2}, one of the relativistic corrections to the static potential is independent of the coupling and is included in semi-phenomenological potentials like the Nijmegen [7] and Paris [8] potentials. There is, in addition, a second relativistic correction which depends on the πNN coupling. If one calculates with wavefunctions deduced from a potential which does *not* include this second correction, then one needs to take into account the change in the wavefunction which is induced by the omitted piece of the potential. It turns out that because of its particular form, it is possible to separate from this piece of the potential a $1\pi E$ part. The changes in the initial and final state wavefunctions which are induced in this way can be translated into a contribution to the charge density which is of $1\pi E$ form and which depends on the πNN coupling. This derivation of an effective $1\pi E$ charge density from a piece of the $1\pi E$ NN potential which has a particular form in momentum space is central to our discussion of $1\pi E$ effects. Its inclusion in $\rho(1\pi E)$ is essential if one is to be able to relate $\rho(1\pi E)$ for PS and for PV coupling via the equivalence theorem and, as we shall discuss later in this introduction, also if one wishes to make a correct practical calculation using wavefunctions derived from an NN potential whose $1\pi E$ part is independent of the coupling.

In Section V we derive all the other parts of the $1\pi E$ and $2\pi E$ charge densities. The section begins with the derivation of a coupling dependent contribution to $\rho(1\pi E)$ which arises in a subtle way when the quasipotential formalism is developed covariantly. All the other contributions in Section V arise either from particular Feynman diagrams or from the retardation in the $1\pi E$ potential. By this we mean the way in which the $1\pi E$ potential depends on the time-component of the 4-momentum transfer. In Section VI we prove the result described earlier for the difference between $\rho(1\pi E)$ for PS and for PV coupling. It is important to make this check that our formalism satisfies the constraints of the equivalence theorem, as Friar [9] has emphasized.

It is a general property of the quasipotential method that it generates an energy dependent quasipotential; in our case the uncrossed and crossed $2\pi E$ potentials are energy dependent, in the sense that they depend explicitly on the time component of the total 4-momentum of the two nucleon system. This energy dependence means that the bound state wavefunction is no longer normalized to unity. It is possible, however, to use a method given by Friar [6] to extract from an energy dependent potential an energy independent part, in terms of which a

modified wavefunction satisfies the usual bound state equation with the same bound state energy. It turns out that this modified wavefunction is normalized to unity, and that, when the two-nucleon 4-current is taken between modified wavefunctions, it has itself to be modified by the addition of extra pieces. In the particular case of the $2\pi E$ charge density, one finds that when these pieces are added, the resulting uncrossed and crossed $2\pi E$ charge densities separately have the property that the total charge is zero. This discussion of the energy dependence of the quasipotential and its consequences is given in Section VII.

The detailed expressions for exchange operators in the quasipotential formalism are in general different from those derived using the unitary transformation method. In particular, the $1\pi E$ and uncrossed $2\pi E$ charge densities are different. We show in Section VIII that these differences arise from the different retardation prescriptions for the $1\pi E$ potential. In this context, we shall clear up a misunderstanding in the paper of Sato, Kobayashi and Ohtsubo [10] (hereafter referred to as SKO). We shall also show how our results for $\rho(1\pi E)$ relate to those derived by Hyuga and Gari [11] (hereafter referred to as HG) using the unitary transformation method.

It will be clear from the comparison between our results and those of HG that it is not correct to use their $1\pi E$ charge density in practical calculations. Apart from an error which arises in their work because they do not have a fully covariant treatment, there are terms in their $1\pi E$ potential which are coupling dependent and which need to be interpreted as effective $1\pi E$ charge densities according to the method which we develop in Section IV. These should then be added to the explicit expression which HG give for $\rho(1\pi E)$, if one wishes to make a practical calculation using a semiphenomenological NN potential whose $1\pi E$ part is coupling independent. There are treatments in recent literature of the form factors of the deuteron and of the forward photodisintegration of the deuteron which are incorrect because they use expressions for $\rho(1\pi E)$ which are incompatible with a coupling independent $1\pi E$ NN potential. We shall discuss these papers, and present our own calculations, in future publications.

Zuilhof and Tjon [12] point out that there are conceptual difficulties with the definition of the deuteron current in the BBS model, and draw attention to the lack of a consistent treatment of both the two-nucleon system and its electromagnetic properties in a quasipotential approach. We give such a treatment in this paper, and develop a formalism which is compact and consistent, which is fully relativistic and does not require an expansion in powers of c^{-2} and in which relativistic and meson exchange effects can be calculated in a straightforward and unified way. Moreover, it is the only formalism available for the two nucleon system which can be used directly for practical calculations of deuteron properties and reactions.

II. Basic formalism

The starting point of the formalism which we shall use for meson exchange processes is the BBS reduction of the Bethe–Salpeter equation. For the NN scattering amplitude W 'without legs' this equation is, in symbolic operator notation,

$$W = U + UGW = U + WGU. \tag{2.1}$$

The quantity U is the amplitude obtained by summing all irreducible two-nucleon diagrams and has a meaning only in the context of some field theoretic description of the NN interaction. The Green function G is well known, and does not need to be written here. The products UGW and WGU in (2.1) imply sums over spinor indices and an integration over an internal relative 4-momentum.

In order to apply (2.1) to bound state problems which are described by a single-time wavefunction, as is the case in nuclear physics, the quasipotential method has been developed, in which the 4-dimensional equation (2.1) is reduced to a 3-dimensional equation by using a new Green function g which restricts the time-component of the internal momentum to a fixed value. Equation (2.1) is then replaced by

$$W = V + VgW = V + WgV, \tag{2.2}$$

where U, V are connected by

$$V - U = V(G - g)U = U(G - g)V. \tag{2.3}$$

Equation (2.2) is now a simpler 3-dimensional integral equation, but the complexity of equation (2.1) has been transferred to equation (2.3). In practice one hopes that the quasipotential V can be obtained by an iterative procedure, in which U and V are expanded in series which consist of terms which correspond to the number of bosons exchanged between the interacting nucleons.

There are many choices of g in the literature, but we shall use the BBS choice in its covariant form (see for example Woloshyn and Jackson [13]), which leads naturally to the Lippmann–Schwinger equation in momentum space and thence to the Schrödinger equation in coordinate space. The BBS Green function belongs to that class which restricts the internal summation over spinor indices to positive energy states only (for each nucleon). We now fix our notation in detail by considering the process

$$N_1(\tfrac{1}{2}P + p', \lambda_1') + N_2(\tfrac{1}{2}P - p', \lambda_2') \to N_1(\tfrac{1}{2}P + p'', \lambda_1'') + N_2(\tfrac{1}{2}P - p'', \lambda_2'').$$

Primed and doubly primed quantities will invariably be quantities associated with the initial and the final state respectively. The subscripts 1 and 2 will always label quantities associated with nucleon 1 and nucleon 2 respectively. The total 4-momentum is $P = (P_0, \vec{P})$ and the initial and final relative 4-momenta are p', p'' respectively. The quantities λ are the helicities. We now define an amplitude $\hat{V}$ with positive energy spinors attached:

$$\begin{aligned}\hat{V}_{\lambda_1''\lambda_2'';\lambda_1'\lambda_2'}&(P; p'', p') \\ &= \bar{u}_1^{(\lambda_1'')}(\tfrac{1}{2}\vec{P} + \vec{p}'')\bar{u}_2^{(\lambda_2'')}(\tfrac{1}{2}\vec{P} - \vec{p}'')V(P; p'', p')u_1^{(\lambda_1')}(\tfrac{1}{2}\vec{P} + \vec{p}')u_2^{(\lambda_2')}(\tfrac{1}{2}\vec{P} - \vec{p}'),\end{aligned} \tag{2.4}$$

and similarly for $\hat{W}$. The sums over spinor indices have been suppressed in (2.4). It is not necessary in what follows to exhibit the helicities explicitly, and we shall suppress them from now on.

The covariant BBS prescription fixes the time-component of the internal

relative momentum according to the BBS condition $P \cdot \hat{k} = 0$. Thus

$$\hat{k}_0 = \frac{\vec{P} \cdot \vec{k}}{P_0} = \frac{E(\frac{1}{2}\vec{P} + \vec{k})^2 - E(\frac{1}{2}\vec{P} - \vec{k})^2}{2P_0}, \tag{2.5}$$

where $E(\vec{p}) = \sqrt{m^2 + \vec{p}^2}$. If $k = (k_0, \vec{k})$ we shall use $\hat{k}$ to denote the restricted 4-vector $\hat{k} = (\hat{k}_0, \vec{k})$. The covariant BBS Green function g is given by

$$g(P; k) = 2\pi\, \delta(k_0 - \hat{k}_0)\hat{g}(P; \hat{k})\Lambda_1^+(\tfrac{1}{2}\vec{Q} + \vec{k})\Lambda_2^+(\tfrac{1}{2}\vec{Q} - \vec{k}), \tag{2.6}$$

where Λ^+ is the usual positive energy projection operator,

$$\hat{g}(P; \hat{k}) = \frac{4m^2}{\tilde{E}(P; \hat{k})[P^2 - 4(m^2 - \hat{k}^2)]}, \tag{2.7}$$

$$Q = Q(P; \hat{k}) = 2(P/\sqrt{P^2})\sqrt{m^2 - \hat{k}^2} \tag{2.8}$$

$$\tilde{E}(P; \hat{k}) = Q_0/2 = (P_0\sqrt{P^2})\sqrt{m^2 - \hat{k}^2} \tag{2.9}$$

and Q has the property that $(Q/2 \pm \hat{k})^2 = m^2$, since $Q \cdot \hat{k} = 0$ according to the BBS condition. Therefore the projection operators in equation (2.6) for $g(P; k)$ remain positive energy projection operators under Lorentz transformations.

Using (2.6), the full statement of (2.2) is

$$\begin{aligned} &\hat{W}(P; p'', p') \\ &\quad = \hat{V}(P; p'', p') + (2\pi)^{-3} \int d^3k \hat{V}(P; p'', \hat{k})\hat{g}(P; \hat{k})\hat{W}(P; \hat{k}, p') \\ &\quad = \hat{V}(P; p'', p') + (2\pi)^{-3} \int d^3k \hat{W}(P; p'', \hat{k})\hat{g}(P; \hat{k})\hat{V}(P; \hat{k}, p'). \end{aligned} \tag{2.10}$$

Care is required in interpreting (2.10). The $\hat{W}$ and $\hat{V}$ which stand alone are defined in (2.4), using spinors which we shall in future refer to as *standard* spinors. However, the appearance of $\hat{k}$ in the quantities which stand in the integrals in (2.10) signals that, on the side in which $\hat{k}$ appears, *BBS* spinors are used. Thus, for example, $\hat{V}(P; \hat{k}, p')$ stands for

$$\hat{V}(P; \hat{k}, p') = \bar{u}_1(\tfrac{1}{2}\vec{Q} + \vec{k})\bar{u}_2(\tfrac{1}{2}\vec{Q} - \vec{k})V(P; \hat{k}, p')u_1(\tfrac{1}{2}\vec{P} + \vec{p}')u_2(\tfrac{1}{2}\vec{P} - \vec{p}').$$

We shall need (2.10) later, but now we need a more restrictive equation in which p'', p' are fixed at $\hat{p}''$, $\hat{p}'$ respectively and at the same time the corresponding standard spinors are replaced by BBS spinors. We therefore define a quantity $\hat{V}(P; \hat{p}'', \hat{p}')$ by

$$\hat{V}(P; \hat{p}'', \hat{p}') = \bar{u}_1(\tfrac{1}{2}\vec{Q}'' + \vec{p}'')\bar{u}_2(\tfrac{1}{2}\vec{Q}'' - \vec{p}'')V(P; \hat{p}'', \hat{p}')u_1(\tfrac{1}{2}\vec{Q}' + \vec{p}')u_2(\tfrac{1}{2}\vec{Q}' - \vec{p}'), \tag{2.11}$$

where $\vec{Q}'$, $\vec{Q}''$ stand for $\vec{Q}(P; \hat{p}')$, $\vec{Q}(P; \hat{p}'')$ respectively. We shall sometimes use the symbol $\hat{V}_0$ as a shorthand for the fully restricted potential defined by equation (2.11). The quantity $\hat{W}_0$ is defined in the same way as $\hat{V}_0$ in (2.11) and from

(2.10) the integral equations it satisfies are

$$\begin{aligned}\hat{W}(P;\hat{p}'',\hat{p}') &= \hat{V}(P;\hat{p}'',\hat{p}')+(2\pi)^{-3}\int d^3k\hat{V}(P;\hat{p}'',\hat{k})\hat{g}(P;\hat{k})\hat{W}(P;\hat{k},\hat{p}') \\ &= \hat{V}(P;\hat{p}'',\hat{p}')+(2\pi)^{-3}\int d^3k\hat{W}(P;\hat{p}'',\hat{k})\hat{g}(P;\hat{k})\hat{V}(P;\hat{k},\hat{p}').\end{aligned} \tag{2.12}$$

Now if the amplitude $\hat{W}_0$ has a pole at $P^2=M^2$ corresponding to a bound state of mass M, the equations satisfied by the bound state vertex function $\hat{\Gamma}$ follow directly from (2.12):

$$\begin{aligned}\hat{\Gamma}(P;\hat{p}'') &= (2\pi)^{-3}\int d^3k\hat{V}(P;\hat{p}'',\hat{k})\hat{g}(P;\hat{k})\hat{\Gamma}(P;\hat{k}), \\ \overline{\hat{\Gamma}(P;\hat{p}')} &= (2\pi)^{-3}\int d^3k\overline{\hat{\Gamma}(P;\hat{k})}\hat{g}(P;\hat{k})\hat{V}(P;\hat{k},\hat{p}').\end{aligned} \tag{2.13}$$

In equations like (2.13) involving bound state functions it will always be understood that $P_0=\sqrt{M^2+\vec{P}^2}$. The normalization condition for $\hat{\Gamma}$ is

$$\begin{aligned}1 = 4m\Big[&-(2\pi)^{-3}\int d^3p\overline{\hat{\Gamma}(P;\hat{p})}\frac{\partial\hat{g}(P;\hat{p})}{\partial(P^2)}\hat{\Gamma}(P;\hat{p}) \\ &-(2\pi)^{-6}\int d^3(p'',p')\overline{\hat{\Gamma}(P;\hat{p}'')}\hat{g}(P;\hat{p}'')\frac{\partial\hat{V}(P;\hat{p}'',\hat{p}')}{\partial(P^2)}\hat{g}(P;\hat{p}')\hat{\Gamma}(P;\hat{p}')\Big],\end{aligned} \tag{2.14}$$

where the derivative is evaluated at $P^2=M^2$. Now note that $\hat{V}(P;\hat{p}'',\hat{p}')$, defined in (2.11), is Lorentz invariant, as also is $(\sqrt{P^2/P_0})\,d^3p$, on account of the invariance of $\delta(P\cdot k)\,d^4k$. Here we have made use of the covariant BBS condition (2.5). Further, it follows from the definitions of $\hat{g}$, $\tilde{E}$ in (2.7), (2.9) respectively that $(P_0/\sqrt{P^2})\hat{g}$ and $(\sqrt{P^2}/P_0)\tilde{E}$ are also invariant. The normalization condition (2.14) then shows that the bound state vertex function $\hat{\Gamma}(P;\hat{p})$ is Lorentz invariant.

Now define a wavefunction ϕ in terms of $\hat{\Gamma}$ by

$$\phi(\vec{P};\vec{p}) = \sqrt{\frac{\tilde{E}(P;\hat{p})}{m}}\,\hat{g}(P;\hat{p})\hat{\Gamma}(P;\hat{p}) \tag{2.15}$$

and a modified quasipotential $\tilde{V}$ by

$$\tilde{V}(P;\vec{p}'',\vec{p}') = \sqrt{\frac{m}{\tilde{E}(P;\hat{p}'')}}\,\hat{V}(P;\hat{p}'',\hat{p}')\sqrt{\frac{m}{\tilde{E}(P;\hat{p}')}}, \tag{2.16}$$

where $\tilde{E}$ is defined in (2.9), $\hat{g}$ in (2.7) and it is of course understood that $P_0=\sqrt{M^2+\vec{P}^2}$. Using these definitions, equation (2.13) becomes

$$\phi(\vec{P};\vec{p}) = \hat{g}(P;\hat{p})\frac{\tilde{E}(P;\hat{p})}{m}(2\pi)^{-3}\int d^3k\tilde{V}(P;\vec{p},\vec{k})\phi(\vec{P};\vec{k}) \tag{2.17}$$

or, in more detail,

$$(4m)^{-1}[M^2-4m^2+4\hat{p}^2]\phi(\vec{P};\vec{p})=(2\pi)^{-3}\int d^3k\tilde{V}(P;\vec{p},\vec{k})\phi(\vec{P};\vec{k}), \tag{2.18}$$

with a similar equation for $\bar{\phi}$. From the invariance properties already established for $\tilde{E}$, $\hat{g}$ and $\hat{\Gamma}$, it follows from the definition (2.15) that $(P_0/\sqrt{P^2})^{1/2}\phi(\vec{P};\vec{p})$ is a Lorentz invariant. This result is also clear from the normalization condition for ϕ, which may be deduced from (2.14). Using the definitions (2.7), (2.9), (2.15) and (2.16), this condition may be shown to be

$$1=(2\pi)^{-3}\int d^3p\,|\phi(\vec{P};\vec{p})|^2$$
$$-\frac{2m}{\sqrt{M^2+\vec{P}^2}}\int d^3(p'',p')\overline{\phi(\vec{P};\vec{p}'')}\frac{\partial\tilde{V}(P;\vec{p}'',\vec{p}')}{\partial P_0}\phi(\vec{P};\vec{p}), \tag{2.19}$$

with the derivative evaluated at $P_0=\sqrt{M^2+\vec{P}^2}$. Equation (2.19) shows that, when the quasipotential $\tilde{V}$ depends explicitly on P_0, the wavefunction ϕ derived from this quasipotential is no longer normalized to 1.

We now transform the wave equation (2.18) to the rest frame by considering the rotationless Lorentz transformation which maps the 4-vector $P=(P_0,\vec{P})$ to $\mathring{P}=(\sqrt{P^2},\vec{0})$. At the same time it maps the 4-vector $\hat{p}=(\hat{p}_0,\vec{p})$ to $\mathring{p}=(0,\mathring{\vec{p}})$, where

$$\mathring{\vec{p}}=\vec{p}-\frac{\vec{p}\cdot\vec{P}\vec{P}}{P_0(P_0+\sqrt{P^2})}\approx\vec{p}-\frac{\vec{p}\cdot\vec{P}\vec{P}}{8m^2}. \tag{2.20}$$

In the bound state case, $P^2=M^2$ and $P_0=\sqrt{M^2+\vec{P}^2}$. We showed above that $(P_0/\sqrt{P^2})^{1/2}\phi(\vec{P};\vec{p})$ is Lorentz invariant, so that

$$\phi(\vec{0};\mathring{\vec{p}})=\left(\frac{P_0}{\sqrt{P^2}}\right)^{1/2}\phi(\vec{P};\vec{p}). \tag{2.21}$$

Since $\hat{p}^2=\mathring{p}^2=-\mathring{\vec{p}}^2$, we find, using the invariance properties given after (2.14), that (2.18) becomes

$$(4m)^{-1}(M^2-4m^2-4\mathring{\vec{p}}^2)\phi(0;\mathring{\vec{p}})=(2\pi)^{-3}\int d^3\mathring{k}\tilde{V}(\mathring{P};\mathring{\vec{p}},\mathring{\vec{k}})\phi(\vec{0};\mathring{\vec{k}}). \tag{2.22}$$

The above system of equations constitutes a covariant framework for the description of the deuteron. The relativistic equation of motion (2.18) for the moving deuteron transforms by means of a Lorentz boost into equation (2.22) in the rest frame. This equation is recognized as the nonrelativistic Schrödinger equation in momentum space and can be solved in the usual way. The eigenvalue in our formalism is $(M^2-4m^2)/4m$, which in the case of weak binding reduces to $(M-2m)$. In the case of the deuteron, this difference is of no importance. The relationship between the bound state wavefunction in the frame with total 3-momentum $\vec{P}$ and that in the rest frame is given exactly by equation (2.21), and the shift of the relative momentum is given by equation (2.20). Note that the transformation property (2.21) of ϕ cannot be established using the wave equation alone, since it is homogeneous. The normalization condition (2.19) is required.

