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SEARCH FOR TWO-DIMENSIONAL MELTING IN A LATTICE OF SUPERCONDUCTING VORTICES

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ABSTRACT : The dynamic shear response Z of the vortex lattice in superconducting granular Al films shows interesting features near T_M, the Kosterlitz-Thouless temperature for dislocation-mediated melting. The T-dependence of Z is interpreted in terms of the coupled motion of displacement field and dislocations in an elastic continuum. Pinning of the vortices by inhomogeneities seems to play an important role.

I. INTRODUCTION

It has been proposed [1,2] that a lattice of quantized vortices in thin superconducting films can be considered as a two-dimensional (2D) crystal undergoing a transition from a solid-like to a fluid-like phase at a melting temperature T_M determined by the Kosterlitz-Thouless [3] criterion for the existence of topological order in two dimensions. According to this theory, the mechanism driving the melting transition is believed to be the unbinding of bound pairs of dislocations, which, together with phonons, represent the thermal excitations of