Our next task is to sketch the way in which two-nucleon continuum states are treated in the quasipotential formalism. Consider such a state with 3-momentum $\vec{P}$ and relative 3-momentum $\vec{p}_0$, so that

$$P_0 = E(\tfrac{1}{2}\vec{P}+\vec{p}_0)+E(\tfrac{1}{2}\vec{P}-\vec{p}_0) = 2\bar{E}(\vec{P};\vec{p}_0),$$

where

$$\bar{E}(\vec{P};\vec{k}) = \tfrac{1}{2}[E(\tfrac{1}{2}\vec{P}+\vec{k})+E(\tfrac{1}{2}\vec{P}-\vec{k})]. \tag{2.23}$$

Note from (2.7) that, when $P_0 = 2\bar{E}(\vec{P};\vec{p}_0)$, $\hat{g}(P;\hat{k})$ has a singularity when $\vec{k}$ lies in a 2-dimensional manifold, and it is necessary to specify the behaviour of $\hat{g}$ in the neighbourhood of this singularity. We define

$$\hat{g}^{(\pm)}(P;\hat{k}) = \frac{4m^2}{\bar{E}(P;\hat{k})[4\bar{E}(\vec{P};\vec{p}_0)^2-\vec{P}^2-4(m^2-\hat{k}^2)\pm i\varepsilon]}. \tag{2.24}$$

Defining $\tilde{W}$ from $\hat{W}_0$ exactly as $\tilde{V}$ is defined in (2.16), we note that $\tilde{W}(P;\vec{p}''\ \vec{p}')$ has a cut along the real axis in the P_0-plane for $P_0 \geqslant 2\bar{E}(\vec{P};\vec{0}) = 2\sqrt{m^2+\vec{P}^2/4}$. Denoting by $\tilde{W}^{(\pm)}(P;\vec{p}'',\vec{p}')$ the boundary values as P_0 approaches $2\bar{E}(\vec{P};\vec{p}_0)$ from above $(+)$ and from below $(-)$, we see from (2.12) that $\tilde{W}^{(\pm)}$ satisfy the singular integral equations

$$\begin{aligned}\tilde{W}^{(\pm)}&(P;\vec{p}'',\vec{p}')\\ &= \tilde{V}(P;\vec{p}'',\vec{p}')+(2\pi)^{-3}\int d^3k\,\tilde{V}(P;\vec{p}'',\vec{k})\frac{\tilde{E}(P;\hat{k})}{m}\,\hat{g}^{(\pm)}(P;\hat{k})\tilde{W}^{(\pm)}(P;\vec{k},\vec{p}')\\ &= \tilde{V}(P;\vec{p}'',\vec{p}')+(2\pi)^{-3}\int d^3k\,\tilde{W}^{(\pm)}(P;\vec{p}'',\vec{k})\frac{\tilde{E}(P;\hat{k})}{m}\,\hat{g}^{(\pm)}(P;\hat{k})\tilde{V}(P;\vec{k},\vec{p}').\end{aligned} \tag{2.25}$$

Also,

$$\overline{\tilde{W}^{(\pm)}(P;\vec{p}'',\vec{p}')} = \tilde{W}^{(\mp)}(P;\vec{p}',\vec{p}''),\ \overline{\tilde{V}(P;\vec{p}'',\vec{p}')} = \tilde{V}(P;\vec{p}',\vec{p}''). \tag{2.26}$$

Now define scattering wavefunctions $\phi^{(\pm)}(\vec{P},\vec{p}_0;\vec{p})$ in the usual way by

$$\begin{aligned}\phi^{(\pm)}&(\vec{P},\vec{p}_0;\vec{p})\\ &= \sqrt{\frac{\tilde{E}(P;\hat{p}_0)}{m}}\left[(2\pi)^3\,\delta^{(3)}(\vec{p}-\vec{p}_0)+\frac{\tilde{E}(P;\hat{p})}{m}\,\hat{g}^{(\pm)}(P;\hat{p})\tilde{W}^{(\pm)}(P;\vec{p},\vec{p}_0)\right],\end{aligned} \tag{2.27}$$

$\tilde{W}^{(\pm)}$ in (2.27) being half on-shell. It follows by standard methods of scattering theory that $\phi^{(\pm)}$ satisfy the inhomogeneous equations

$$\begin{aligned}\phi^{(\pm)}&(\vec{P},\vec{p}_0;\vec{p})\\ &= \sqrt{\frac{\tilde{E}(P;\hat{p}_0)}{m}}(2\pi)^3\,\delta^{(3)}(\vec{p}-\vec{p}_0)+\frac{\tilde{E}(P;\hat{p})}{m}\,\hat{g}^{(\pm)}(P;\hat{p})\\ &\qquad\times(2\pi)^{-3}\int d^3k\,\tilde{V}(P;\vec{p},\vec{k})\phi^{(\pm)}(\vec{P},\vec{p}_0;\vec{k}).\end{aligned} \tag{2.28}$$

By complex conjugating (2.28) and using (2.26) we deduce that

$$(2\pi)^3\,\delta^{(3)}(\vec{p}_0-\vec{k}'')=(2\pi)^{-3}\int d^3p''\,\overline{\phi^{(\mp)}(\vec{P},\vec{p}_0;\vec{p}'')}\sqrt{\frac{m}{\tilde{E}(P;\hat{p}'')}}$$
$$\times[(2\pi)^3\,\delta^{(3)}(\vec{p}''-\vec{k}'')-\hat{V}(P;\hat{p}'',\hat{k}'')\hat{g}^{(\pm)}(P;\hat{k}'')]. \quad (2.29)$$

Since (2.28) is an inhomogeneous equation, the transformation of the wavefunction and of the equation itself follow directly. Equation (2.28) becomes, in the rest frame,

$$\phi^{(\pm)}(\vec{0},\mathring{\vec{p}}_0;\mathring{\vec{p}})$$
$$=\sqrt{\frac{E(\mathring{\vec{p}}_0)}{m}}\,(2\pi)^3\,\delta^{(3)}(\mathring{\vec{p}}-\mathring{\vec{p}}_0)+\frac{m}{(\mathring{p}_0^2-\mathring{\vec{p}}^2\pm i\varepsilon)}(2\pi)^{-3}$$
$$\times\int d^3\mathring{k}\,\tilde{V}(\mathring{P};\mathring{\vec{p}},\mathring{\vec{k}})\phi^{(\pm)}(\vec{0},\mathring{\vec{p}}_0;\mathring{\vec{k}}), \quad (2.30)$$

where, just as in (2.21),

$$\phi^{(\pm)}(\vec{0},\mathring{\vec{p}}_0;\mathring{\vec{p}})=(P_0/\sqrt{P^2})^{1/2}\phi^{(\pm)}(\vec{P},\vec{p}_0;\vec{p}),$$

with $\mathring{\vec{p}}$ given by (2.20), while $\mathring{\vec{p}}_0$ is defined analogously. Equation (2.30) is the standard integral equation for the incoming and outgoing scattering wavefunctions in the two-nucleon rest system. For completeness we give also the corresponding Schrödinger equation in momentum space

$$m^{-1}(\mathring{p}_0^2-\mathring{\vec{p}}^2)\phi^{(\pm)}(\vec{0},\mathring{\vec{p}}_0;\mathring{\vec{p}})=(2\pi)^{-3}\int d^3\mathring{k}\,\tilde{V}(\mathring{P};\mathring{\vec{p}},\mathring{\vec{k}})\phi^{(\pm)}(\vec{0},\mathring{\vec{p}}_0;\mathring{\vec{k}}).$$

We turn now to the formalism for *meson exchange effects* in the two-nucleon system, and begin by considering the amplitude for the process

$$\gamma(q)+N_1(\tfrac{1}{2}P'+p')+N_2(\tfrac{1}{2}P'-p')\to N_1(\tfrac{1}{2}P''+p'')+N_2(\tfrac{1}{2}P''-p'').$$

Using the standard reduction procedure of field theory, the S-matrix element for this process may be written in terms of the electromagnetic current operator $J_\mu(0)$:

$$\langle P'';p''|S|P';p';q,\varepsilon\rangle$$
$$=-i(2\pi)^{-3/2}(2\pi)^4\,\delta^{(4)}(P''-P'-q)\varepsilon_\mu\langle P'';p''|J^\mu(0)|P';p'\rangle. \quad (2.31)$$

Single particle plane wave states are normalized according to

$$\langle\vec{k}''|\vec{k}'\rangle=2E(\vec{k}')\,\delta^{(3)}(\vec{k}''-\vec{k}')$$

and ε is the polarization 4-vector of the photon. The quantity on the left side of (2.31) is evaluated using standard Feynman rules. Now define the quantity $\hat{M}^\mu$ by

$$\langle P'';p''|J^\mu(0)|P';p'\rangle=(2\pi)^{-6}(4m^2)\hat{M}^\mu(P'',P';p'',p'), \quad (2.32)$$

where $\hat{M}^\mu$ has standard spinors on both sides.

We are interested first in the matrix element of the current $J^\mu(0)$, not between states containing free nucleons, but between two-nucleon bound states. We shall denote the state vector for such a bound state by $|d;\vec{P}\rangle$, since the

deuteron is the only such bound state. The 3-momentum of the deuteron is $\vec{P}$ and its total energy is $P_0=\sqrt{M^2+\vec{P}^2}$. Adopting for bound states the same relativistic normalization as before, we find, using for example the arguments in Ref. 14, that

$$\begin{aligned}
&\langle d;\vec{P}''|\,J^\mu(0)\,|d;\vec{P}'\rangle\\
&\quad=(2\pi)^{-3}(4m)(2\pi)^{-6}\int d^3(p'',p')\,\overline{\hat{\Gamma}(P'';\hat{p}'')}\hat{g}(P'';\hat{p}'')\hat{\Lambda}^\mu(P'',P';\hat{p}'',\hat{p}')\\
&\qquad\times\hat{g}(P';\hat{p}')\hat{\Gamma}(P';\hat{p}')\\
&\quad=(2\pi)^{-3}(4m)(2\pi)^{-6}\int d^3(p'',p')\,\overline{\phi(\vec{P}'';\vec{p}'')}\,\tilde{\Lambda}^\mu(P'',P';\vec{p}'',\vec{p}')\phi(\vec{P}';\vec{p}'),
\end{aligned}\tag{2.33}$$

where

$$\begin{aligned}
&\hat{\Lambda}^\mu(P'',P';\hat{p}'',\hat{p}')\\
&\quad=(2\pi)^{-6}\int d^3(k'',k')[(2\pi)^3\,\delta^{(3)}(\vec{p}''-\vec{k}'')-\hat{V}(P'';\hat{p}'',\hat{k}'')\hat{g}(P'';\hat{k}'')]\\
&\qquad\times\hat{M}^\mu(P'',P';\hat{k}'',\hat{k}')[(2\pi)^3\,\delta^{(3)}(\vec{k}'-\vec{p}')-\hat{g}(P';\hat{k}')\hat{V}(P';\hat{k}',\hat{p}')],
\end{aligned}\tag{2.34}$$

$$\tilde{\Lambda}^\mu(P'',P';\vec{p}'',\vec{p}')=\sqrt{\frac{m}{\tilde{E}(P'';\hat{p}'')}}\,\hat{\Lambda}^\mu(P'',P';\hat{p}'',\hat{p}')\,\sqrt{\frac{m}{\tilde{E}(P';\hat{p}')}}.\tag{2.35}$$

We shall use the symbol $\hat{M}_0^\mu$ as a shorthand for $\hat{M}^\mu(P'',P';\hat{p}'',\hat{p}')$, and similarly for $\hat{\Lambda}_0^\mu$. While $\hat{M}^\mu$ in (2.32) has standard spinors attached, $\hat{M}_0^\mu$ in (2.34) has BBS spinors instead. Moreover, $\hat{M}_0^\mu$ has poles at $P_0'=\sqrt{M^2+\vec{P}'^2}$, $P_0''=\sqrt{M^2+\vec{P}''^2}$ which are compensated by the zeros which arise, according to (2.13), from the quantities in square brackets, when the matrix element indicated in (2.33) is taken. From (2.33) we see that, when one calculates matrix elements using nonrelativistic wavefunctions, the correct two-nucleon current to use is $\tilde{\Lambda}^\mu$, which is given in terms of $\hat{\Lambda}_0^\mu$ by (2.35). In turn, $\hat{\Lambda}_0^\mu$ is given in terms of $\hat{M}_0^\mu$ by (2.34), and $\hat{M}_0^\mu$ is obtained by applying the BBS prescription to the field theoretic amplitude defined by (2.31) and (2.32).

We wish now to indicate the modifications which are required when we take the matrix element of the current $J^\mu(0)$ between an initial two-nucleon bound state and a final continuum state, as in the case of deuteron photodisintegration. We normalize two-nucleon states according to

$$\langle 2N;\vec{p}_1''\vec{p}_2''\,|\,2N;\vec{p}_1'\vec{p}_2'\rangle=4E(\vec{p}_1')E(\vec{p}_2')\,\delta^{(3)}(\vec{p}_2''-\vec{p}_2')\,\delta^{(3)}(\vec{p}_1''-\vec{p}_1').\tag{2.36}$$

We shall label continuum states as before using the total 3-momentum $\vec{P}$ and the relative 3-momentum $\vec{p}_0$. With the normalization (2.36) the equation analogous to (2.33) is

$$\begin{aligned}
&\langle 2N;\vec{P}'',\vec{p}_0|\,J^\mu(0)\,|d;\vec{P}'\rangle\\
&\quad=(2\pi)^{-21/2}4m^{3/2}\int d^3(k',p')\hat{M}^\mu(P'',P';\hat{p}_0,\hat{k}')\\
&\qquad\times[(2\pi)^3\,\delta^{(3)}(\vec{k}'-\vec{p}')-\hat{g}(P';\hat{k}')\hat{V}(P';\hat{k}',\hat{p}')]\hat{g}(P';\hat{p}')\hat{\Gamma}(P';\hat{p}').
\end{aligned}\tag{2.37}$$

Equation (2.37) can be rewritten, using (2.29) to give

$$\langle 2N; \vec{P}'', \vec{p}_0 | J^\mu(0) | d; \vec{P}' \rangle$$
$$= (2\pi)^{-21/2} 4m^{3/2} \int d^3(p'', p') \, \overline{\phi^{(-)}(\vec{P}'', \vec{p}_0; \vec{p}'')} \, \tilde{\Lambda}^\mu(P'', P'; \vec{p}'', \vec{p}') \phi(\vec{P}'; \vec{p}'), \tag{2.38}$$

where $\tilde{\Lambda}^\mu$ is defined by (2.34) and (2.35), with $\hat{g}(P'', \hat{k}'')$ in this case being $\hat{g}^{(+)}(P''; \hat{k}'')$.

To proceed further we decompose the complete set of diagrams which contribute to M^μ (which has the spinors removed) in a way which is exactly like the decomposition which gives the Bethe–Salpeter equation (2.1). Using symbolic operator notation again, the result is that M^μ can be written as

$$M^\mu = K^\mu + WGK^\mu + K^\mu GW + WGK^\mu GW. \tag{2.39}$$

Equation (2.39) defines K^μ, which is obtained by summing all irreducible diagrams. The NN scattering amplitude W and Green function G are exactly as in (2.1). The irreducible diagrams whose contributions we shall consider are given in Fig. 1. There are two diagrams for each of (a), (b), (d) and (e), corresponding to absorption of the photon on nucleon 1 and on nucleon 2. In (d), the intermediate state is the $\Delta(1232)$ isobar. There is another diagram like (d), in which the pion is exchanged first, but this gives no contribution when there is a deuteron initial state.

For the present we consider what happens when the matrix element is taken between bound states. When we use (2.34) and split $\hat{\Lambda}_0^\mu$ into four parts according to the decomposition of M^μ in (2.39), the matrix elements of the first three parts

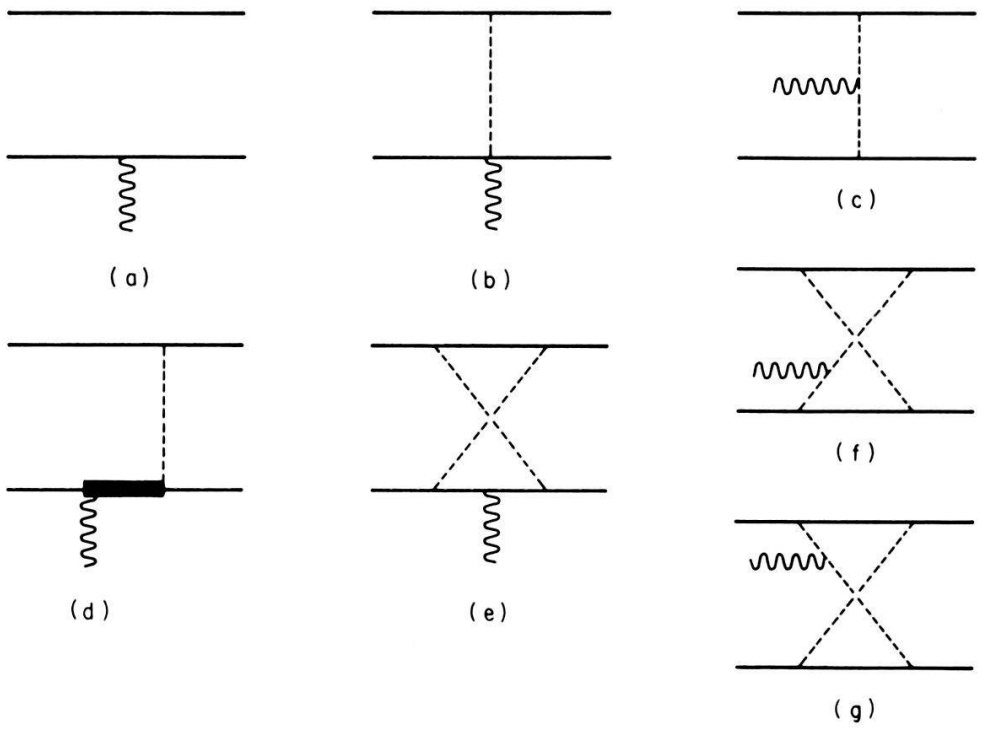

Figure 1
Irreducible diagrams which contribute to K^μ, defined in equation (2.39), in impulse approximation and one- and two-pion exchange approximation.

vanish when they are calculated using (2.33). This follows directly from (2.13), since the first three terms on the right side of (2.39) do *not* have poles in *both* P_0' and P_0'' to compensate for the zeros which arise from the equations for the initial and final vertex functions. Therefore, for the purpose of calculating matrix elements between bound states according to (2.33), one can replace M^μ by the final term on the right side of (2.39) and write

$$M^\mu \doteq WGK^\mu GW. \tag{2.40}$$

(The notation $\doteq$ will mean 'equal for the purpose of calculating matrix elements'.) Using the identity

$$\begin{aligned} &E(\vec{q})(\gamma_0 q_0 - \vec{\gamma}\cdot\vec{q} + m) \\ &\quad = m(q_0 + E(\vec{q}))\sum_r u^{(r)}(\vec{q})\bar{u}^{(r)}(\vec{q}) + m(q_0 - E(\vec{q}))\sum_r v^{(r)}(-\vec{q})\bar{v}^{(r)}(-\vec{q}), \end{aligned}$$

the Green function G can be split into four parts. This means that the right side of (2.40) decomposes into sixteen terms, just one of which contains four internal positive energy states. Each of the other fifteen terms contains at least one negative energy state. Thus, attaching positive energy spinors, we have

$$\begin{aligned} &\hat{M}^\mu(P'', P'; \hat{p}'', \hat{p}') \\ &\quad = (2\pi)^{-8}\int d^4(k'', k')\hat{W}(P''; \hat{p}'', k'')G_{++}(P''; k'')\hat{K}^\mu(P'', P'; k'', k') \\ &\qquad \times G_{++}(P'; k')\hat{W}(P'; k', \hat{p}') + \cdots, \end{aligned} \tag{2.41}$$

the remainder on the right side denoting the other fifteen terms. For an internal two-nucleon state with total 4-momentum P and relative 4-momentum p, the function G_{++} is given by

$$G_{++}(P; p) = \frac{1}{i}\frac{m^2}{E(\frac{1}{2}\vec{P}+\vec{p})E(\frac{1}{2}\vec{P}-\vec{p})}\frac{1}{(p_0 - x_1(P;\vec{p}) + i\varepsilon)(p_0 + x_2(P;\vec{p}) - i\varepsilon)}, \tag{2.42}$$

where

$$\begin{aligned} x_1(P;\vec{p}) &= E(\tfrac{1}{2}\vec{P}+\vec{p}) - \tfrac{1}{2}P_0, \\ x_2(P;\vec{p}) &= E(\tfrac{1}{2}\vec{P}-\vec{p}) - \tfrac{1}{2}P_0. \end{aligned} \tag{2.43}$$

The two impulse approximation diagrams are slightly different. Each of them gives a contribution which splits only into eight terms, and the term containing only positive energy internal states has only one propagator for particle 2 when the photon is absorbed on nucleon 1, and vice versa. This term for the impulse approximation charge density, when the photon is asorbed on nucleon 1, is given in (3.6).

We now discuss the terms in (2.41) which involve at least one negative energy internal state. The only terms which contribute at $1\pi E$ level are those in which K^μ is given by the impulse approximation diagrams and there is *one* negative energy state adjacent to the vertex where the photon is absorbed. These terms give what is called the pair current (PC) and they are important for PS πNN coupling. The use of PV πNN coupling results in pair suppression, and these

terms are not important in that case. However, for PV coupling the SG diagrams (Fig. 1(b)) have to be included in K^μ; they are absent for PS coupling. All the other terms in (2.41) which involve at least one negative energy internal state are at least at $2\pi E$ level. Some of these terms are very important if one calculates them naively using PS πNN coupling. However, these important contributions are cancelled to a large extent by a special class of $2\pi E$ contributions which are generated by a $\pi\pi NN$ contact interaction which arises quite naturally in a chiral invariant theory. This interaction is usually simulated by a σNN interaction. So one hopes that there is no unusually big effect due to $2\pi E$, and notes that there would be serious problems if one tried to do a full $2\pi E$ calculation using PS πNN coupling. For PV coupling all these negative energy contributions are strongly suppressed, so that one obtains only the $2\pi E$ contributions which come from the positive energy term which we have written explicitly in (2.41). These contributions are the same for PS and PV coupling; we shall consider them later in order to compare the results which come from the BBS formalism with those from the unitary transformation method. From now on then we consider only the term which we have written explicitly in (2.41). Inserting this in (2.34) and using (2.10), we arrive at the key result

$$\begin{aligned}\hat{\Lambda}^\mu(P'', P'; \hat{p}'', \hat{p}') &\doteq (2\pi)^{-8} \int d^4(k'', k')\hat{V}(P''; \hat{p}'', k'')G_{++}(P''; k'') \\ &\quad \times \hat{K}^\mu(P'', P'; k'', k')G_{++}(P'; k')\hat{V}(P'; k', \hat{p}')\end{aligned} \tag{2.44a}$$

or, in shorthand notation,

$$\hat{\Lambda}_0^\mu \doteq \hat{V}G_{++}\hat{K}^\mu G_{++}\hat{V}. \tag{2.44b}$$

It will be seen that the right side of (2.44) is free of zeros and poles.

We complete this section by indicating what happens to the preceding arguments when the matrix element is taken between a bound state and a continuum state. Now the third and fourth terms of (2.39) remain, so that

$$M^\mu \doteq K^\mu GW + WGK^\mu GW.$$

Keeping only positive energy states as before, we have

$$\hat{M}_0^\mu \doteq \hat{K}^\mu G_{++}\hat{W} + \hat{W}^{(+)}G_{++}\hat{K}^\mu G_{++}\hat{W}. \tag{2.45}$$

Note that it is $\hat{W}^{(+)}$ which appears in the second term on the right side of (2.45); for this reason, the matrix element in (2.38) involves $\phi^{(-)}$. Using (2.34), it follows from (2.10) that

$$\hat{\Lambda}_0^\mu \doteq (\mathbb{1} - \hat{V}_0\hat{g}^{(+)})\hat{K}^\mu G_{++}\hat{V} + \hat{V}G_{++}\hat{K}^\mu G_{++}\hat{V}. \tag{2.46}$$

Note that there is a cut for $P_0'' \geqslant \sqrt{4m^2 + \vec{P}''^2}$ in $\hat{M}_0^\mu$ due to the appearance of $\hat{W}^{(+)}$, but that this cut has disappeared from $\hat{\Lambda}_0^\mu$. If matrix elements are calculated between bound states using (2.44), and between a bound state and a continuum state using (2.46), one finds formally identical results for the two cases. This can be proved using the methods which will be developed in the next section. We shall therefore for convenience use equation (2.44) as the basis of all our calculations.

III. The impulse approximation with relativistic corrections

In this section we shall calculate the two-nucleon charge density and current density for the impulse approximation diagrams. To do this, we start from (2.44) and write each $\hat{V}$ in the form

$$\hat{V} = \hat{V}_0 + (\hat{V} - \hat{V}_0), \tag{3.1}$$

but take only the term in which each $\hat{V}$ in (2.44) is replaced by $\hat{V}_0$. We shall consider the effect of the second term of the decomposition (3.1) in later sections. Using the definitions of $\hat{V}_0$, $\tilde{V}$ and $\tilde{\Lambda}^\mu$ in (2.11), (2.16) and (2.35) respectively, we have for the term which we are now considering

$$\tilde{\Lambda}^\mu(P'', P'; \vec{p}'', \vec{p}') = (2\pi)^{-8} \int d^4(k'', k') \tilde{V}(P''; \vec{p}'', \vec{k}'') \sqrt{\frac{\tilde{E}(P''; \hat{k}'')}{m}} G_{++}(P''; k'') \times \hat{K}^\mu(P'', P'; k'', k') G_{++}(P'; k') \sqrt{\frac{\tilde{E}(P'; \hat{k}')}{m}} \tilde{V}(P'; \vec{k}', \vec{p}'). \tag{3.2}$$

From now we shall denote $\tilde{\Lambda}^0$ by ρ and the 3-vector $(\tilde{\Lambda}^1, \tilde{\Lambda}^2, \tilde{\Lambda}^3)$ by $\vec{j}$. All expressions in this paper for the various parts of ρ will have the opposite sign to those given in JW. It will be seen from (2.31) that, in computing $\hat{M}^\mu$ from a particular diagram, a factor $+i$ has to be included as well as the various factors given by the Feynman rules. In JW a factor $-i$ was used, which arose from the unconventional Fourier transformation between coordinate and momentum space which was used there (see (2.9) and the equation after (2.10) of JW). In this paper we use the standard Fourier transform convention. The results given in JW for the electric dipole operator in *coordinate* space are of course correct, and independent of the convention used.

We now consider the impulse approximation charge density ρ(IA). This is obtained from (3.2) by taking for $\hat{K}^0$ the contribution of the impulse approximation diagrams of Fig. 1(a). The charge density ρ(IA) may be split into two terms:

$$\rho(\mathrm{IA}) = \rho_1(\mathrm{IA}) + \rho_2(\mathrm{IA}), \tag{3.3}$$

where ρ_1 and ρ_2 correspond to absorption of the photon by nucleon 1 and nucleon 2 respectively. For the γNN vertex we need to include the Pauli term which is conventionally introduced to simulate the anomalous magnetic properties of the nucleons. Then

$$V(\gamma NN) = -ie\left(\hat{e}\gamma^\mu + \frac{i\hat{\kappa}\sigma^{\mu\nu}q_\nu}{2m}\right), \tag{3.4}$$

where the isospin operators $\hat{e}$, $\hat{\kappa}$ are defined by

$$\hat{e} = \tfrac{1}{2}(\mathbb{1} + \tau_z), \qquad \hat{\kappa} = \tfrac{1}{2}(\kappa_S \mathbb{1} + \kappa_V \tau_z), \tag{3.5}$$

with

$$1 + \kappa_S = \mu_p + \mu_n, \qquad 1 + \kappa_V = \mu_p - \mu_n.$$

The 4-vector q^μ is the 4-momentum of the incoming photon. Then, using (3.2),

(2.42), (2.43) and (3.4) and the Feynman rules, we have

$$\begin{aligned}
&\rho_1(\mathrm{IA}; P'', P'; \vec{p}'', \vec{p}') \\
&= -ie(2\pi)^{-4} \int d^4(k'', k') \tilde{V}(P''; \vec{p}'', \vec{k}'') \sqrt{\frac{\tilde{E}(P''; \hat{k}'')}{m}} \\
&\quad \times \bar{u}_1(\tfrac{1}{2}\vec{P}'' + \vec{k}'')\left(\hat{e}_1\gamma_0 - \frac{\hat{\kappa}_1 \vec{\gamma}\cdot\vec{q}\gamma_0}{2m}\right) u_1(\tfrac{1}{2}\vec{P}' + \vec{k}') \\
&\quad \times \sqrt{\frac{\tilde{E}(P'; \hat{k}')}{m}}\, \tilde{V}(P'; \vec{k}', \vec{p}') \\
&\quad \times \frac{m^3}{E(\frac{1}{2}\vec{P}'' + \vec{k}'')E(\frac{1}{2}\vec{P}' + \vec{k}')E(\frac{1}{2}\vec{P}' - \vec{k}')} \; \frac{\delta^{(4)}(k'' - k' - \frac{1}{2}q)}{(k_0'' - x_1'' + i\varepsilon)(k_0' - x_1' + i\varepsilon)(k_0' + x_2' - i\varepsilon)},
\end{aligned} \tag{3.6}$$

where

$$\begin{aligned}
x_1' &= E(\tfrac{1}{2}\vec{P}' + \vec{k}') - \tfrac{1}{2}P_0', \\
x_1'' &= E(\tfrac{1}{2}\vec{P}'' + \vec{k}'') - \tfrac{1}{2}P_0'', \\
x_2' &= E(\tfrac{1}{2}\vec{P}' - \vec{k}') - \tfrac{1}{2}P_0'.
\end{aligned} \tag{3.7}$$

Note that

$$P'' - P' = q,$$

so that, because of the δ-function in (3.6),

$$\tfrac{1}{2}P'' - k'' = \tfrac{1}{2}P' - k',$$

and so

$$E(\tfrac{1}{2}\vec{P}' - \vec{k}') = E(\tfrac{1}{2}\vec{P}'' - \vec{k}''), \qquad k_0' + x_2' = k_0'' + x_2'', \tag{3.8}$$

where

$$x_2'' = E(\tfrac{1}{2}\vec{P}'' - \vec{k}'') - \tfrac{1}{2}P_0''. \tag{3.9}$$

Now the integration over k_0'', k_0' can be simply performed by using the δ-function and then by closing the contour in the upper half-plane, with the result

$$\begin{aligned}
&\frac{1}{2\pi i}\int d(k_0'', k_0') \frac{\delta(k_0'' - k_0' - \frac{1}{2}q_0)}{(k_0'' - x_1'' + i\varepsilon)(k_0' - x_1' + i\varepsilon)(k_0' + x_2' - i\varepsilon)} \\
&= \frac{1}{[P_0'' - 2\bar{E}(\vec{P}''; \vec{k}'')][P_0' - 2\bar{E}(\vec{P}'; \vec{k}')]},
\end{aligned} \tag{3.10}$$

where we have used equations (3.7)–(3.9) and the definition of $\bar{E}$ in (2.23). Using (3.10), we now write (3.6) in the form

$$\begin{aligned}
&\rho_1(\mathrm{IA}; P'', P'; \vec{p}'', \vec{p}') \\
&= (2\pi)^{-6} \int d^3(k'', k') \tilde{V}(P''; \vec{p}'', \vec{k}'') \frac{\tilde{E}(P''; \hat{k}'')}{m}\, g_r(P''; \vec{k}'') \\
&\quad \times e(2\pi)^3\, \delta^{(3)}(\vec{k}'' - \vec{k}' - \tfrac{1}{2}\vec{q}) \frac{(P_0''P_0')^{1/2}}{2m}
\end{aligned}$$

$$\times\sqrt{\frac{m}{E(\frac{1}{2}\vec{P}''+\vec{k}'')}}\,\bar{u}_1(\tfrac{1}{2}\vec{P}''+\vec{k}'')\left(\hat{e}_1\gamma_0-\frac{\hat{\kappa}_1\vec{\gamma}\cdot\vec{q}\gamma_0}{2m}\right)u_1(\tfrac{1}{2}\vec{P}'+\vec{k}')$$

$$\times\sqrt{\frac{m}{E(\frac{1}{2}\vec{P}'+\vec{k}')}}\,g_r(P';\vec{k}')\frac{\tilde{E}(P';\hat{k}')}{m}\,\tilde{V}(P';\vec{k}',\vec{p}'), \tag{3.11}$$

where

$$g_r(P;\vec{k})=\frac{\sqrt{2}\,m^2}{[\tilde{E}(P;\hat{k})E(\frac{1}{2}\vec{P}+\vec{k})E(\frac{1}{2}\vec{P}-\vec{k})]^{1/2}P_0^{1/2}[P_0-2\bar{E}(\vec{P};\vec{k})]}. \tag{3.12}$$

From the definition (2.9) of $\tilde{E}$, it follows that, to terms of order m^{-4},

$$\tilde{E}(P;\hat{k})\approx\frac{E(\frac{1}{2}\vec{P}+\vec{k})E(\frac{1}{2}\vec{P}-\vec{k})}{\bar{E}(\vec{P};\vec{k})}\left[1+\frac{(\vec{k}^2-\delta)\vec{P}^2}{8m^4}+\frac{(\vec{P}\cdot\vec{k})^2}{4m^4}\right], \tag{3.13}$$

where

$$\delta=-mB\quad\text{(bound state)},\qquad \delta=\vec{p}_0^{\,2}\quad\text{(continuum state)}, \tag{3.14}$$

B being the binding energy of the bound state. Also, from the definition (2.7) of $\hat{g}$, it may be shown that

$$\hat{g}(P;\hat{k})\approx\frac{4m^2}{\tilde{E}(P;\hat{k})[P_0^2-4\bar{E}(\vec{P};\vec{k})^2][1-(\vec{P}\cdot\vec{k})^2/4m^4]}. \tag{3.15}$$

Putting together equations (3.12), (3.13) and (3.15), we find that

$$g_r(P;\vec{k})\approx\hat{g}(P;\hat{k})\left[1-\frac{(\vec{P}\cdot\vec{k})^2}{8m^4}+\frac{(\vec{k}^2-\delta)\vec{P}^2}{16m^4}+\frac{(\vec{k}^2-\delta)^2}{32m^4}\right],$$

so that g_r and $\hat{g}$ may be identified in the approximation to which we are working. Thus, forming the matrix element of $\rho_1(\mathrm{IA})$ in (3.10) between two-body wavefunctions according to (2.33) and using (2.17), it follows that

$$\begin{aligned}&\rho_1(\mathrm{IA};P'',P';\vec{p}'',\vec{p}')\\&\quad\doteq e(2\pi)^3\,\delta^{(3)}(\vec{p}''-\vec{p}'-\tfrac{1}{2}\vec{q})(P_0''P_0')^{1/2}/2m\\&\qquad\times\sqrt{\frac{m}{E(\frac{1}{2}\vec{P}''+\vec{p}'')}}\,\bar{u}_1(\tfrac{1}{2}\vec{P}''+\vec{p}'')\left(\hat{e}_1\gamma_0-\frac{\hat{\kappa}_1\vec{\gamma}\cdot\vec{q}\gamma_0}{2m}\right)\\&\qquad\times u_1(\tfrac{1}{2}\vec{P}'+\vec{p}')\sqrt{\frac{m}{E(\frac{1}{2}\vec{P}'+\vec{p}')}}.\end{aligned} \tag{3.16}$$

As we indicated at the end of Section II, when a physical photon produces transitions from a bound state to a continuum state it may be shown that the expression for $\rho_1(\mathrm{IA})$ in (3.16) remains the same.

Performing the spinor reduction in (3.16) and noting that

$$\vec{P}''-\vec{P}'=\vec{q},\qquad \vec{p}''-\vec{p}'=\tfrac{1}{2}\vec{q},$$

we have, to order m^{-2},

$$\rho_1(\mathrm{IA}; P'', P'; \vec{p}'', \vec{p}') \doteq e(2\pi)^3\, \delta^{(3)}(\vec{p}''-\vec{p}'-\tfrac{1}{2}\vec{q})(P_0''P_0')^{1/2}/2m \times\left[\hat{e}_1-\frac{i(\hat{e}_1+2\hat{\kappa}_1)}{4m^2}\,\vec{q}\cdot\{\vec{\sigma}_1\times(\vec{p}'+\tfrac{1}{4}\vec{P})\}-\frac{(\hat{e}_1+2\hat{\kappa}_1)}{8m^2}\,\vec{q}^{\,2}\right], \tag{3.17}$$

where

$$\vec{P}=\vec{P}'+\vec{P}''. \tag{3.18}$$

In a frame for which $\vec{P}$ is parallel to $\vec{q}$, $\frac{1}{4}\vec{P}$ may be removed from the spin-orbit term in (3.17) and our result then agrees with that given in Appendix D of Ref. 4. The expression for $\rho_2(\mathrm{IA})$ in (3.3) is obtained from that for $\rho_1(\mathrm{IA})$ in (3.17) by making the changes $\hat{e}_1\to\hat{e}_2$, $\hat{\kappa}_1\to\hat{\kappa}_2$, $\vec{\sigma}_1\Leftrightarrow\vec{\sigma}_2$, $\vec{p}'\to-\vec{p}'$, $\vec{p}''\to-\vec{p}''$. We note here too that our derivation of $\rho(\mathrm{IA})$ using the Green function g_r of (3.12) shows that the 'one pion exchange' charge density of JW is in fact absent.

The right side of (3.16) should in fact be sandwiched between helicity spinors

$$\chi_1^{(\lambda_1'')}(\tfrac{1}{2}\vec{P}''+\vec{p}'')^*\chi_2^{(\lambda_2'')}(\tfrac{1}{2}\vec{P}''-\vec{p}'')^*\cdots\chi_1^{(\lambda_1')}(\tfrac{1}{2}\vec{P}'+\vec{p}')\chi_2^{(\lambda_2')}(\tfrac{1}{2}\vec{P}'-\vec{p}'),$$

which come from the spinor reduction in (3.16). Helicity spinors depend of course only on spatial direction, but the notation we have just used is convenient. We next remark that one may replace these standard helicity spinors by BBS helicity spinors:

$$\chi_1^{(\lambda_1'')}(\tfrac{1}{2}\vec{Q}''+\vec{p}'')^*\chi_2^{(\lambda_2'')}(\tfrac{1}{2}\vec{Q}''-\vec{p}'')^*\cdots\chi_1^{(\lambda_1')}(\tfrac{1}{2}\vec{Q}'+\vec{p}')\chi_2^{(\lambda_2')}(\tfrac{1}{2}\vec{Q}'-\vec{p}'),$$

where

$$\vec{Q}''=\vec{Q}(P''; P'), \qquad \vec{Q}'=\vec{Q}(\hat{p}; \hat{p}').$$

One can show that, in making this replacement, one is neglecting $1\pi E$ contributions to the charge density which are down by $O(m^{-2})$ compared with the $1\pi E$ contributions which are considered later in the paper. The argument is easy to construct using two results which we shall need again later, namely

$$\vec{Q}(P; \hat{p})\approx\vec{P}\left[1-\frac{P_0-2\bar{E}(\vec{P}; \vec{p})}{2m}\right] \tag{3.19}$$

and

$$\chi^{(\lambda)}(\tfrac{1}{2}\vec{Q}\pm\vec{p})\approx\left[1\mp\frac{i\vec{P}\cdot(\vec{\sigma}\times\vec{\mathring{p}})}{8m^2}\right]\chi^{(\lambda)}(\pm\vec{\mathring{p}}), \tag{3.20}$$

which is the familiar expression for the Wigner rotation. In equation (3.20), $\vec{Q}$ stands for $\vec{Q}(P; \hat{p})$ and $\vec{\mathring{p}}$ is related to $\vec{p}$ by (2.20).

Instead of sandwiching the right side of (3.17) between BBS helicity spinors as explained above we can take the matrix element of the charge density in (3.17) between an initial wavefunction

$$\Phi(\vec{P}'; \vec{p}')=\sum_{\lambda_1'\lambda_2'}\chi_1^{(\lambda_1')}(\tfrac{1}{2}\vec{Q}'+\vec{p}')\chi_2^{(\lambda_2')}(\tfrac{1}{2}\vec{Q}'-\vec{p}')\phi^{\lambda_1'\lambda_2'}(\vec{P}'; \vec{p}') \tag{3.21}$$

and a final wavefunction with all primed quantities changed to doubly primed

quantities. Equation (3.21) gives the initial wavefunction which is used in practical calculations. It is chosen to be an eigenfunction of total angular momentum. We now drop the primes from (3.21) and express $\Phi(\vec{P};\vec{p})$ in terms of the wavefunction $\Phi(\vec{0};\vec{p})$ in the rest frame. Using (2.21) and (3.20) we have

$$\left(\frac{P_0}{\sqrt{P^2}}\right)^{1/2}\Phi(\vec{P};\vec{p})\approx\left[1-\frac{i\vec{P}\cdot\{(\vec{\sigma}_1-\vec{\sigma}_2)\times\overset{\circ}{\vec{p}}\}}{8m^2}\right]\Phi(\vec{0};\overset{\circ}{\vec{p}}). \tag{3.22}$$

Using the relation (2.20) between $\vec{p}$ and $\overset{\circ}{\vec{p}}$, we find the desired transformation

$$\left(\frac{P_0}{\sqrt{P^2}}\right)^{1/2}\Phi(\vec{P};\vec{p})\approx\left[1-\frac{i\vec{P}\cdot\{(\vec{\sigma}_1-\vec{\sigma}_2)\times\vec{p}\}}{8m^2}-\frac{\vec{P}\cdot\vec{p}\vec{P}\cdot\nabla_{\vec{p}}}{8m^2}\right]\Phi(\vec{0};\vec{p}). \tag{3.23}$$

Note that, in the order in which we calculate, (3.23) is independent of the potential. This result depends on (2.20), and is an interesting feature of the BBS prescription. We wish to note here that the Gross prescription [15], which treats the two nucleons in an unsymmetrical manner, gives instead of (2.20) the relation

$$\overset{\circ}{\vec{p}}\approx\vec{p}-\vec{p}\cdot\vec{P}\vec{P}/8m^2+(m^2+\vec{p}^{\,2}-\tfrac{1}{4}P^2)\vec{P}/4m^2$$

and the additional term induces dynamic corrections [16]. It was clearly shown by Friar [9] that the dynamic corrections peculiar to the Gross formalism correspond to certain $1\pi E$ (or retardation) corrections in a framework in which the two nucleons are treated symmetrically. This correspondence is true also for the quasipotential formalism. Equation (3.23) agrees with results given by Friar [6] and Coester and Havas [17].

We can now write the expression for the impulse approximation charge density $\tilde{\rho}_1(\mathrm{IA})$ whose matrix element is to be taken between rest frame wavefunctions $\Phi_i(\vec{0};\vec{p}')$ and $\Phi_f(\vec{0};\vec{p}'')$. From equations (3.17) and (3.23) we have

$$\begin{aligned}
&\tilde{\rho}_1(\mathrm{IA};P'',P';\vec{p}'',\vec{p}')\\
&\quad\doteq e(\sqrt{P''^2P'^2})^{1/2}/2m\\
&\qquad\times\Bigg[(2\pi)^3\,\delta^{(3)}(\vec{p}''-\vec{p}'-\tfrac{1}{2}\vec{q})\Bigg\{\hat{e}_1-\frac{(\hat{e}_1+2\hat{\kappa}_1)}{4m^2}\,i\vec{q}\cdot(\vec{\sigma}_1\times(\vec{p}'+\tfrac{1}{4}\vec{P}))-\frac{(\hat{e}_1+2\hat{\kappa}_1)}{8m^2}\,\vec{q}^{\,2}\\
&\qquad+\frac{\hat{e}_1}{8m^2}\,i\vec{q}\cdot((\vec{\sigma}_1-\vec{\sigma}_2)\times(\vec{p}'-\tfrac{1}{4}\vec{P}))-\frac{\hat{e}_1}{8m^2}\,\vec{P}'\cdot\vec{p}'\vec{P}'\cdot\nabla_{\vec{p}'}\Bigg\}\\
&\qquad-\frac{\hat{e}_1}{8m^2}\,\vec{P}''\cdot\nabla_{\vec{p}''}\vec{P}''\cdot\vec{p}''(2\pi)^3\,\delta^{(3)}(\vec{p}''-\vec{p}'-\tfrac{1}{2}\vec{q})+O(m^{-4})\Bigg],
\end{aligned} \tag{3.24}$$

where it is understood that $\nabla_{\vec{p}'}$ acts to the right on the initial wavefunction and $\nabla_{\vec{p}''}$ acts to the left on the final wavefunction.

We conclude this section by giving the result for $\vec{j}\,(\mathrm{IA})$ analogous to that for $\rho(\mathrm{IA})$. To obtain $\vec{j}_1(\mathrm{IA})$, one goes to (3.16) and replaces the central bracket in the spinor reduction by

$$\hat{e}_1\vec{\gamma}+\frac{i\hat{\kappa}_1}{2m}\,\vec{\sigma}_1\times\vec{q}-\frac{q_0}{2m}\,\hat{\kappa}_1\vec{\gamma}\gamma_0.$$

We write the expression for $\tilde{\vec{j}}_1(\mathrm{IA})$, whose matrix element is to be taken between

rest frame wavefunctions, and work this time to first order in $\vec{q}$ and $\vec{P}$. The result is

$$\begin{aligned}
&\tilde{\vec{j}}_1(\mathrm{IA}; P'', P'; \vec{p}'', \vec{p}') \\
&\quad \doteq e(\sqrt{P''^2 P'^2})^{1/2}/2m(2m)^{-1}(2\pi)^3\, \delta^{(3)}(\vec{p}''-\vec{p}'-\tfrac{1}{2}\vec{q}) \\
&\qquad \times\Big[\hat{e}_1(\tfrac{1}{2}(\vec{P}+\vec{q})+2\vec{p}')+(\hat{e}_1+\hat{\kappa}_1)i\vec{\sigma}_1\times\vec{q}-\frac{q_0\hat{\kappa}_1}{2m}(\vec{q}+i\vec{\sigma}_1\times(\tfrac{1}{2}(\vec{P}+\vec{q})+2\vec{p}')) \\
&\qquad -\frac{\hat{e}_1}{2m^2}(\vec{p}'\vec{p}'\cdot(\vec{P}+\vec{q}+2\vec{p}')+\vec{q}\cdot\vec{p}'i\vec{\sigma}_1\times\vec{p}'+\tfrac{1}{2}\vec{p}'^2(\vec{P}+\vec{q}+2i\vec{\sigma}_1\times\vec{q})) \\
&\qquad +\frac{\hat{\kappa}_1}{2m^2}i\vec{q}\times\vec{p}'\vec{\sigma}_1\cdot\vec{p}'+\frac{\hat{e}_1}{4m^2}\vec{p}'i\vec{q}\cdot((\vec{\sigma}_1-\vec{\sigma}_2)\times\vec{p}')+O(m^{-4})\Big]. \qquad (3.25)
\end{aligned}$$

To obtain $\tilde{\rho}_2(\mathrm{IA})$ from $\tilde{\rho}_1(\mathrm{IA})$ and $\tilde{j}_2(\mathrm{IA})$ from $\tilde{j}_1(\mathrm{IA})$, one makes the changes $\hat{e}_1\to\hat{e}_2$, $\hat{\kappa}_1\to\hat{\kappa}_2$, $\vec{\sigma}_1\Leftrightarrow\vec{\sigma}_2$, $\vec{p}'\to-\vec{p}'$, $\vec{p}''\to-\vec{p}''$ in (3.24) and (3.25).

IV. The one-pion exchange potential

Many of the developments in the rest of this paper hinge on a proper understanding of the $1\pi ENN$ potential, and we shall devote this section to its study. For the process

$$N_1(\tfrac{1}{2}P+p')+N_2(\tfrac{1}{2}P-p')\to N_1(\tfrac{1}{2}P+p'')+N_2(\tfrac{1}{2}P-p'')$$

the full field-theoretic $1\pi E$ potential $\hat{V}_\pi(P; p'', p')$ is

$$\hat{V}_\pi(P; p'', p')=\frac{f^2}{m_\pi^2}\vec{\tau}_1\cdot\vec{\tau}_2\frac{N}{-\omega(\vec{p}''-\vec{p}')^2+(p_0''-p_0')^2}, \qquad (4.1)$$

where

$$N(PS)=-4m^2\bar{u}_1(\tfrac{1}{2}\vec{P}+\vec{p}'')\gamma_5 u_1(\tfrac{1}{2}\vec{P}+\vec{p}')\bar{u}_2(\tfrac{1}{2}\vec{P}-\vec{p}'')\gamma_5 u_2(\tfrac{1}{2}\vec{P}-\vec{p}'), \qquad (4.2)$$

$$\begin{aligned}
N(PV)&=\bar{u}_1(\tfrac{1}{2}\vec{P}+\vec{p}'')[\gamma_0(p_0''-p_0')-\vec{\gamma}\cdot(\vec{p}''-\vec{p}')]\gamma_5 u_1(\tfrac{1}{2}\vec{P}+\vec{p}') \\
&\quad\times\bar{u}_2(\tfrac{1}{2}\vec{P}-\vec{p}'')[\gamma_0(p_0''-p_0')-\vec{\gamma}\cdot(\vec{p}''-\vec{p}')]\gamma_5 u_2(\tfrac{1}{2}\vec{P}-\vec{p}'), \qquad (4.3)
\end{aligned}$$

$$\omega(\vec{k})^2=m_\pi^2+\vec{k}^2,$$

and $f^2/4\pi\approx 0.079$. We define $\vec{p}$ by

$$\vec{p}=\vec{p}''-\vec{p}'. \qquad (4.4)$$

For both PS and PV coupling the leading approximation to N is

$$N\approx\vec{\sigma}_1\cdot\vec{p}\vec{\sigma}_2\cdot\vec{p}. \qquad (4.5)$$

For PV coupling there is a term in each spinor reduction which involves $(p_0''-p_0')$. Taking only the term in N which is linear in $(p_0''-p_0')$, we obtain

$$\begin{aligned}
N(\mathrm{PV})\approx\vec{\sigma}_1\cdot\vec{p}\vec{\sigma}_2\cdot\vec{p}-\frac{(p_0''-p_0')}{2m}[&\vec{\sigma}_1\cdot(\vec{P}+\vec{p}''+\vec{p}')\vec{\sigma}_2\cdot\vec{p} \\
&+\vec{\sigma}_1\cdot\vec{p}\vec{\sigma}_2\cdot(\vec{P}-\vec{p}''-\vec{p}')].
\end{aligned}$$

It is convenient to introduce a quantity c such that

$$c=+1 \text{ (PV coupling)}, \qquad c=-1 \text{ (PS coupling)}. \tag{4.6}$$

For the calculation in Section V of $1\pi E$ and $2\pi E$ effects, we shall use the approximation for $\hat{V}_\pi$ which we have just obtained, which neglects terms of order m^{-2} in the spinor reductions, namely

$$\begin{aligned}\hat{V}_\pi(P;p'',p') &\approx \frac{f^2}{m_\pi^2}\vec{\tau}_1\cdot\vec{\tau}_2\frac{1}{(p_0''-p_0')^2-\omega(\vec{p}''-\vec{p}')^2}\\ &\times\left[\vec{\sigma}_1\cdot\vec{p}\vec{\sigma}_2\cdot\vec{p}-\frac{(1+c)}{2}\frac{(p_0''-p_0')}{2m}\{\vec{\sigma}_1\cdot(\vec{P}+\vec{p}''+\vec{p}')\vec{\sigma}_2\cdot\vec{p}\right.\\ &\left.\qquad+\vec{\sigma}_1\cdot\vec{p}\vec{\sigma}_2\cdot(\vec{P}-\vec{p}''-\vec{p}')\}\right].\end{aligned} \tag{4.7}$$

This exhibits explicitly the dependence of $\hat{V}_\pi$ on p_0'' and p_0'. Note that $\hat{V}_\pi$, unlike the $2\pi E$ potential which we shall consider in Section VII, does not depend on P_0.

We next want to consider the quasipotential $\tilde{V}_\pi$ in the two-nucleon rest frame ($\vec{P}=\vec{0}$). From now on we use $\vec{p}'$, $\vec{p}''$ to denote the initial and final 3-momenta in *this* frame. Using (4.1)–(4.3), with

$$\vec{P}=\vec{0}, \qquad \hat{p}_0'=\hat{p}_0''=0,$$

and making the spinor reductions to order m^{-2}, we find using (2.16) that

$$\begin{aligned}\tilde{V}_\pi(\vec{0};\vec{p}'',\vec{p}') &= \frac{f^2}{m_\pi^2}\vec{\tau}_1\cdot\vec{\tau}_2\frac{-1}{\omega(\vec{p})^2}\left[\vec{\sigma}_1\cdot\vec{p}\vec{\sigma}_2\cdot\vec{p}-\frac{(\vec{p}'^2+\vec{p}''^2)}{4m^2}\vec{\sigma}_1\cdot\vec{p}\vec{\sigma}_2\cdot\vec{p}\right.\\ &\left.+\frac{c(\vec{p}''^2-\vec{p}'^2)}{4m^2}(\vec{\sigma}_1\cdot\vec{p}\vec{\sigma}_2\cdot\vec{p}+\vec{\sigma}_1\cdot\vec{p}'\vec{\sigma}_2\cdot\vec{p}+\vec{\sigma}_1\cdot\vec{p}\vec{\sigma}_2\cdot\vec{p}')\right].\end{aligned} \tag{4.8}$$

The quantity c is defined in (4.6) and the difference between PS and PV coupling appears in the third term of (4.8).

Within the framework of the quasipotential formalism it is the $1\pi E$ potential $\tilde{V}_\pi(\vec{0};\vec{p}'',\vec{p}')$ of (4.8) which should be included in a semi-phenomenological potential for computing two-nucleon wavefunctions in the c.m. system. All semi-phenomenological potentials use the leading term in (4.8). The second term gives a relativistic correction to the $1\pi E$ potential which is included in the Nijmegen potential [7] and is partly included in the Paris potential [8]. The Paris potential is energy dependent. However, the linear energy dependence of the central potential can be transformed into a $\vec{p}^2$ dependence which for the $1\pi E$ potential is given exactly by the second term of (4.8). The $\vec{p}^2$ dependence of the tensor potential is neglected. This is a defect of the Paris potential, but this potential is available in the literature in readily usable form and is the one most widely used in recent calculations. The third term in (4.8), which is c-dependent, is not used in any of the semi-phenomenological potentials. It turns out, however, that this part of $\tilde{V}_\pi$ can be dealt with in a very neat way, which has also been indicated by HG [11], and that the resulting corrections to the initial and final state wavefunctions lead to $1\pi E$ contributions to the charge density.

To show this, we write the third term of (4.8) as

$$\Delta\tilde{V}(\vec{p}'', \vec{p}') = c\frac{(\vec{p}''^2 - \vec{p}'^2)}{4m^2} v(\vec{p}'', \vec{p}'), \tag{4.9}$$

where

$$v(\vec{p}'', \vec{p}') = \frac{f^2}{m_\pi^2} \vec{\tau}_1 \cdot \vec{\tau}_2 \frac{-1}{\omega(\vec{p})^2} (\vec{\sigma}_1 \cdot \vec{p}\vec{\sigma}_2 \cdot \vec{p} + \vec{\sigma}_1 \cdot \vec{p}'\vec{\sigma}_2 \cdot \vec{p} + \vec{\sigma}_1 \cdot \vec{p}\vec{\sigma}_2 \cdot \vec{p}'), \tag{4.10}$$

$\vec{p}$ being defined in (4.4). From equation (2.22) for the case of weak binding, the first order correction $\Delta\phi$ to the bound state wavefunction ϕ associated with $\tilde{V}$ is given by the inhomogeneous equation

$$(H_0 + B + \tilde{V})\,\Delta\phi = -\Delta\tilde{V}\phi, \tag{4.11}$$

where we have written H_0 instead of $\vec{p}^2/m$, and $\tilde{V}$ and $\Delta\tilde{V}$ are of course integral operators. The trick which we now use is to take advantage of the special form of $\Delta\tilde{V}$ in (4.9) to separate from $\Delta\phi$ its longest range part. We have

$$\begin{aligned}\Delta\tilde{V}\phi &= \frac{c}{4m}(H_0 v - vH_0)\phi \\ &= \frac{c}{4m}[(H_0 + \tilde{V} + B)v + [v, \tilde{V}]_- - v(H_0 + \tilde{V} + B)]\phi \\ &= \frac{c}{4m}[(H_0 + \tilde{V} + B)v\phi + [v, \tilde{V}]_-\phi],\end{aligned}$$

using the Schrödinger equation for ϕ. Thus (4.11) becomes

$$(H_0 + \tilde{V} + B)\left(\Delta\phi + \frac{c}{4m} v\phi\right) = -\frac{c}{4m}[v, \tilde{V}]_-\phi. \tag{4.12}$$

We next remark that simply adding $\Delta\tilde{V}$ to $\tilde{V}$ will give a potential which changes the binding energy B and asymptotic D/S ratio η of the deuteron. Since the approximation in which we calculate $\Delta\tilde{V}$ breaks down at short distances, the potential needs to be further modified at such distances in order to reproduce the bound state parameters correctly. Since the Paris potential is chosen phenomenologically inside a radius of 0.8 fm, this is a completely justified procedure. The separation between an inner and an outer region is precisely the philosophy of the Paris potential. In fact, it turns out that the simple expedient of cutting off $\Delta\tilde{V}$ in coordinate space at a radius of 0.76 fm leaves B unaltered. What we see in (4.12) is that, while v has a range m_π^{-1}, the commutator on the right side has a range $(2m_\pi)^{-1}$, and any further term on the right side arising from a modification of the potential required to reproduce B and η correctly will also be of range $(2m_\pi)^{-1}$ or less. Thus we expect that the solution of (4.12) which one obtains by integrating in from infinity will be 'small' compared with $(c/4m)v\phi$ until one gets to a radial distance of about $(2m_\pi)^{-1}$. This expectation is borne out by detailed numerical calculations.

We have now shown that, because of the special form of $\Delta\tilde{V}$ in (4.9), it is a good approximation except at short distances to take for $\Delta\phi$ its longest range

part, namely

$$\Delta\phi \approx -\frac{c}{4m}\, v\phi. \tag{4.13}$$

This result can be derived also by means of a unitary transformation [6]. In evaluating the matrix element $\bar{\phi}\rho\phi$ of (2.33), we then see that the change in ϕ given by (4.13) is equivalent to a change $\Delta\rho$ in ρ which is given by

$$\begin{aligned}\Delta\rho(\vec{q};\vec{p}'',\vec{p}') = -\frac{ce}{4m}[&\hat{e}_1 v(\vec{p}''-\tfrac{1}{2}\vec{q},\vec{p}')+\hat{e}_2 v(\vec{p}''+\tfrac{1}{2}\vec{q},\vec{p}')\\ &-v(\vec{p}'',\vec{p}'+\tfrac{1}{2}\vec{q})\hat{e}_1 - v(\vec{p}'',\vec{p}'-\tfrac{1}{2}\vec{q})\hat{e}_2].\end{aligned} \tag{4.14}$$

In writing (4.14) we have used the nonrelativistic impulse approximation for ρ, namely

$$\rho(\mathrm{NRIA};\vec{q};\vec{p}'',\vec{p}') = e(2\pi)^3[\hat{e}_1\,\delta^{(3)}(\vec{p}''-\vec{p}'-\tfrac{1}{2}\vec{q})+\hat{e}_2\,\delta^{(3)}(\vec{p}''-\vec{p}'+\tfrac{1}{2}\vec{q})]. \tag{4.15}$$

To understand the minus sign in the third and fourth terms of (4.14), note that the spin factor in (4.10) may be written as

$$\vec{\sigma}_1\cdot\vec{p}''\vec{\sigma}_2\cdot\vec{p}''-\vec{\sigma}_1\cdot\vec{p}'\vec{\sigma}_2\cdot\vec{p}',$$

so that

$$v(\vec{p}',\vec{p}'') = -v(\vec{p}'',\vec{p}').$$

Since $(\hat{e}_1+\hat{e}_2)$ commutes with $\vec{\tau}_1\cdot\vec{\tau}_2$, it follows from (4.14) that $\Delta\rho(\vec{0};\vec{p}'',\vec{p}')=0$. Expanding $\Delta\rho$ around $\vec{q}=\vec{0}$ and keeping only the terms linear in $\vec{q}$ (the exact dependence on $\vec{q}$ is given in (4.14)), we have

$$\begin{aligned}&\Delta\rho(\vec{q};\vec{p}'',\vec{p}')\\ &\approx\frac{ef^2c}{4mm_\pi^2}[\tfrac{1}{4}(\tau_{1z}-\tau_{2z})\omega(\vec{p})^{-2}(\vec{\sigma}_1\cdot\vec{q}\vec{\sigma}_2\cdot\vec{p}+\vec{\sigma}_1\cdot\vec{p}\vec{\sigma}_2\cdot\vec{q})\\ &\quad+i(\vec{\tau}_1\times\vec{\tau}_2)_z\{\omega(\vec{p})^{-2}(\tfrac{1}{2}\vec{\sigma}_1\cdot\vec{q}\vec{\sigma}_2\cdot\vec{p}+\tfrac{1}{2}\vec{\sigma}_1\cdot\vec{p}\vec{\sigma}_2\cdot\vec{q}+\vec{\sigma}_1\cdot\vec{q}\vec{\sigma}_2\cdot\vec{p}'\\ &\qquad\qquad+\vec{\sigma}_1\cdot\vec{p}'\vec{\sigma}_2\cdot\vec{q})\\ &\quad-2\vec{p}\cdot\vec{q}\omega(\vec{p})^{-4}(\vec{\sigma}_1\cdot\vec{p}\vec{\sigma}_2\cdot\vec{p}+\vec{\sigma}_1\cdot\vec{p}'\vec{\sigma}_2\cdot\vec{p}+\vec{\sigma}_1\cdot\vec{p}\vec{\sigma}_2\cdot\vec{p}')\}].\end{aligned} \tag{4.16}$$

The corrections to the initial and final state wavefunctions which are induced by the c-dependent part of the $1\pi E$ quasipotential $\tilde{V}_\pi$ have now been shown to lead to a change $\Delta\rho$ in ρ which is given by equation (4.16). Provided this change is taken into account, one can calculate matrix elements with wavefunctions obtained in the two-nucleon rest frame from a potential whose $1\pi E$ part includes the c-independent relativistic correction but not the c-dependent correction.

V. One-pion and two-pion exchange processes

We begin by recalling the decomposition of $\hat{V}$ in (3.1), which splits the right side of (2.44) into four terms. The contribution of the first term to $\hat{\Lambda}^\mu$ was given in (3.2). The term which we can write symbolically as

$$(\hat{V}-\hat{V}_0)G_{++}\hat{K}^\mu G_{++}(\hat{V}-\hat{V}_0)$$

can be shown to give as its leading contribution a $2\pi E$ term which is down by a factor of order m_π/m compared with the leading $2\pi E$ terms. It remains to consider the terms

$$\hat{V}_0 G_{++} \hat{K}^\mu G_{++}(\hat{V}-\hat{V}_0)+(\hat{V}-\hat{V}_0)G_{++}\hat{K}^\mu G_{++}\hat{V}_0. \tag{5.1}$$

The factor $(\hat{V}-\hat{V}_0)$ in the first term of (5.1) is, in full,

$$\begin{aligned}\hat{V}-\hat{V}_0 = {} & \bar{u}_1(\tfrac{1}{2}\vec{P}'+\vec{k}')\bar{u}_2(\tfrac{1}{2}\vec{P}'-\vec{k}')V(P';k',\hat{p}')u_1(\tfrac{1}{2}\vec{Q}'+\vec{p}')u_2(\tfrac{1}{2}\vec{Q}'-\vec{p}') \\ & -\bar{u}_1(\tfrac{1}{2}\vec{\tilde{Q}}'+\vec{k}')\bar{u}_2(\tfrac{1}{2}\vec{\tilde{Q}}'-\vec{k}')V(P';\hat{k}',\hat{p}')u_1(\tfrac{1}{2}\vec{Q}'+\vec{p}')u_2(\tfrac{1}{2}\vec{Q}'-\vec{p}'),\end{aligned} \tag{5.2}$$

where

$$\vec{Q}'=\vec{Q}(P';\hat{p}'), \qquad \vec{\tilde{Q}}'=\vec{Q}(P';\hat{k}').$$

We now insert directly to the left of V in the first term of (5.2) the identity operator in the form

$$\begin{aligned}\mathbb{1} = {} & \left[\sum u_1(\tfrac{1}{2}\vec{\tilde{Q}}'+\vec{k}')\bar{u}_1(\tfrac{1}{2}\vec{\tilde{Q}}'+\vec{k}')-\sum v_1(\tfrac{1}{2}\vec{\tilde{Q}}'+\vec{k}')\bar{v}_1(\tfrac{1}{2}\vec{\tilde{Q}}'+\vec{k}')\right] \\ & \times\left[\sum u_2(\tfrac{1}{2}\vec{\tilde{Q}}'-\vec{k}')\bar{u}_2(\tfrac{1}{2}\vec{\tilde{Q}}'-\vec{k}')-\sum v_2(\tfrac{1}{2}\vec{\tilde{Q}}'-\vec{k}')\bar{v}_2(\tfrac{1}{2}\vec{\tilde{Q}}'-\vec{k}')\right].\end{aligned} \tag{5.3}$$

The sums in (5.3) are over helicities and $\hat{V}$ in the first term of (5.2) is split by means of (5.3) into the sum of four terms:

$$\hat{V}=\hat{V}^{++}+\hat{V}^{-+}+\hat{V}^{+-}+\hat{V}^{--}, \tag{5.4}$$

where

$$\begin{aligned}\hat{V}^{++} = {} & \sum \bar{u}_1(\tfrac{1}{2}\vec{\tilde{Q}}'+\vec{k}')\bar{u}_2(\tfrac{1}{2}\vec{\tilde{Q}}'-\vec{k}')V(P';k',\hat{p}')u_1(\tfrac{1}{2}\vec{Q}'+\vec{p}')u_2(\tfrac{1}{2}\vec{Q}'-\vec{p}') \\ & \times\bar{u}_1(\tfrac{1}{2}\vec{P}'+\vec{k}')u_1(\tfrac{1}{2}\vec{\tilde{Q}}'+\vec{k}')\bar{u}_2(\tfrac{1}{2}\vec{P}'-\vec{k}')u_2(\tfrac{1}{2}\vec{\tilde{Q}}'-\vec{k}'),\end{aligned} \tag{5.5}$$

$$\begin{aligned}\hat{V}^{-+} = {} & -\sum \bar{v}_1(\tfrac{1}{2}\vec{\tilde{Q}}'+\vec{k}')\bar{u}_2(\tfrac{1}{2}\vec{\tilde{Q}}'-\vec{k}')V(P';k',\hat{p}')u_1(\tfrac{1}{2}\vec{Q}'+\vec{p}')u_2(\tfrac{1}{2}\vec{Q}'-\vec{p}') \\ & \times\bar{u}_1(\tfrac{1}{2}\vec{P}'+\vec{k}')v_1(\tfrac{1}{2}\vec{\tilde{Q}}'+\vec{k}')\bar{u}_2(\tfrac{1}{2}\vec{P}'-\vec{k}')u_2(\tfrac{1}{2}\vec{\tilde{Q}}'-\vec{k}').\end{aligned} \tag{5.6}$$

There is no need to write the full expressions for $\hat{V}^{+-}$ and $\hat{V}^{--}$. The sums in (5.5) and (5.6) are over the helicities of the spinors which depend on $\frac{1}{2}\vec{\tilde{Q}}'\pm\vec{k}'$.

Now use (3.19) to write

$$\vec{\tilde{Q}}'-\vec{P}'=\vec{Q}(P';\hat{k}')-\vec{P}'\approx-\vec{P}'\,\frac{P_0'-2\bar{E}(\vec{P}';\vec{k}')}{2m}. \tag{5.7}$$

Note further that

$$P_0'-2\bar{E}(\vec{P}';\vec{k}')\approx(\delta-\vec{k}'^2)/m,$$

where δ is defined in (3.14), so that $\vec{\tilde{Q}}'-\vec{P}'$ is in fact $O(m^{-2})$. Then

$$\bar{u}_1(\tfrac{1}{2}\vec{P}'+\vec{k}')u_1(\tfrac{1}{2}\vec{\tilde{Q}}'+\vec{k}')\approx 1-\frac{i}{8m^2}\vec{\sigma}_1\cdot\{(\vec{P}'-\vec{\tilde{Q}}')\times\vec{k}'\}\approx 1$$

and, from (5.5),

$$\hat{V}^{++}(P';k',\hat{p}')\approx\bar{u}_1(\tfrac{1}{2}\vec{\tilde{Q}}'+\vec{k}')\bar{u}_2(\tfrac{1}{2}\vec{\tilde{Q}}'-\vec{k}')V(P';k',\hat{p}')u_1(\tfrac{1}{2}\vec{Q}'+\vec{p}')u_2(\tfrac{1}{2}\vec{Q}'-\vec{p}'). \tag{5.8}$$

The quantities $\hat{V}^{-+}$ and $\hat{V}^{+-}$ need to be considered only for PS πNN coupling

(for PV coupling the expressions are down by a factor of order $(m_\pi/m)^2$) and are given by

$$\hat{V}^{-+}(P';k',\hat{p}') = -\frac{[P_0'-2\bar{E}(\vec{P}';\vec{k}')]}{4m}\frac{f^2}{m_\pi^2}\frac{(1-c)}{2} \times \vec{\tau}_1\cdot\vec{\tau}_2\frac{\vec{\sigma}_1\cdot\vec{P}'\vec{\sigma}_2\cdot(\vec{k}'-\vec{p}')}{\omega(\vec{k}'-\vec{p}')^2}+O(m^{-4}), \quad (5.9)$$

$$\hat{V}^{+-}(P';k',\hat{p}') = \frac{[P_0'-2\bar{E}(\vec{P}';\vec{k}')]}{4m}\frac{f^2}{m_\pi^2}\frac{(1-c)}{2} \times \vec{\tau}_1\cdot\vec{\tau}_2\frac{\vec{\sigma}_1\cdot(\vec{k}'-\vec{p}')\vec{\sigma}_2\cdot\vec{P}'}{\omega(\vec{k}'-\vec{p}')^2}+O(m^{-4}). \quad (5.10)$$

Going again to symbolic notation, from (5.4) we have

$$\hat{V}-\hat{V}_0 = (\hat{V}^{++}-\hat{V}_0)+\hat{V}^{-+}+\hat{V}^{+-}+\hat{V}^{--}, \quad (5.11)$$

where $\hat{V}^{++}$, $\hat{V}_0$, $\hat{V}^{-+}$ and $\hat{V}^{+-}$ are given in full by (5.8), the second term of (5.2), (5.9) and (5.10), while $\hat{V}^{--}$ can be neglected in the order of approximation to which we are working.

We shall at first investigate that part of (5.1) which is given by

$$\hat{V}_0 G_{++}\hat{K}^\mu G_{++}(\hat{V}^{-+}+\hat{V}^{+-}+\hat{V}^{--})+(\hat{V}^{-+}+\hat{V}^{+-}+\hat{V}^{--})G_{++}\hat{K}^\mu G_{++}\hat{V}_0.$$

We shall obtain the effective charge density ($\mu=0$), considering only the impulse approximation diagrams for $\hat{K}^0$ and taking $\hat{V}$ as the $1\pi E$ potential $\hat{V}_\pi$. The actual calculation is similar to the one described in Section III and we can use the right side of (3.11) as a guide, insert $(\hat{V}^{-+}+\hat{V}^{+-})$ from (5.9) and (5.10) instead of $\hat{V}$ on the far right, keep only the nonrelativistic impulse approximation in the middle and use the equation for the final state wavefunction on the left. Since

$$g_r(P';\vec{k}')\approx[P_0'-2\bar{E}(\vec{P}';\vec{k}')]^{-1}$$

and factors like $\bar{E}/m$ can be set equal to 1, the resulting charge density is similar to (4.14), namely

$$\rho(\mathrm{PS};\vec{P}'',\vec{P}';\vec{p}'',\vec{p}') = e\frac{(1-c)}{8m}[\hat{e}_1 X(\vec{P}';\vec{p}''-\tfrac{1}{2}\vec{q},\vec{p}')+\hat{e}_2 X(\vec{P}';\vec{p}''+\tfrac{1}{2}\vec{q},\vec{p}') - X(\vec{P}'';\vec{p}'',\vec{p}'+\tfrac{1}{2}\vec{q})\hat{e}_1 - X(\vec{P}'';\vec{p}'',\vec{p}'-\tfrac{1}{2}\vec{q})\hat{e}_2], \quad (5.12)$$

where

$$X(\vec{P};\vec{p}'',\vec{p}') = \frac{f^2}{m_\pi^2}\vec{\tau}_1\cdot\vec{\tau}_2\frac{1}{\omega(\vec{p})^2}(\vec{\sigma}_1\cdot\vec{p}\vec{\sigma}_2\cdot\vec{P}-\vec{\sigma}_1\cdot\vec{P}\vec{\sigma}_2\cdot\vec{p}). \quad (5.13)$$

Expanding $\rho(\mathrm{PS})$ and keeping only terms linear in the external momenta, we have

$$\rho(\mathrm{PS};\vec{P}'',\vec{P}';\vec{p}'',\vec{p}') \approx \frac{f^2}{4mm_\pi^2}\frac{(1-c)}{2}[\vec{\tau}_1\cdot\vec{\tau}_2+\tfrac{1}{2}(\tau_{1z}+\tau_{2z})]\frac{1}{\omega(\vec{p})^2}(\vec{\sigma}_1\cdot\vec{q}\vec{\sigma}_2\cdot\vec{p}-\vec{\sigma}_1\cdot\vec{p}\vec{\sigma}_2\cdot\vec{q}), \quad (5.14)$$

since

$$(\hat{e}_1+\hat{e}_2)\vec{\tau}_1\cdot\vec{\tau}_2=\vec{\tau}_1\cdot\vec{\tau}_2(\hat{e}_1+\hat{e}_2)=\vec{\tau}_1\cdot\vec{\tau}_2+\tfrac{1}{2}(\tau_{1z}+\tau_{2z}).$$

We now work out the contribution to the charge density from the remaining terms in (5.1) which involve the retardation in $\hat{V}$, namely

$$\hat{V}_0G_{++}\hat{K}^0G_{++}(\hat{V}^{++}-\hat{V}_0)+(\hat{V}^{++}-\hat{V}_0)G_{++}\hat{K}^0G_{++}\hat{V}_0.$$

Let us denote this charge density by

$$\rho(\mathrm{RET})=\rho'(\mathrm{RET})+\rho''(\mathrm{RET}).$$

Since we are interested only in $1\pi E$ and $2\pi E$ effects, we approximate $\hat{V}$ by $\hat{V}_\pi$ and take for $\hat{K}^0$ the contribution of the impulse approximation diagrams of Fig. 1(a). Thus $\rho(\mathrm{RET})$ may be split into two terms,

$$\rho(\mathrm{RET})=\rho_1(\mathrm{RET})+\rho_2(\mathrm{RET}),$$

just as $\rho(\mathrm{IA})$ was split in (3.3). Using (4.7) for $\hat{V}_\pi$, the expression for $\rho_1'(\mathrm{RET})$ is

$$\begin{aligned}
&\rho_1'(\mathrm{RET};P'',P';\vec{p}'',\vec{p}')\\
&\approx e(2\pi)^{-3}\int d^3(k'',k')\tilde{V}(P'';\vec{p}'',\vec{k}'')\sqrt{\frac{\tilde{E}(P'';\hat{k}'')}{m}}\\
&\times\bar{u}_1(\tfrac{1}{2}\vec{P}''+\vec{k}'')\left(\hat{e}_1\gamma_0-\frac{\hat{\kappa}_1\vec{\gamma}\cdot\vec{q}\gamma_0}{2m}\right)u_1(\tfrac{1}{2}\vec{P}'+\vec{k}')\sqrt{\frac{m}{\tilde{E}(P';\hat{p}')}}\\
&\times\frac{m^3}{E(\frac{1}{2}\vec{P}''+\vec{k}'')E(\frac{1}{2}\vec{P}'+\vec{k}')E(\frac{1}{2}\vec{P}'-\vec{k}')}\frac{f^2}{m_\pi^2}\vec{\tau}_1\cdot\vec{\tau}_2\\
&\times\frac{\delta^{(3)}(\vec{k}''-\vec{k}'-\frac{1}{2}\vec{q})}{[\omega(\vec{k}'-\vec{p}')^2-(\hat{k}_0'-\hat{p}_0')^2]}\\
&\times\Bigg[\vec{\sigma}_1\cdot(\vec{k}'-\vec{p}')\vec{\sigma}_2\cdot(\vec{k}'-\vec{p}')I_1\\
&-\left(\frac{1+c}{2}\right)\frac{1}{2m}\{\vec{\sigma}_1\cdot(\vec{P}'+\vec{k}'+\vec{p}')\vec{\sigma}_2\cdot(\vec{k}'-\vec{p}')\\
&+\vec{\sigma}_1\cdot(\vec{k}'-\vec{p}')\vec{\sigma}_2\cdot(\vec{P}'-\vec{k}'-\vec{p}')\}I_2\Bigg],
\end{aligned}\tag{5.15}$$

where

$$\begin{aligned}
I_1=&\frac{1}{2\pi i}\int d(k_0'',k_0')\\
&\times\frac{\delta(k_0''-k_0'-\frac{1}{2}q_0)(k_0'-\hat{k}_0')(k_0'+\hat{k}_0'-2\hat{p}_0')}{(k_0''-x_1''+i\varepsilon)(k_0'-x_1'+i\varepsilon)(k_0'+x_2'-i\varepsilon)[(k_0'-\hat{p}_0')^2-\omega(\vec{k}'-\vec{p}')^2+i\varepsilon]},
\end{aligned}\tag{5.16}$$

$$\begin{aligned}
I_2=&\frac{1}{2\pi i}\int d(k_0'',k_0')\\
&\times\frac{\delta(k_0''-k_0'-\frac{1}{2}q_0)(k_0'-\hat{k}_0')[(k_0'-\hat{p}_0')(\hat{k}_0'-\hat{p}_0')+\omega(\vec{k}'-\vec{p}')^2]}{(k_0''-x_1''+i\varepsilon)(k_0'-x_1'+i\varepsilon)(k_0'+x_2'-i\varepsilon)[(k_0'-\hat{p}_0')^2-\omega(\vec{k}'-\vec{p}')^2+i\varepsilon]}.
\end{aligned}\tag{5.17}$$

The quantities x_1', x_1'' and x_2' are defined in (3.7).

To obtain the leading contributions to $\rho_1'(\mathrm{RET})$, we approximate the spinor reduction on the second line of (5.15) by $\hat{e}_1$ and replace all factors m/E or $m/\tilde{E}$ by 1. Further, we approximate (2.5) by

$$\hat{k}_0 \approx \tfrac{1}{2}[E(\tfrac{1}{2}\vec{P}' + \vec{k}') - E(\tfrac{1}{2}\vec{P}' - \vec{k}')] = \tfrac{1}{2}(x_1' - x_2'),$$

again neglecting terms of order m^{-2} compared with the leading term. In the same way, we neglect the quantity $(\hat{k}_0' - \hat{p}_0')^2$ in (5.15), since it is of order m^{-2}. The integration over k_0'', k_0' in (5.16) and (5.17) can be performed as before by eliminating the δ-function and then closing the contour in the upper half-plane. Using (3.8) and neglecting terms which lead to effective $2\pi E$ contributions which are down by a factor of order (m_π/m) on the leading $2\pi E$ contributions, we find that

$$\begin{aligned} I_1 &\approx \frac{x_1'' - \frac{1}{2}(P_0'' - P_0') + \hat{k}_0' - 2\hat{p}_0'}{2(x_1'' + x_2'')\omega(\vec{k}' - \vec{p}')^2} \\ &= \frac{1}{4\omega(\vec{k}' - \vec{p}')^2} + \frac{\frac{1}{2}(x_1'' - x_2'') - \frac{1}{2}(P_0'' - P_0') + \hat{k}_0' - 2\hat{p}_0'}{2(x_1'' + x_2'')\omega(\vec{k}' - \vec{p}')^2}, \end{aligned} \tag{5.18}$$

$$I_2 \approx \frac{1}{2(x_1'' + x_2'')}, \tag{5.19}$$

where x_2'' is defined in (3.9). From (5.15), (5.18) and (5.19) we obtain three parts of $\rho_1'(\mathrm{RET})$. Taking the first term in (5.18) and approximating $\tilde{V}$ in (5.15) by $\tilde{V}_\pi$ and $\tilde{V}_\pi$ by the first term in (4.9), we arrive at part of the leading $2\pi E$ contribution

$$\begin{aligned} &\rho_1'(2\pi E, u; \vec{q}; \vec{p}'', \vec{p}') \\ &= -e\frac{f^4}{m_\pi^4}\vec{\tau}_1 \cdot \vec{\tau}_2 \hat{e}_1 \vec{\tau}_1 \cdot \vec{\tau}_2 \\ &\quad \times (2\pi)^{-3} \int d^3(k'', k') \\ &\quad \times \frac{\delta^{(3)}(\vec{k}'' - \vec{k}' - \frac{1}{2}\vec{q})\vec{\sigma}_1 \cdot (\vec{p}'' - \vec{k}'')\vec{\sigma}_2 \cdot (\vec{p}'' - \vec{k}'')\vec{\sigma}_1 \cdot (\vec{k}' - \vec{p}')\vec{\sigma}_2 \cdot (\vec{k}' - \vec{p}')}{4\omega(\vec{p}'' - \vec{k}'')^2\omega(\vec{k}' - \vec{p}')^4}. \end{aligned} \tag{5.20}$$

On the left side of (5.20), 'u' denotes the uncrossed $2\pi E$ process. In an analogous manner $\rho_1''(2\pi E, u)$ may be derived from $\rho_1''(\mathrm{RET})$. We find for $\rho_1(2\pi E, u) = \rho_1'(2\pi E, u) + \rho_1''(2\pi E, u)$ the result

$$\begin{aligned} \rho_1(2\pi E, u; \vec{q}; \vec{p}'', \vec{p}') = &\frac{ef^4}{m_\pi^4}\vec{\tau}_1 \cdot \vec{\tau}_2 \hat{e}_1 \vec{\tau}_1 \cdot \vec{\tau}_2 \\ &\times (2\pi)^{-3} \int d^3(k'', k')\, \delta^{(3)}(\vec{k}'' - \vec{k}' - \tfrac{1}{2}\vec{q}) J_u(\vec{k}'', \vec{k}'; \vec{p}'', \vec{p}') \\ &\times \vec{\sigma}_1 \cdot (\vec{p}'' - \vec{k}'')\vec{\sigma}_2 \cdot (\vec{p}'' - \vec{k}'')\vec{\sigma}_1 \cdot (\vec{k}' - \vec{p}')\vec{\sigma}_2 \cdot (\vec{k}' - \vec{p}'), \end{aligned} \tag{5.21}$$

where

$$J_u(\vec{k}'', \vec{k}'; \vec{p}'', \vec{p}') = -\frac{\omega(\vec{p}'' - \vec{k}'')^2 + \omega(\vec{k}' - \vec{p}')^2}{4\omega(\vec{p}'' - \vec{k}'')^4\omega(\vec{k}' - \vec{p}')^4}. \tag{5.22}$$

To deal with the second term in (5.18), we note that, from (3.7) and (3.9),

$$\frac{-1}{(x_1''+x_2'')}=\frac{1}{P_0''-2\bar{E}(\vec{P}'';\vec{k}'')}\approx\hat{g}(P'';\hat{k}'')\frac{\tilde{E}(P'';\hat{k}'')}{m}, \tag{5.23}$$

using (3.15). When the matrix element $\bar{\phi}\rho\phi$ is taken, one can therefore use (2.17) for the final state wavefunction. Also, in leading approximation,

$$\tfrac{1}{2}(x_1''-x_2'')\approx\vec{P}''\cdot\vec{k}''/2m,\qquad \hat{k}_0'\approx\vec{P}'\cdot\vec{k}'/2m,\qquad \hat{p}_0'\approx\vec{P}'\cdot\vec{p}'/2m. \tag{5.24}$$

In the numerator of the second term of (5.18) there is also the term $-\frac{1}{2}(P_0''-P_0')=-\frac{1}{2}q_0$. When the matrix element is taken between bound states, and one works to first order in $\vec{P}'$ and $\vec{P}''$, $P_0'\approx P_0''\approx M$ and this term does not contribute. However, when a physical photon induces transitions between a bound state and a continuum state, it needs to be taken into account. We leave a discussion of this term to the paper on deuteron photodisintegration. Using (5.15), (5.23) and (5.24), the second term in (5.18) leads to the $1\pi E$ charge density

$$\begin{aligned}&\rho_1'(1\pi E;\vec{P}'',\vec{P}';\vec{p}'',\vec{p}')\\&=\frac{ef^2}{4mm_\pi^2}\frac{\hat{e}_1\vec{\tau}_1\cdot\vec{\tau}_2}{\omega(\vec{p}-\frac{1}{2}\vec{q})^4}\vec{\sigma}_1\cdot(\vec{p}-\tfrac{1}{2}\vec{q})\vec{\sigma}_2\cdot(\vec{p}-\tfrac{1}{2}\vec{q})\\&\quad\times[-\vec{q}\cdot\vec{p}'-\vec{P}\cdot\vec{p}+\tfrac{1}{4}\vec{q}\cdot(\vec{P}-\vec{q})],\end{aligned} \tag{5.25}$$

where $\vec{P},\vec{p}$ are defined in (3.18), (4.4) respectively. In the same way we may calculate $\rho_1''(1\pi E)$ from $\rho_1''(\mathrm{RET})$, with the result

$$\begin{aligned}&\rho_1''(1\pi E;\vec{P}'',\vec{P}';\vec{p}'',\vec{p}')\\&=\frac{ef^2}{4mm_\pi^2}\frac{\vec{\tau}_1\cdot\vec{\tau}_2\hat{e}_1}{\omega(\vec{p}-\frac{1}{2}\vec{q})^4}\vec{\sigma}_1\cdot(\vec{p}-\tfrac{1}{2}\vec{q})\vec{\sigma}_2\cdot(\vec{p}-\tfrac{1}{2}\vec{q})\\&\quad\times[\vec{q}\cdot\vec{p}''+\vec{P}\cdot\vec{p}-\tfrac{1}{4}\vec{q}\cdot(\vec{P}+\vec{q})].\end{aligned} \tag{5.26}$$

Working to first order in $\vec{P}$ and $\vec{q}$, we find using (5.25) and (5.26) that $\rho_1(1\pi E)=\rho_1'(1\pi E)+\rho_1''(1\pi E)$ is given by

$$\begin{aligned}\rho_1(1\pi E;\vec{P}'',\vec{P}';\vec{p}'',\vec{p}')\approx\frac{ef^2}{4mm_\pi^2}\frac{\vec{\sigma}_1\cdot p\vec{\sigma}_2\cdot\vec{p}}{\omega(\vec{p})^4}&[\tfrac{1}{2}(\vec{\tau}_1\cdot\vec{\tau}_2+\tau_{2z})\vec{q}\cdot\vec{p}\\&+i(\vec{\tau}_1\times\vec{\tau}_2)_z(\tfrac{1}{2}\vec{q}\cdot(\vec{p}''+\vec{p}')+\vec{P}\cdot\vec{p})],\end{aligned} \tag{5.27}$$

where use has been made of the identities

$$\begin{aligned}\hat{e}_1\vec{\tau}_1\cdot\vec{\tau}_2&=\tfrac{1}{2}(\vec{\tau}_1\cdot\vec{\tau}_{2z})-\tfrac{1}{2}i(\vec{\tau}_1\times\vec{\tau}_2)_z,\\\vec{\tau}_1\cdot\vec{\tau}_2\hat{e}_1&=\tfrac{1}{2}(\vec{\tau}_1\cdot\vec{\tau}_2+\tau_{2z})+\tfrac{1}{2}i(\vec{\tau}_1\times\vec{\tau}_2)_z.\end{aligned} \tag{5.28}$$

This is part of the operator which in JW we called CORR; we now call it $1\pi E$ since this is the operator which is normally referred to in the literature as the one-pion exchange operator.

The third part of $\rho_1(\mathrm{RET})$ comes from I_2 in (5.19). From (5.15), (5.19) and

(5.23) we obtain a contribution to ρ of the form

$$\begin{aligned}
&\rho_1(\mathrm{PV}; \vec{P}'', \vec{P}'; \vec{p}'', \vec{p}') \\
&= \frac{ef^2}{4mm_\pi^2}\left(\frac{1+c}{2}\right)\frac{1}{\omega(\vec{p}-\frac{1}{2}\vec{q})^2} \\
&\quad \times[\hat{e}_1\vec{\tau}_1\cdot\vec{\tau}_2\{\vec{\sigma}_1\cdot(\tfrac{1}{2}\vec{P}-\vec{q}+\vec{p}''+\vec{p}')\vec{\sigma}_2\cdot(\vec{p}-\tfrac{1}{2}\vec{q}) \\
&\qquad\qquad +\vec{\sigma}_1\cdot(\vec{p}-\tfrac{1}{2}\vec{q})\vec{\sigma}_2\cdot(\tfrac{1}{2}\vec{P}-\vec{p}''-\vec{p}')\} \\
&\quad -\vec{\tau}_1\cdot\vec{\tau}_2\hat{e}_1\{\vec{\sigma}_1\cdot(\tfrac{1}{2}\vec{P}+\vec{q}+\vec{p}''+\vec{p}')\vec{\sigma}_2\cdot(\vec{p}-\tfrac{1}{2}\vec{q}) \\
&\qquad\qquad +\vec{\sigma}_1\cdot(\vec{p}-\tfrac{1}{2}\vec{q})\vec{\sigma}_2\cdot(\tfrac{1}{2}\vec{P}-\vec{p}''-\vec{p}')\}].
\end{aligned} \tag{5.29}$$

We use the label PV because this term arises from the PV πNN vertex. To first order in $\vec{P}$ and $\vec{q}$ we find, using (5.28) again,

$$\begin{aligned}
&\rho_1(\mathrm{PV}; \vec{P}'', \vec{P}'; \vec{p}'', \vec{p}') \\
&\approx \frac{ef^2}{4mm_\pi^2}\left(\frac{1+c}{2}\right)[-(\vec{\tau}_1\cdot\vec{\tau}_2+\tau_{2z})\omega(\vec{p})^{-2}\vec{\sigma}_1\cdot\vec{q}\vec{\sigma}_2\cdot\vec{p} \\
&\quad +i(\vec{\tau}_1\times\vec{\tau}_2)_z\{\omega(\vec{p})^{-2}(2\vec{\sigma}_1\cdot\vec{p}\sigma_2\cdot\vec{p}'-2\vec{\sigma}_1\cdot\vec{p}'\vec{\sigma}_2\cdot\vec{p}-\tfrac{1}{2}\vec{\sigma}_1\cdot\vec{P}\vec{\sigma}_2\cdot\vec{p} \\
&\quad -\tfrac{1}{2}\vec{\sigma}_1\cdot\vec{p}\vec{\sigma}_2\cdot\vec{P}-\tfrac{1}{2}\vec{\sigma}_1\cdot\vec{q}\vec{\sigma}_2\cdot(\vec{p}''+\vec{p}')+\tfrac{1}{2}\vec{\sigma}_1\cdot(\vec{p}''+\vec{p}')\vec{\sigma}_2\cdot\vec{q}) \\
&\quad +2\vec{p}\cdot\vec{q}\omega(\vec{p})^{-4}(\vec{\sigma}_1\cdot\vec{p}\vec{\sigma}_2\cdot\vec{p}'-\vec{\sigma}_1\cdot\vec{p}'\vec{\sigma}_2\cdot\vec{p})\}].
\end{aligned} \tag{5.30}$$

In the second part of this section, we consider the charge density given by (3.2) for each of the diagrams (b)–(g) of Fig. 1. In fact the results for SG (Fig. 1(b)), πC (Fig. 1(c)), $1\pi E$ with $\Delta(1232)$ intermediate state (Fig. 1(d)) and crossed $2\pi E$ (Fig. 1(e)) are given in JW, although the results for πC and SG need to be generalized to an arbitrary frame of reference and that for SG is misprinted in equation (4.3) of JW. It is instructive to see how these results come from the formalism which we developed in Section II. We take the πC diagram as an example. Omitting the factors $\sqrt{\bar{E}/m}$ in (3.2) and the factors m/E in G_{++} in (2.42), and using (4.5) for the πNN vertex, equation (3.2) gives

$$\begin{aligned}
&\rho(\pi C; \vec{P}'', \vec{P}'; \vec{p}'', \vec{p}') \\
&\approx (2\pi)^{-6}\int d^3(k'', k')\tilde{V}(P''; \vec{p}'', \vec{k}'')\left(\frac{-2ef^2}{m_\pi^2}\right) \\
&\quad \times i(\vec{\tau}_1\times\vec{\tau}_2)_z\vec{\sigma}_1\cdot(\vec{k}+\tfrac{1}{2}\vec{q})\vec{\sigma}_2\cdot(\vec{k}-\tfrac{1}{2}\vec{q})\tilde{V}(P'; \vec{k}', \vec{p}') \\
&\quad \times(2\pi i)^{-2}\int d(k_0'', k_0')\frac{(k_0''-k_0')}{(k_0''-x_1''+i\varepsilon)(k_0''+x_2''-i\varepsilon)(k_0'-x_1'+i\varepsilon)(k_0'+x_2'-i\varepsilon)} \\
&\quad \times\frac{1}{[(k_0''-k_0')^2-\omega_+^2+i\varepsilon][(k_0''-k_0')^2-\omega_-^2+i\varepsilon]},
\end{aligned} \tag{5.31}$$

where x_1', x_2', x_1'', x_2'' are defined in (3.7) and (3.9) and

$$\vec{k}=\vec{k}''-\vec{k}', \qquad \omega_\pm^2=m^2+(\vec{k}\pm\tfrac{1}{2}\vec{q})^2.$$

The integral over k_0'', k_0' in (5.31) may be evaluated exactly, with the result

$$\frac{(x_1''-x_2''-x_1'+x_2')[1+(\omega_++\omega_-)^{-1}(x_1''+x_2''+x_1'+x_2')]}{(x_1''+x_2'')(x_1'+x_2')2(\omega_++x_1''+x_2')(\omega_++x_1'+x_2'')(\omega_-+x_1''+x_2')(\omega_-+x_1'+x_2'')}$$
$$\approx\frac{(x_1''-x_2''-x_1'+x_2')}{2\omega_+^2\omega_-^2(x_1''+x_2'')(x_1'+x_2')}$$

in leading approximation. As in (5.24),

$$x_1''-x_2''\approx\vec{P}''\cdot\vec{k}''/m,\qquad x_1'-x_2'\approx\vec{P}'\cdot\vec{k}'/m.$$

Using (5.23) and the similar equation with singly primed quantities, taking the matrix element between two-body wavefunctions and using (2.17), Eq. (5.31) gives

$$\rho(\pi C;\vec{P}'',\vec{P}';\vec{p}'',\vec{p}')\doteq-\frac{ef^2}{2mm_\pi^2}i(\vec{\tau}_1\times\vec{\tau}_2)_z$$
$$\times\frac{\vec{\sigma}_1\cdot(\vec{p}+\frac{1}{2}\vec{q})\vec{\sigma}_2\cdot(\vec{p}-\frac{1}{2}\vec{q})}{\omega(\vec{p}+\frac{1}{2}\vec{q})^2\omega(\vec{p}-\frac{1}{2}\vec{q})^2}[\vec{P}\cdot\vec{p}+\vec{q}\cdot(\vec{p}''+\vec{p}')].\quad(5.32)$$

The correct result for the SG diagram (Fig. 1(b)) is

$$\rho_1(\mathrm{SG};\vec{P}'',\vec{P}';\vec{p}'',\vec{p}')$$
$$\approx\frac{ef^2}{4mm_\pi^2}\left(\frac{1+c}{2}\right)\frac{i(\vec{\tau}_1\times\vec{\tau}_2)_z}{\omega(\vec{p}-\frac{1}{2}\vec{q})^2}$$
$$\times[2\vec{\sigma}_1\cdot(\vec{p}''+\vec{p}')\vec{\sigma}_2\cdot\vec{p}-\vec{\sigma}_1\cdot(\vec{p}''+\vec{p}')\vec{\sigma}_2\cdot\vec{q}$$
$$+\vec{\sigma}_1\cdot\vec{P}\vec{\sigma}_2\cdot\vec{p}-\tfrac{1}{2}\vec{\sigma}_1\cdot\vec{P}\vec{\sigma}_2\cdot\vec{q}].\quad(5.33)$$

We now show that $\rho(1\pi E)+\rho(\pi C)$ is a function of $\vec{q}$ only, and does not depend on $\vec{P}$ if only terms linear in $\vec{q}$ and $\vec{P}$ are considered. Returning to (5.27) and noting that $\rho_2(1\pi E)$ is obtained from $\rho_1(1\pi E)$ by the replacements $\vec{\sigma}_1\leftrightarrow\vec{\sigma}_2$, $\vec{\tau}_1\leftrightarrow\vec{\tau}_2$, $\vec{p}'\to-\vec{p}$, $\vec{p}''\to-\vec{p}''$, we have for the sum of $\rho_1(1\pi E)$ and $\rho_2(1\pi E)$

$$\rho(1\pi E;\vec{P}'',\vec{P}';\vec{p}'',\vec{p}')$$
$$\approx\frac{ef^2}{4mm_\pi^2}\frac{\vec{\sigma}_1\cdot\vec{p}\vec{\sigma}_2\cdot\vec{p}}{\omega(\vec{p})^4}[-\tfrac{1}{2}(\tau_{1z}-\tau_{2z})\vec{q}\cdot\vec{p}+i(\vec{\tau}_1\times\vec{\tau}_2)_z(\vec{q}\cdot(\vec{p}''+\vec{p}')+2\vec{P}\cdot\vec{p})].$$
$$(5.34)$$

Adding the charge densities in (5.32) and (5.34),

$$\rho(\pi C+1\pi E;\vec{P}'',\vec{P}';\vec{p}'',\vec{p}')\approx-\frac{ef^2}{4mm_\pi^2}\frac{\vec{\sigma}_1\cdot\vec{p}\vec{\sigma}_2\cdot\vec{p}}{\omega(\vec{p})^4}$$
$$\times\vec{q}\cdot[\tfrac{1}{2}(\tau_{1z}-\tau_{2z})\vec{p}+i(\vec{\tau}_1\times\vec{\tau}_2)_z(2\vec{p}'+\vec{p})],\quad(5.35)$$

which is independent of $\vec{P}$.

Next we show that $\rho(\mathrm{SG})+\rho(\mathrm{PV})$ does not depend on $\vec{P}$ in the linear

approximation. Adding the charge densities in (5.30) and (5.33), we have

$$\begin{aligned}
&\rho_1(\mathrm{SG}+\mathrm{PV};\vec{P}'',\vec{P}';\vec{p}'',\vec{p}') \\
&\approx \frac{ef^2}{4mm_\pi^2}\left(\frac{1+c}{2}\right)[\omega(\vec{p})^{-2}\{-(\vec{\tau}_1\cdot\vec{\tau}_2+\tau_{2z})\vec{\sigma}_1\cdot\vec{q}\vec{\sigma}_2\cdot\vec{p} \\
&\quad +i(\vec{\tau}_1\times\vec{\tau}_2)_z(2\vec{\sigma}_1\cdot\vec{p}\vec{\sigma}_2\cdot\vec{p}'+2\vec{\sigma}_1\cdot\vec{p}'\vec{\sigma}_2\cdot\vec{p}+2\vec{\sigma}_1\cdot\vec{p}\vec{\sigma}_2\cdot\vec{p} \\
&\quad -\tfrac{1}{2}\vec{\sigma}_1\cdot\vec{q}\vec{\sigma}_2\cdot(\vec{p}+2\vec{p}')-\tfrac{1}{2}\vec{\sigma}_1\cdot(\vec{p}+2\vec{p}')\vec{\sigma}_2\cdot\vec{q} \\
&\quad +\tfrac{1}{2}\vec{\sigma}_1\cdot\vec{P}\vec{\sigma}_2\cdot\vec{p}-\tfrac{1}{2}\vec{\sigma}_1\cdot\vec{p}\vec{\sigma}_2\cdot\vec{P})\} \\
&\quad +2\vec{q}\cdot\vec{p}\omega(\vec{p})^{-4}i(\vec{\tau}_1\times\vec{\tau}_2)_z(\vec{\sigma}_1\cdot\vec{p}\vec{\sigma}_2\cdot\vec{p}'+\vec{\sigma}_1\cdot\vec{p}'\vec{\sigma}_2\cdot\vec{p}+\vec{\sigma}_1\cdot\vec{p}\vec{\sigma}_2\cdot\vec{p})]. \qquad (5.36)
\end{aligned}$$

On making the same replacements as before to obtain $\rho_2(\mathrm{SG}+\mathrm{PV})$ and then adding, we find that

$$\begin{aligned}
&\rho(\mathrm{SG}+\mathrm{PV};\vec{P}'',\vec{P}';\vec{p}'',\vec{p}') \\
&\approx \frac{ef^2}{4mm_\pi^2}\left(\frac{1+c}{2}\right)[\omega(\vec{p})^{-2}\{(\tfrac{1}{2}(\tau_{1z}+\tau_{2z})+\vec{\tau}_1\cdot\vec{\tau}_2)(\vec{\sigma}_1\cdot\vec{p}\vec{\sigma}_2\cdot\vec{q}-\vec{\sigma}_1\cdot\vec{q}\vec{\sigma}_2\cdot\vec{p}) \\
&\quad +\tfrac{1}{2}(\tau_{1z}-\tau_{2z})(\vec{\sigma}_1\cdot\vec{p}\vec{\sigma}_2\cdot\vec{q}+\vec{\sigma}_1\cdot\vec{q}\vec{\sigma}_2\cdot\vec{p}) \\
&\quad -i(\vec{\tau}_1\times\vec{\tau}_2)_z(\vec{\sigma}_1\cdot\vec{q}\vec{\sigma}_2\cdot(\vec{p}+2\vec{p}')+\vec{\sigma}_1\cdot(\vec{p}+2\vec{p}')\vec{\sigma}_2\cdot\vec{q})\} \\
&\quad +4\vec{q}\cdot\vec{p}\omega(\vec{p})^{-4}i(\vec{\tau}_1\times\vec{\tau}_2)_z(\vec{\sigma}_1\cdot\vec{p}\vec{\sigma}_2\cdot\vec{p}'+\vec{\sigma}_1\cdot\vec{p}'\vec{\sigma}_2\cdot\vec{p}+\vec{\sigma}_1\cdot\vec{p}\vec{\sigma}_2\cdot\vec{p})], \qquad (5.37)
\end{aligned}$$

which is independent of $\vec{P}$. Note also that, from (5.35) and (5.37),

$$\begin{aligned}
&\rho(\pi C+1\pi E;\vec{q}=\vec{0};\vec{p}'',\vec{p}')=0, \\
&\rho(\mathrm{SG}+\mathrm{PV};\vec{q}=\vec{0};\vec{p}'',\vec{p}')=0.
\end{aligned} \qquad (5.38)$$

Equation (5.38) means that these parts of the total charge vanish.

This completes the study of $1\pi E$ effects, except for $1\pi E$ with $\Delta(1232)$ as the intermediate state and the pair current (PC) contribution for PS coupling which was discussed near the end of Section II. The charge density for the latter is

$$\begin{aligned}
&\rho(\mathrm{PC};\vec{q};\vec{p}'',\vec{p}') \\
&= -\frac{ef^2}{2mm_\pi^2}\left(\frac{1-c}{2}\right)[\omega(\vec{p}-\tfrac{1}{2}\vec{q})^{-2}\vec{\sigma}_1\cdot\vec{q}\vec{\sigma}_2\cdot(\vec{p}-\tfrac{1}{2}\vec{q})\{(1+\kappa_S)\vec{\tau}_1\cdot\vec{\tau}_2+(1+\kappa_V)\tau_{2z}\} \\
&\qquad -\omega(\vec{p}+\tfrac{1}{2}\vec{q})^{-2}\vec{\sigma}_1\cdot(\vec{p}+\tfrac{1}{2}\vec{q})\vec{\sigma}_2\cdot\vec{q}\{(1+\kappa_S)\vec{\tau}_1\cdot\vec{\tau}_2+(1+\kappa_V)\tau_{1z}\}]. \qquad (5.39)
\end{aligned}$$

The study of the $1\pi E$ diagrams with a $\Delta(1232)$ intermediate state (Fig. 1(d)) is quite separate from that of the $1\pi E$ effects we have considered so far. It is made in great detail in the latter part of Section 4 of JW and there is no need to add anything to the discussion given there.

We have already given in (5.21) and (5.22) the leading contribution to $\rho_1(2\pi E, u)$. Obtaining $\rho_2(2\pi E, u)$ from $\rho_1(2\pi E, u)$ by the usual substitutions and

adding, we have

$$\rho(2\pi E; u; \vec{q}; \vec{p}'', \vec{p}') = \frac{ef^4}{m_\pi^4}(2\pi)^{-3}\int d^3(k'', k')\vec{\tau}_1\cdot\vec{\tau}_2[\hat{e}_1\,\delta^{(3)}(\vec{k}''-\vec{k}'-\tfrac{1}{2}\vec{q}) + \hat{e}_2\,\delta^{(3)}(\vec{k}''-\vec{k}'+\tfrac{1}{2}\vec{q})]\vec{\tau}_1\cdot\vec{\tau}_2 \times J_u(\vec{k}'', \vec{k}'; \vec{p}'', \vec{p}')\vec{\sigma}_1\cdot(\vec{p}''-\vec{k}'')\vec{\sigma}_2\cdot(\vec{p}''-\vec{k}'')\vec{\sigma}_1\cdot(\vec{k}'-\vec{p}')\vec{\sigma}_2\cdot(\vec{k}'-\vec{p}'), \tag{5.40}$$

where J_u is given by (5.22). This of course is the result given in Equations (3.2) and (3.6) of JW, and it is the *correct* result within the framework of the BBS formalism. SKO [10] give a result which is *twice* that given by (5.40) and claim that our result is incorrect. We shall explain in Section VIII how the difference between the two results arises from the different retardation in the $1\pi E$ potential in the two cases. For the crossed $2\pi E$ process (Fig. 1(e)) we are in agreement with SKO, as one would expect, since the result comes from a direct calculation of the crossed diagram and does not involve subtle details of the retardation in $\hat{V}_\pi$. The result is given in equations (3.9) and (3.10) of JW and is

$$\rho(2\pi E, c; \vec{q}; \vec{p}'', \vec{p}') = \frac{ef^4}{m_\pi^4}(2\pi)^{-3}\int d^3(k'', k')\sum_{\alpha,\beta}[\tau_{1\alpha}\hat{e}_1\tau_{1\beta}\tau_{2\beta}\tau_{2\alpha}\,\delta^{(3)}(\vec{k}''-\vec{k}'-\tfrac{1}{2}\vec{q}) + \tau_{1\alpha}\tau_{1\beta}\tau_{2\beta}\hat{e}_2\,\tau_{2\alpha}\,\delta^{(3)}(\vec{k}''-\vec{k}'+\tfrac{1}{2}\vec{q})] \times J_c(\vec{k}'', \vec{k}'; \vec{p}'', \vec{p}')\vec{\sigma}_1\cdot(\vec{p}''-\vec{k}'')\vec{\sigma}_1\cdot(\vec{k}'-\vec{p}')\vec{\sigma}_2\cdot(\vec{k}'-\vec{p}')\vec{\sigma}_2\cdot(\vec{p}''-\vec{k}''), \tag{5.41}$$

where

$$J_c(\vec{k}'', \vec{k}'; \vec{p}'', \vec{p}') = -2J_u(\vec{k}'', \vec{k}'; \vec{p}'', \vec{p}'). \tag{5.42}$$

Finally, the charge density arising from the diagrams of Fig. 1(f) and (g), in which the photon is absorbed on one of the intermediate pions, is easily found to be

$$\rho(2\pi E, \pi C; \vec{q}; \vec{p}'', \vec{p}') = \frac{ef^4}{m^4}(2\tau_{1z}+2\tau_{2z}+i(\vec{\tau}_1\times\vec{\tau}_2)_z) \times(2\pi)^{-3}\int d^3k \times\frac{\vec{\sigma}_1\cdot(\vec{p}''-\vec{k})\vec{\sigma}_1\cdot(\vec{k}-\vec{p}'+\tfrac{1}{2}\vec{q})\vec{\sigma}_2\cdot(\vec{k}-\vec{p}'-\tfrac{1}{2}\vec{q})\vec{\sigma}_2\cdot(\vec{p}''-\vec{k})}{\omega(\vec{p}''-\vec{k})^2\omega(\vec{k}-\vec{p}'+\tfrac{1}{2}\vec{q})^2\omega(\vec{k}-\vec{p}'-\tfrac{1}{2}\vec{q})^2} + \frac{ef^4}{m_\pi^4}(2\tau_{1z}+2\tau_{2z}-i(\vec{\tau}_1\times\vec{\tau}_2)_z) \times(2\pi)^{-3}\int d^3k \times\frac{\vec{\sigma}_1\cdot(\vec{p}''-\vec{k}+\tfrac{1}{2}\vec{q})\vec{\sigma}_1\cdot(\vec{k}-\vec{p}')\vec{\sigma}_2\cdot(\vec{k}-\vec{p}')\vec{\sigma}_2\cdot(\vec{p}''-\vec{k}-\tfrac{1}{2}\vec{q})}{\omega(\vec{p}''-\vec{k}+\tfrac{1}{2}\vec{q})^2\omega(\vec{p}''-\vec{k}-\tfrac{1}{2}\vec{q})^2\omega(\vec{k}-\vec{p}')^2}. \tag{5.43}$$

SKO call this charge density the $2\pi E$ boson charge density; we use the notation πC in analogy with that for the corresponding $1\pi E$ process.

VI. The equivalence theorem and the difference between PS and PV coupling

Adding the expressions for ρ given in (4.16), (5.14), (5.35), (5.37) and (5.39), the result is

$$\begin{aligned}\rho(\vec{q};\vec{p}'',\vec{p}') = \frac{ef^2}{4mm_\pi^2}\Big[&(1+(1-c)\kappa_S)\vec{\tau}_1\cdot\vec{\tau}_2\omega(\vec{p})^{-2}(\vec{\sigma}_1\cdot p\vec{\sigma}_2\cdot\vec{q}-\vec{\sigma}_1\cdot\vec{q}\vec{\sigma}_2\cdot\vec{p})\\ &+\tfrac{1}{2}(1+(1-c)\kappa_V)(\tau_{1z}+\tau_{2z})\omega(\vec{p})^{-2}(\vec{\sigma}_1\cdot\vec{p}\vec{\sigma}_2\cdot q-\vec{\sigma}_1\cdot\vec{q}\vec{\sigma}_2\cdot\vec{p})\\ &+\tfrac{1}{2}(1+(1-c)\kappa_V)(\tau_{1z}-\tau_{2z})\omega(\vec{p})^{-2}(\vec{\sigma}_1\cdot\vec{p}\vec{\sigma}_2\cdot\vec{q}+\vec{\sigma}_1\cdot\vec{q}\vec{\sigma}_2\cdot\vec{p})\\ &+\tfrac{1}{4}(\tau_{1z}-\tau_{2z})\left(\frac{\vec{\sigma}_1\cdot\vec{p}\vec{\sigma}_2\cdot\vec{q}+\vec{\sigma}_1\cdot\vec{q}\vec{\sigma}_2\cdot\vec{p}}{\omega(\vec{p})^2}-\frac{2\vec{\sigma}_1\cdot\vec{p}\vec{\sigma}_2\cdot\vec{p}\vec{p}\cdot\vec{q}}{\omega(\vec{p})^4}\right)\\ &-\tfrac{1}{2}i(\vec{\tau}_1\times\vec{\tau}_2)_z\omega(\vec{p})^{-2}(\vec{\sigma}_1\cdot\vec{q}\vec{\sigma}_2\cdot(2\vec{p}'+\vec{p})+\vec{\sigma}_1\cdot(2\vec{p}'+\vec{p})\vec{\sigma}_2\cdot\vec{q})\\ &+i(\vec{\tau}_1\times\vec{\tau}_2)_z\omega(\vec{p})^{-4}\{2\vec{p}\cdot\vec{q}(\vec{\sigma}_1\cdot\vec{p}\vec{\sigma}_2\cdot\vec{p}+\vec{\sigma}_1\cdot\vec{p}'\vec{\sigma}_2\cdot\vec{p}+\vec{\sigma}_1\cdot\vec{p}\vec{\sigma}_2\cdot\vec{p}')\\ &-\vec{\sigma}_1\cdot\vec{p}\vec{\sigma}_2\cdot\vec{p}(2\vec{p}'+\vec{p})\cdot\vec{q}\}\Big].\end{aligned} \tag{6.1}$$

One sees that in the final result for ρ there are three c-dependent terms. The difference between PS and PV coupling is located in just these terms. We now show how it is these terms which are singled out by the equivalence theorem, which connects the PV and PS interaction Hamiltonians for the $\gamma\pi N$ system.

The equivalence theorem in just the form we want is given by Friar [6]. He shows that, if $H(\mathrm{PS})$ and $H(\mathrm{PV})$ are the Hamiltonian operators for the $\gamma\pi N$, system, then a unitary Dyson-Foldy transformation gives

$$\begin{aligned}H(\mathrm{PS})\to H'(\mathrm{PS}) = {}& H(\gamma NN;\mathrm{charge})+H(\gamma NN;mm)+H(\pi NN;\mathrm{PV})\\ &+H(\gamma\pi NN;\mathrm{SG})+H(\gamma\pi NN;mm)\\ &+H(\pi\pi NN)+\cdots.\end{aligned} \tag{6.2}$$

There is an infinite sum of multipion vertices generated by the equivalence transformation, as indicated by the last term and the dots in (6.2). The γNN charge interaction generates the $\gamma\pi NN$ contact interaction which we have labelled $H(\gamma\pi NN;\mathrm{SG})$ and have used in the calculations of Section V. However, the γNN anomalous magnetic moment interaction,

$$H(\gamma NN;mm) = -\frac{e}{2m}\bar{N}\sigma_{\mu\nu}\tfrac{1}{2}(\kappa_S\mathbb{1}+\kappa_V\tau_z)N\,\partial^\nu A^\mu, \tag{6.3}$$

generates a second $\gamma\pi NN$ contact interaction which we have called $H(\gamma\pi NN;mm)$. It is given by

$$H(\gamma\pi NN;mm) = \frac{ief}{2mm_\pi}\bar{N}\sigma_{\mu\nu}\gamma_5(\kappa_S\tau_a+\kappa_V\,\delta_{3a})N\,\partial^\nu A^\mu\phi_a,$$

where there is a sum on the isospin index a. The corresponding vertex is

$$V(\gamma\pi NN;\, mm) = -\frac{ief}{2mm_\pi}\,\sigma^{\mu\nu}q_\nu\gamma_5(\kappa_S\tau_a + \kappa_V\,\delta_{3a}), \tag{6.4}$$

where q is the 4-momentum of the incoming photon and a is the isospin index of the pion.

Using the vertex in (6.4) one may quickly derive the expression for $\rho_1(mm)$ from the seagull diagram in which the photon is absorbed on nucleon 1. It is

$$\begin{aligned}\rho_1(mm;\vec{P}'',\vec{P}';\vec{p}'',\vec{p}') \approx{}& \left(\frac{-ief}{2mm_\pi}\right)\left(\frac{f}{m_\pi}\right)\frac{1}{m_\pi^2+(\vec{p}''-\vec{p}'-\frac{1}{2}\vec{q})^2}(\kappa_S\tau_{1a}+\kappa_V\,\delta_{3a})\tau_{2a}\\ &\times\bar{u}_1(\tfrac{1}{2}\vec{P}''+\vec{p}'')\gamma_5(i\vec{\gamma}\cdot\vec{q})\gamma_0 u_1(\tfrac{1}{2}\vec{P}'+\vec{p}')\\ &\times\bar{u}_2(\tfrac{1}{2}\vec{P}''-\vec{p}'')\vec{\gamma}\cdot(\vec{p}''-\vec{p}'-\tfrac{1}{2}\vec{q})\gamma_5 u_2(\tfrac{1}{2}\vec{P}'-\vec{p}').\end{aligned}$$

Working to first order in $\vec{P}'$ and $\vec{P}''$,

$$\rho_1(mm;\vec{P}'',\vec{P}';\vec{p}'',\vec{p}') = -\left(\frac{ef^2}{2mm_\pi^2}\right)(\kappa_S\vec{\tau}_1\cdot\vec{\tau}_2+\kappa_V\tau_{2z})\frac{\vec{\sigma}_1\cdot\vec{q}\vec{\sigma}_2\cdot\vec{p}}{\omega(\vec{p})^2}, \tag{6.5}$$

which is independent of $\vec{P}$. One obtains $\rho_2(mm)$ from $\rho_1(mm)$ in the usual way, giving

$$\begin{aligned}&\rho(mm;\vec{q};\vec{p}'',\vec{p}')\\ &= \frac{f^2}{4mm_\pi^2}\left(\frac{1+c}{2}\right)\frac{1}{\omega(\vec{p})^2}[\{2\kappa_S\vec{\tau}_1\cdot\vec{\tau}_2+\kappa_V(\tau_{1z}+\tau_{2z})\}(\vec{\sigma}_1\cdot\vec{p}\vec{\sigma}_2\cdot\vec{q}-\vec{\sigma}_1\cdot\vec{q}\vec{\sigma}_2\cdot\vec{p})\\ &\qquad+\kappa_V(\tau_{1z}-\tau_{2z})(\vec{\sigma}_1\cdot\vec{p}\vec{\sigma}_2\cdot\vec{q}+\vec{\sigma}_1\cdot\vec{q}\vec{\sigma}_2\cdot\vec{p})].\end{aligned} \tag{6.6}$$

The factor $(1+c)/2$ has been included in (6.6) because the content of the equivalence theorem is that, if the contact interaction $H(\gamma\pi NN;\, mm)$ is added to

$$H(\mathrm{PV}) = H(\gamma NN;\,\mathrm{charge}) + H(\gamma NN;\, mm) + H(\pi NN;\,\mathrm{PV}) + H(\gamma\pi NN;\,\mathrm{SG}),$$

then, provided that the $1\pi E$ processes have been correctly considered, the final result will be independent of whether PS or PV coupling is used for the calculation. On adding the charge densities in (6.1) and (6.6), one sees that this is indeed the case since the sum is independent of c. Thus the difference between PS and PV coupling for the calculations leading to (6.1) is correctly accounted for by the equivalence theorem. HG [11] have also found that PS and PV coupling are equivalent except for the Pauli term, which is the same result. Incidentally, this discussion shows that the splitting of the c-dependent part of $\tilde{V}_\pi$ into a $1\pi E$ part and a $2\pi E$ part, which leads from (4.11) to (4.12), is a very natural step when looked at from the point of view of the equivalence theorem. The effect of the $2\pi E$ part $(c/4m)[v, \tilde{V}]_-$ of $\Delta\tilde{V}$, which is fortunately very small, must be discussed together with all other $2\pi E$ processes.

What then does one do when calculating the charge density for $1\pi E$ processes? It is often argued that the correct result is that obtained using PS coupling, on the grounds that this, being renormalizable, is 'more fundamental'. The result for ρ would then be the c-independent expression obtained by adding (6.1) and (6.6). From this point of view, it would be claimed that PV coupling is less fundamental, though it may be useful for low energy phenomenology.

However, we wish to point out that there is a weakness in the formalism, namely the Pauli term $H(\gamma NN; mm)$ of (6.3), which is conventionally introduced to simulate the anomalous magnetic properties of the nucleons. It is this term which generates the equivalence breaking term $H(\gamma\pi NN; mm)$. It is not clear, for example, how this Pauli term should be used in higher order perturbation theory. Moreover, as we remarked in JW, Olsson and Osypowski [18] have shown that it is possible to obtain a consistent phenomenology of low energy processes involving γ, π and N, using only the gauge invariant Hamiltonian H(PV) and omitting the contact interaction $H(\gamma\pi NN; mm)$. In addition, our preliminary study of the photodisintegration of the deuteron in the forward direction [1] and also the work of Cambi et al. [3, 19] seem to indicate that the experimental information favours a PV coupling theory.

We now wish to interpret these results in a slightly different way, namely, that there is no fundamental difference between PS and PV coupling theories and that these results rather indicate that the Pauli term cannot be used in connection with virtual negative energy states. The PV coupling theory naturally suppresses negative energy states, and that is why it permits a more reliable description of $\gamma\pi N$ dynamics. To summarize then, we take the view that a reasonable description of physical effects is possible only if (a) for PV coupling, the contact interaction $H(\gamma\pi NN; mm)$ is not used, which means just the usual PV coupling theory, and (b) for PS coupling, the Pauli term is permitted to connect only virtual positive energy states. Under these conditions, PS and PV coupling theories produce the same results and ρ is given by putting $c = +1$ in (6.1), which makes κ_S and κ_V disappear from the final expression. It is this charge density which we shall use to calculate the $1\pi E$ corrections to the electric matrix elements for low energy deuteron photodisintegration.

VII. Energy dependence of the two-pion exchange potential and its consequences

The quasipotential method described in Section II leads to a potential which is energy dependent in the sense that it depends explicitly on the time-component P_0 of the total 4-momentum P. From (2.14) or (2.19), we see that this leads to a vertex function $\hat{\Gamma}$ or a wave-function ϕ for a bound state which is no longer normalized to unity. This normalization correction was included in JW as an additional correction to the cross section for deuteron photodisintegration. In this section we obtain first the dependence on P_0 of the $2\pi E$ potential for both uncrossed and crossed processes. We then discuss how one can remove this energy dependence of the potential and arrive at wavefunctions derived from an energy independent potential. It turns out that such a bound state wavefunction has to be normalized to unity, and that the two-nucleon current $\tilde{\Lambda}^\mu$ has to be modified when its matrix element is taken between such wavefunctions.

For the uncrossed $2\pi E$ quasipotential one has to proceed with care. If exchange of pions only is considered, the iteration procedure applied to (2.3) gives

$$V^{(1)} = U^{(1)}, \qquad V^{(2)} = U^{(2)} + U^{(1)}(G - g)U^{(1)},$$

where $U^{(1)}$ is given by the $1\pi E$ diagram and $U^{(2)}$ by the crossed $2\pi E$ diagram.

The quantity $U^{(1)}(G-g)U^{(1)}$ is the uncrossed $2\pi E$ quasipotential. As we discussed between (2.40) and (2.41), the Green function G can be split into four parts. When we compute the quasipotential with nucleon spinors attached (that is, for $NN \to NN$), the leading part for PV coupling is given by taking G_{++}. In writing $\hat{U}^{(1)}G_{++}\hat{U}^{(1)}$ we do not need to include the two factors m/E in (2.42) at the level of approximation in which we are interested. We therefore write

$$(\hat{U}^{(1)}G_{++}\hat{U}^{(1)})(P;p'',p') = \left(\frac{f^2}{m_\pi^2}\right)^2 (\vec{\tau}_1\cdot\vec{\tau}_2)^2(2\pi)^{-3}$$
$$\times \int d^3k\vec{\sigma}_1\cdot\vec{k}''\vec{\sigma}_2\cdot\vec{k}''\vec{\sigma}_1\cdot\vec{k}'\vec{\sigma}_2\cdot\vec{k}' I(P;p'',p';\vec{k}), \quad (7.1)$$

where

$$I(P;p'',p';\vec{k}) = (2\pi i)^{-1}\int dk_0[(k_0-x_1+i\varepsilon)(k_0+x_2-i\varepsilon)(k_0'^2-\omega'^2+i\varepsilon)$$
$$\times(k_0''^2-\omega''^2+i\varepsilon)]^{-1}, \quad (7.2)$$

$$k'=p'-k, \qquad k''=p''-k,$$
$$\omega'=\omega(\vec{k}'), \qquad \omega''=\omega(\vec{k}''),$$
$$x_1=E(\tfrac{1}{2}\vec{P}+\vec{k})-\tfrac{1}{2}P_0, \qquad x_2=E(\tfrac{1}{2}\vec{P}-\vec{k})-\tfrac{1}{2}P_0.$$

We also introduce the total internal energy $W(\vec{P};\vec{k})$, which is

$$W(\vec{P};\vec{k}) = E(\tfrac{1}{2}\vec{P}+\vec{k})+E(\tfrac{1}{2}\vec{P}-\vec{k}) = 2\bar{E}(\vec{P};\vec{k}).$$

To find the leading approximation to the integral I in (7.2), it is best to evaluate it by taking the mean of the expressions obtained by closing the contour in the upper and in the lower half-plane. One then comes to the result

$$I(P;p'',p';\vec{k}) \approx \frac{1}{P_0-W(\vec{P};\vec{k})}\frac{1}{\omega'^2\omega''^2}$$
$$\times\left[1+\frac{(p_0'-x_1)^2+(p_0'+x_2)^2}{2\omega'^2}+\frac{(p_0''-x_1)^2+(p_0''+x_2)^2}{2\omega''^2}\right]$$
$$+\frac{\omega'^2+\omega'\omega''+\omega''^2}{2\omega'^3\omega''^3(\omega'+\omega'')}. \quad (7.3)$$

We use the leading approximation to the BBS prescription (2.5) for $\hat{k}_0$, namely

$$\hat{k}_0 \approx \tfrac{1}{2}[E(\tfrac{1}{2}\vec{P}+\vec{k})-E(\tfrac{1}{2}\vec{P}-\vec{k})] = \tfrac{1}{2}(x_1-x_2).$$

It follows that

$$(p_0'-x_1)^2+(p_0'+x_2)^2 = 2(p_0'-\hat{k}_0)^2+\tfrac{1}{2}(x_1+x_2)^2$$
$$= 2(p_0'-\hat{k}_0)^2+\tfrac{1}{2}[P_0-W(\vec{P};\vec{k})]^2. \quad (7.4)$$

The uncrossed quasipotential is then

$$\hat{V}(2\pi E, u; P; p'', p') = (\hat{U}^{(1)} G_{++} \hat{U}^{(1)})(P; p'', p') - (2\pi)^{-3}$$
$$\times \int d^3k \hat{U}^{(1)}(p'', \hat{k}) \hat{g}(P; \hat{k}) \hat{U}^{(1)}(\hat{k}, p')$$
$$= \left(\frac{f^2}{m_\pi^2}\right)^2 (\vec{\tau}_1 \cdot \vec{\tau}_2)^2 (2\pi)^{-3} \int d^3k \vec{\sigma}_1 \cdot \vec{k}'' \vec{\sigma}_2 \cdot \vec{k}'' \vec{\sigma}_1 \cdot \vec{k}' \vec{\sigma}_2 \cdot \vec{k}'$$
$$\times \left[\frac{\omega'^2 + \omega'\omega'' + \omega''^2}{2\omega'^3 \omega''^3 (\omega' + \omega'')} + \{P_0 - W(\vec{P}; \vec{k})\} \frac{\omega'^2 + \omega''^2}{4\omega'^4 \omega''^4}\right]. \quad (7.5)$$

Equation (7.5) follows from (7.1), (7.3) and (7.4), if we approximate $\hat{g}(P; \hat{k})$ by $[P_0 - W(\vec{P}; \vec{k})]^{-1}$ and use also (4.1) and (4.5) to approximate $\hat{U}^{(1)}(\hat{k}, p')$ by

$$\hat{U}^{(1)}(\hat{k}, p') \approx \frac{f^2}{m_\pi^2} \vec{\tau}_1 \cdot \vec{\tau}_2 \frac{\vec{\sigma}_1 \cdot (\vec{k} - \vec{p}') \vec{\sigma}_2 \cdot (\vec{k} - \vec{p}')}{\omega(\vec{k} - \vec{p}')^2} \left[1 + \frac{(\hat{k}_0 - p_0')^2}{\omega(\vec{k} - \vec{p}')^2}\right],$$

and similarly for $\hat{U}^{(1)}(p'', \hat{k})$. The leading approximation to $\hat{V}(2\pi E, u)$ given in (7.5) does not depend on p_0' and p_0'' and so we may take the right side of (7.5) also as the BBS uncrossed $2\pi E$ potential $\tilde{V}(2\pi E, u; P; \vec{p}'', \vec{p}')$. Note that we have not used m^{-2} terms in spinor reductions, so we have not distinguished between standard and BBS spinors.

The crossed $2\pi E$ potential comes from direct evaluation of the crossed $2\pi E$ diagram. A fairly long but straightforward calculation shows that the leading approximation is

$$\hat{V}(2\pi E, c; P; p'', p') = \left(\frac{f^2}{m_\pi^2}\right)^2 \sum_{\alpha,\beta} \tau_{1\alpha}\tau_{1\beta}\tau_{2\beta}\tau_{2\alpha} (2\pi)^{-3}$$
$$\times \int d^3k \vec{\sigma}_1 \cdot \vec{k}'' \vec{\sigma}_1 \cdot \vec{k}' \vec{\sigma}_2 \cdot \vec{k}' \vec{\sigma}_2 \cdot \vec{k}''$$
$$\times \left[-\frac{\omega'^2 + \omega'\omega'' + \omega''^2}{2\omega'^3 \omega''^3 (\omega' + \omega'')} - \{P_0 - W_c(\vec{P}; \vec{p}'', \vec{p}'; \vec{k})\} \frac{\omega'^2 + \omega''^2}{2\omega'^4 \omega''^4}\right.$$
$$\left. + \frac{(p_0'' - p_0')(\omega''^2 - \omega'^2)}{2\omega'^4 \omega''^4}\right], \quad (7.6)$$

where $\vec{k}', \vec{k}'', \omega', \omega''$ have the same meaning as before and

$$W_c(\vec{P}; \vec{p}'', \vec{p}'; \vec{k}) = E(\tfrac{1}{2}\vec{P} + \vec{k}) + E(\tfrac{1}{2}\vec{P} + \vec{k} - \vec{p}'' - \vec{p}).$$

We do not need to consider the final term on the right side of (7.6) because its matrix element between arbitrary initial and final wavefunctions always vanishes. To show this, we note that when the matrix element is taken between coordinate space wavefunctions, it is of the form

$$(\sigma_{1i}\sigma_{1j}\sigma_{2k}\sigma_{2l} - \sigma_{1j}\sigma_{1i}\sigma_{2l}\sigma_{2k}) \int d^3x\, \overline{\psi_f(\vec{x})} \frac{\partial^2 f(x)}{\partial x_i\, \partial x_l} \frac{\partial^2 g(x)}{\partial x_j\, \partial x_k} \psi_i(\vec{x}).$$

The precise forms of f, g can readily be given, but the only important point is that

they depend only on $x = |\vec{x}|$. Now

$$(\sigma_{1i}\sigma_{1j}\sigma_{2k}\sigma_{2l} - \sigma_{1j}\sigma_{1i}\sigma_{2l}\sigma_{2k}) = 2i(\varepsilon_{ijm}\,\delta_{kl}\sigma_{1m} + \varepsilon_{klm}\,\delta_{ij}\sigma_{2m}),$$

$$\frac{\partial^2 f(x)}{\partial x_i\,\partial x_l} = \delta_{il}x^{-1}f'(x) + \hat{x}_i\hat{x}_l(f''(x) - x^{-1}f'(x)),$$

and similarly for g. The kernel of the matrix element is then quickly shown to be zero. Thus $\hat{V}(2\pi E, c)$ is obtained by taking the first two terms in square brackets in (7.6) and is independent of p_0' and p_0''. Thus we may take the right side of (7.6), without the third term in square brackets, as the BBS crossed $2\pi E$ potential $\tilde{V}(2\pi E, c; P; \vec{p}'', \vec{p}')$.

We have now derived the P_0 dependence of both the uncrossed and crossed $2\pi E$ potentials in the BBS formalism. This means that the wavefunction ϕ obtained from an NN potential which incorporates the $2\pi E$ potentials correctly will not be normalized to unity. Instead, using (2.19), putting $\vec{P} = \vec{0}$ and neglecting the very small difference between $2m$ and M, we have

$$1 = (2\pi)^{-3}\int d^3p\,|\phi(\vec{p})|^2 - (2\pi)^{-6}\int d^3(p'', p')\,\overline{\phi(\vec{p}'')}\,\frac{\partial\tilde{V}(P_0; \vec{p}'', \vec{p}')}{\partial P_0}\,\phi(\vec{p}'). \tag{7.7}$$

For reasons which will appear shortly, we write $\tilde{V}(P_0)$ in the form

$$\tilde{V}(P_0) = \tilde{V} + \tfrac{1}{2}\{P_0 - H_0, \tilde{V}'\}_+, \tag{7.8}$$

where $\tilde{V}$ and $\tilde{V}'$ are independent of P_0. In (7.8), H_0 is just the nonrelativistic free two-nucleon Hamiltonian $2m + \vec{p}^2/m$. From (7.5) and (7.6) we see that

$$\begin{aligned}
&\tilde{V}'(2\pi E, u + c; \vec{p}'', \vec{p}') \\
&= \left(\frac{f^2}{m_\pi^2}\right)^2 (\vec{\tau}_1\cdot\vec{\tau}_2)^2(2\pi)^{-3}\int d^3k\,\vec{\sigma}_1\cdot\vec{k}''\vec{\sigma}_1\cdot\vec{k}'\vec{\sigma}_2\cdot\vec{k}''\vec{\sigma}_2\cdot\vec{k}'\,\frac{(\omega'^2+\omega''^2)}{4\omega'^4\omega''^4} \\
&\quad + \left(\frac{f^2}{m_\pi^2}\right)^2 \sum_{\alpha,\beta}\tau_{1\alpha}\tau_{1\beta}\tau_{2\beta}\tau_{2\alpha}(2\pi)^{-3} \\
&\quad \times \int d^3k\,\vec{\sigma}_1\cdot\vec{k}''\vec{\sigma}_1\cdot\vec{k}'\vec{\sigma}_2\cdot\vec{k}'\vec{\sigma}_2\cdot\vec{k}''\,\frac{-(\omega'^2+\omega''^2)}{2\omega'^4\omega''^4}.
\end{aligned} \tag{7.9}$$

On comparing (7.9) with (5.40)–(5.42) and (5.22), we have

$$\tilde{V}'(2\pi E, u; \vec{p}'', \vec{p}') = -e^{-1}\rho_S(2\pi E, u; \vec{q} = \vec{0}; \vec{p}'', \vec{p}'), \tag{7.10}$$

$$\tilde{V}'(2\pi E, c; \vec{p}'', \vec{p}') = -e^{-1}\rho_S(2\pi E, c; \vec{q} = \vec{0}; \vec{p}'', \vec{p}'), \tag{7.11}$$

where the subscript S on the right sides of (7.10) and (7.11) denotes the isoscalar charge density (that is, the part which does not depend on the nucleon isospin operators τ_{1z}, τ_{2z}). The results in (7.10) and (7.11) are special cases of the result that the normalization expression Z on the right side of (7.7) may be written as

$$Z = e^{-1}(2\pi)^{-6}\int d^3(p'', p')\,\overline{\phi(\vec{p}'')}\rho_S\,(\vec{q} = \vec{0}; \vec{p}'', \vec{p}')\phi(\vec{p}'). \tag{7.12}$$

The first term on the right side of (7.7) comes from the impulse approximation isoscalar charge density

$$\rho_S(\mathrm{IA}; \vec{q} = \vec{0}; \vec{p}'', \vec{p}') = e(2\pi)^3\,\delta^{(3)}(\vec{p}'' - \vec{p}').$$

This method of calculating Z was used in our former papers [14, 20]. We also see that, because the uncrossed and crossed $2\pi E$ potentials are energy dependent, the isoscalar parts of the charge densities for the uncrossed and crossed $2\pi E$ processes do *not* have the property that the total charge vanishes. The criticism of SKO, that the charge densities given in JW and rederived here cannot be correct because they fail to have this property, arises from a misunderstanding of the quasipotential formalism.

Now all semiphenomenological potentials which are used in practical calculations are energy independent. Thus, in order to present a consistent calculation of meson exchange effects using wavefunctions derived from a potential such as the Paris potential, one must remove the energy dependence of the $2\pi E$ potential. This can be done by a method described by Friar [6], starting from the form of the potential given in (7.8). We denote by $\phi(M)$ the bound state wavefunction obtained from $\tilde{V}(P_0=M)$, M being the mass of the bound state. Then

$$M\phi(M)=[H_0+\tilde{V}(P_0=M)]\phi(M).$$

Using (7.8) for $\tilde{V}(P_0=M)$, we have

$$M(\mathbb{1}-\tilde{V}')\phi(M)=[H_0+\tilde{V}-\tfrac{1}{2}\{H_0,\tilde{V}'\}_+]\phi(M).$$

Now define a wavefunction ϕ by

$$\phi=(\mathbb{1}-\tilde{V}')^{1/2}\phi(M). \tag{7.13}$$

Then

$$\begin{aligned} M\phi &= (\mathbb{1}-\tilde{V}')^{-1/2}[H_0+\tilde{V}-\tfrac{1}{2}\{H_0,\tilde{V}'\}_+](\mathbb{1}-\tilde{V}')^{-1/2}\phi \\ &\approx (H_0+\tilde{V})\phi+\tfrac{1}{2}\{\tilde{V},\tilde{V}'\}_+\phi, \end{aligned} \tag{7.14}$$

working to first order in $\tilde{V}'$. If we now assume that the energy dependence is sufficiently weak for the second term on the right side of (7.14) to be able to be neglected compared with the first, we see that ϕ is the bound state wavefunction obtained from the energy *independent* potential $\tilde{V}$, with the *same* bound state mass M. When we use an energy independent potential, therefore, the wavefunction ϕ which we calculate differs from $\phi(M)$ calculated with the energy dependent potential, the relation between ϕ and $\phi(M)$ being given by (7.13).

Now the normalization condition in (7.7) applies to $\phi(M)$, and may be written symbolically as

$$1=\bar{\phi}(M)\phi(M)-\bar{\phi}(M)\tilde{V}'\phi(M).$$

It follows immediately from (7.13) that

$$\bar{\phi}\phi=1,$$

so that the wavefunction ϕ derived from the energy independent potential is to be normalized to 1. The normalization correction which we included in JW should therefore be omitted in a consistent treatment of meson exchange processes using an energy independent potential.

At the same time the two-nucleon current $\tilde{\Lambda}^\mu$, which in (2.33) was taken between states $\phi(M)$ calculated with an energy dependent potential, has to be modified. Using (7.13) to write, to first order in $\tilde{V}'$,

$$\phi(M)\approx(\mathbb{1}+\tfrac{1}{2}\tilde{V}')\phi,$$

we have

$$\tilde{\Lambda}^{\mu} \to \tilde{\Lambda}^{\mu} + \tfrac{1}{2}\tilde{V}'\tilde{\Lambda}^{\mu} + \tfrac{1}{2}\tilde{\Lambda}^{\mu}\tilde{V}'.$$

In calculating the extra pieces which need to be added to $\tilde{\Lambda}^{\mu}$ it is clearly sufficient to take for $\tilde{\Lambda}^{\mu}$ the nonrelativistic impulse approximation. Corresponding to $\tilde{V}'$ given in (7.9) for the uncrossed and crossed $2\pi E$ potentials we then have extra pieces to be added to the uncrossed and crossed $2\pi E$ charge densities in (5.40) and (5.41). The expression in (5.40) is to be changed by replacing $\vec{\tau}_1 \cdot \vec{\tau}_2 \hat{e}_1 \vec{\tau}_1 \cdot \vec{\tau}_2$ by

$$\vec{\tau}_1 \cdot \vec{\tau}_2 \hat{e}_1 \vec{\tau}_1 \cdot \vec{\tau}_2 - \tfrac{1}{2}\hat{e}_1(\vec{\tau}_1 \cdot \vec{\tau}_2)^2 - \tfrac{1}{2}(\vec{\tau}_1 \cdot \vec{\tau}_2)^2 \hat{e}_1 = -2(\tau_{1z} - \tau_{2z})$$

and $\vec{\tau}_1 \cdot \vec{\tau}_2 \hat{e}_2 \vec{\tau}_1 \cdot \vec{\tau}_2$ by $+2(\tau_{1z} - \tau_{2z})$. In the same way, $\sum_{\alpha\beta} \tau_{1\alpha}\hat{e}_1\tau_{1\beta}\tau_{2\beta}\tau_{2\alpha}$ in (5.41) is to be replaced by

$$\sum_{\alpha\beta} (\tau_{1\alpha}\hat{e}_1\tau_{1\beta} - \tfrac{1}{2}\hat{e}_1\tau_{1\alpha}\tau_{1\beta} - \tfrac{1}{2}\tau_{1\alpha}\tau_{1\beta}\hat{e}_1)\tau_{2\beta}\tau_{2\alpha} = -2(\tau_{1z} + \tau_{2z})$$

and $\sum_{\alpha\beta} \tau_{1\alpha}\tau_{1\beta}\tau_{2\beta}\hat{e}_2\tau_{2\alpha}$ by the same quantity. Thus the uncrossed and crossed $2\pi E$ charge operators which are to be used when matrix elements are taken between wavefunctions obtained from an energy *independent* potential are

$$\begin{aligned}
&\rho(2\pi E, u; \vec{q}; \vec{p}'', \vec{p}') \\
&= -\frac{2ef^4}{m_\pi^4}(\tau_{1z} - \tau_{2z})(2\pi)^{-3} \int d^3(k'', k')[\delta^{(3)}(\vec{k}'' - \vec{k}' - \tfrac{1}{2}\vec{q}) - \delta^{(3)}(\vec{k}'' - \vec{k}' + \tfrac{1}{2}\vec{q})] \\
&\quad \times J_u(\vec{k}'', \vec{k}'; \vec{p}'', \vec{p}')\vec{\sigma}_1 \cdot (\vec{p}'' - \vec{k}'')\vec{\sigma}_2 \cdot (\vec{p}'' - \vec{k}'')\vec{\sigma}_1 \cdot (\vec{k}' - \vec{p}')\vec{\sigma}_2 \cdot (\vec{k}' - \vec{p}'), \qquad (7.15)
\end{aligned}$$

$$\begin{aligned}
&\rho(2\pi E, c; \vec{q}; \vec{p}'', \vec{p}') \\
&= -\frac{2ef^4}{m_\pi^4}(\tau_{1z} + \tau_{2z})(2\pi)^{-3} \int d^3(k'', k')[\delta^{(3)}(\vec{k}'' - \vec{k}' - \tfrac{1}{2}\vec{q}) + \delta^{(3)}(\vec{k}'' - \vec{k}' + \tfrac{1}{2}\vec{q})] \\
&\quad \times J_c(\vec{k}'', \vec{k}'; \vec{p}'', \vec{p}')\vec{\sigma}_1 \cdot (\vec{p}'' - \vec{k}'')\vec{\sigma}_1 \cdot (\vec{k}' - \vec{p}')\vec{\sigma}_2 \cdot (\vec{k}' - \vec{p}')\vec{\sigma}_2 \cdot (\vec{p}'' - \vec{k}''), \qquad (7.16)
\end{aligned}$$

where J_u and J_c are given by (5.22) and (5.42).

From (7.15) one sees directly that

$$\rho(2\pi E, u; \vec{q} = \vec{0}; \vec{p}'', \vec{p}') = 0, \qquad (7.17)$$

while from (7.16) and (5.28) it follows that

$$\begin{aligned}
&\rho(2\pi E, c + \pi C; \vec{q} = \vec{0}; \vec{p}'', \vec{p}') \\
&= \frac{ef^4}{m_\pi^4} i(\vec{\tau}_1 \times \vec{\tau}_2)_z (2\pi)^{-3} \int d^3k \vec{\sigma}_1 \cdot (\vec{p}'' - \vec{k})\vec{\sigma}_1 \cdot (\vec{k} - \vec{p}')\vec{\sigma}_2 \cdot (\vec{k} - \vec{p}')\vec{\sigma}_2 \cdot (\vec{p}'' - \vec{k}) \\
&\quad \times \frac{\omega(\vec{p}'' - \vec{k})^2 - \omega(\vec{k} - \vec{p}')^2}{\omega(\vec{p}'' - \vec{k})^4 \omega(\vec{k} - \vec{p}')^4}.
\end{aligned}$$

But we have already seen after (7.6) that the right side of this equation has zero matrix elements between arbitrary initial and final states. Thus

$$\rho(2\pi E; c + \pi C; \vec{q} = \vec{0}; \vec{p}'', \vec{p}') \doteq 0. \qquad (7.18)$$

We therefore see from (7.17) and (7.18) that the modified total $2\pi E$ charge operator vanishes when taken between wavefunctions derived from an energy *independent* potential. This is the so-called 'charge conservation law' of SKO, but we emphasize again that it holds only in a formalism which has an energy independent potential. There is still one remaining difference between us and SKO. The uncrossed $2\pi E$ charge operator given in (7.15) is exactly $\frac{1}{2}$ of that given by SKO. We shall explain the reason for this difference in the final section.

VIII. Comparison with other work

Meson exchange effects are usually discussed using the unitary transformation method, which is well known to be equivalent to an S-matrix treatment. In this section we shall show that the essential difference between the transformation method and the quasipotential formalism which we have developed in this paper lies in the treatment of the retardation (or nucleon recoil) effect in the $1\pi E$ potential $\hat{V}_\pi$. When this is understood correctly, the reason for the difference between our result and that of SKO for the leading approximation to the uncrossed $2\pi E$ charge density becomes clear. Moreover, at the same time it turns out that the correction which in Section V we labelled $1\pi E$ is different in the two methods and this difference is the cause of the difference between our result for the total $1\pi E$ charge density and that of HG [11].

To see the difference between the two methods, we go back to the integral I_1 in (5.18) and write it as

$$I_1 = \frac{1}{2\omega(\vec{k}'-\vec{p}')^2} + \frac{-\frac{1}{2}(x_1'+x_2')+2(\hat{k}_0'-\hat{p}_0')}{2(x_1''+x_2'')\omega(\vec{k}'-\vec{p}')^2}. \tag{8.1}$$

In (8.1) we have explicitly separated from I_1 the piece which gives the uncrossed $2\pi E$ charge density of SKO. In the quasipotential method we wrote I_1 in the form given in (5.18), with the first term leading to the uncrossed $2\pi E$ charge density of (5.20) and the second term leading to the expression for $\rho_1'(1\pi E)$ in (5.25). However, in the transformation method the decomposition of I_1 in (8.1) is the natural one because in that method the quantity $(x_1'+x_2')$ in the numerator of the second term in (8.1) vanishes. This comes about because the $1\pi E$ potential is characterized by the condition that both nucleons are on their mass shells. In the transformation method, therefore, the first term on the right side of (8.1) leads to an expression for $\rho_1'(2\pi E, u)$ which is *twice* that given by (5.20). Since the method leads also to an energy independent uncrossed $2\pi E$ potential, it follows that the final result for $\rho(2\pi E, u)$ is also twice that given by (7.15) and (5.22), and satisfies the condition that the total uncrossed $2\pi E$ charge vanishes.

In addition, the second term in (8.1) leads to a different $1\pi E$ charge density in the transformation method. By (5.24),

$$2(\hat{k}_0'-\hat{p}_0') \approx \vec{P}' \cdot (\vec{k}'-\vec{p}')/m,$$

and this result holds also in the transformation method. Thus (5.25) is to be replaced in that method by a similar equation in which the factor in brackets on the right side is replaced by $[-\vec{P}\cdot\vec{p}+\vec{q}\cdot\vec{p}+\frac{1}{2}\vec{q}\cdot(\vec{P}-\vec{q})]$. When one makes a

similar modification to $\rho_1''(1\pi E)$ and then obtains $\rho_2(1\pi E)$ and finally $\rho(1\pi E)$, one finds to first order in $\vec{q}$ and $\vec{P}$ exactly the expression given by the sum of the first term in equation (33) and the second part of the third term in equation (34) of HG. The equation which replaces (5.34) is

$$\rho(1\pi E(\mathrm{HG}); \vec{P}'', \vec{P}'; \vec{p}'', \vec{p}') = \frac{ef^2}{4mm_\pi^2} \frac{\vec{\sigma}_1 \cdot \vec{p}\vec{\sigma}_2 \cdot \vec{p}}{\omega(\vec{p})^4} \times [-(\tau_{1z} - \tau_{2z})\vec{q} \cdot \vec{p} + 2i(\vec{\tau}_1 \times \vec{\tau}_2)_z \vec{P} \cdot \vec{p}]. \quad (8.2)$$

Unfortunately HG have an incorrect sign in writing part of the πC charge density; the second term in their equation (34) should have the opposite sign. When this correction is made, their πC charge density becomes the one we have given in (5.32). The result which follows from (5.32), (5.34) and (8.2) is that, in the usual linear approximation,

$$\rho(\pi C + 1\pi E(\mathrm{HG})) = 2\rho(\pi C + 1\pi E(\mathrm{JW})).$$

However, the $1\pi E$ potential used in HG contains a retardation term which is proportional to $(\vec{p}''^2 - \vec{p}'^2)/m$. The method developed in Section IV can therefore be used to remove this retardation term from the potential and to interpret it as an effective charge density. It can be shown that the sum of this charge density and $\rho(\pi C + 1\pi E(\mathrm{HG}))$ is equal to $\rho(\pi C + 1\pi E(\mathrm{JW}))$. We note too that one obtains also the $1\pi E$ charge density proportional to q_0, which appears also in our formalism and was discussed briefly after equation (5.24). The first part of the second term in equation (33) of HG is just $\rho(\mathrm{PC})$. The second part of this term is part of what we called $\rho(\mathrm{SG}+\mathrm{PV})$. When it is added to the first term in equation (34), the expression which we give in (5.37) is reproduced exactly. The last two terms in equation (34) of HG are frame dependent terms quadratic in $\vec{P}$ and $\vec{q}$ which we have not considered.

The HG $1\pi E$ potential contains also two c-dependent terms. We shall not consider their term proportional to $T_1 - T_2$ (see Friar's criticism in footnote 53 of Ref. 9), but shall discuss the remaining term. In the c.m. frame it equals the c-dependent part of our potential (4.8), which generates the effective charge density $\Delta\rho$ given in (4.16). In the same way the $\vec{P}$-dependent part of this term can be interpreted as another effective charge density which can be shown to be equal to $\rho(\mathrm{PS})$ as given in (5.14), except that the factor $(1-c)$ is replaced by $-c$. However, the $\vec{P}$-dependent part of the potential must be treated in a covariant framework and this is not done in HG. For this reason $\rho(\mathrm{PS})$ is not obtained correctly in the HG formalism. To summarize, apart from this discrepancy, the various contributions to the $1\pi E$ charge density which we derived in Sections IV and V agree with the results of HG if the c-dependent and retardation terms in their potential are intepreted as effective charge densities. A method for doing this, which is similar to ours, has been sketched also by HG for the isoscalar charge density.

To conclude, then, the detailed expressions for the meson exchange charge and current densities depend upon the potential used and are different for the BBS and SKO or HG potentials. SKO claim that we are incorrect because they miss this crucial fact. The SKO potential is characterized by putting both nucleons on their mass shells and is naturally generated in the Fukuda–Sawada–Taketani formalism for the construction of an energy independent potential. In principle,

any potential can be used and no one knows which is the 'best' prescription. However, the $1\pi E$ potential in the transformation method is very complicated, as can be seen in equation (23) of HG, and has never been used as part of a semiphenomenological potential for practical calculations. While we have indicated how the retardation and c-dependent terms can be removed from the HG $1\pi E$ potential, the situation is certainly much more complicated for the $2\pi E$ potential. On the other hand, the BBS potential is distinguished by the property that there is no retardation in the c.m. frame (see also Friar [9], equation (16)). This is an obvious advantage, since the wavefunctions used in calculations are derived from a potential which has this property. We see the BBS formalism as providing a very natural framework for applications to the two-nucleon system, and the Paris potential as the most satisfactory one available, for reasons which were discussed in Section IV. Within the BBS formalism, we have derived in this paper the expressions for the $1\pi E$ and $2\pi E$ charge densities which it is correct to use in conjunction with wavefunctions calculated in the c.m. frame from an energy independent potential.

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REFERENCES

[1] W. Jaus and W. S. Woolcock, Nucl. Phys. *A365*, 477 (1981).
[2] R. Blankenbecler and R. Sugar, Phys. Rev. *142*, 1051 (1966).
[3] A. Cambi, B. Mosconi and P. Ricci, Phys. Rev. Lett. *48*, 462 (1982).
[4] T. De Forest Jr. and J. D. Walecka, Adv. in Phys. *15*, 1 (1966).
[5] E. Truhlík, private communication.
[6] J. L. Friar, Ann. of Phys. *104*, 380 (1977).
[7] M. M. Nagels, T. A. Rijken and J. J. De Swart, Phys. Rev. *D17*, 768 (1978) and Lecture Notes in Physics *82*, 17 (1978).
[8] M. Lacombe, B. Loiseau, J. M. Richard, R. Vinh Mau, J. Côté, P. Pires and R. De Tourreil, Phys. Rev. *C21*, 861 (1980).
[9] J. L. Friar, Phys. Rev. *C22*, 796 (1980).
[10] T. Sato, K. Kobayashi and H. Ohtsubo, Prog. Theor. Phys. *68*, 840 (1982).
[11] H. Hyuga and M. Gari, Nucl. Phys. *A274*, 333 (1976).
[12] M. J. Zuilhof and J. A. Tjon, Phys. Rev. *C26*, 1277 (1982).
[13] R. M. Woloshyn and A. D. Jackson, Nucl. Phys. *B64*, 269 (1973).
[14] W. Jaus, Helv. Phys. Acta *49*, 475 (1976).
[15] F. Gross, Phys. Rev. *186*, 1448 (1969); *C26*, 2203 (1982), where further references to the extensive work of Gross can be found.
[16] F. Coester and A. Ostebee, Phys. Rev. *C11*, 1836 (1975).
[17] F. Coester and P. Havas, Phys. Rev. *D14*, 2556 (1976).
[18] M. G. Olsson and E. T. Osypowski, Nucl. Phys. *B87*, 399 (1975).
[19] A. Cambi, B. Mosconi and P. Ricci, Invited paper at the Workshop on *Perspectives in Nuclear Physics at Intermediate Energies*, Trieste, 1983.
[20] W. Jaus, Nucl. Phys. *A314*, 287 (1979).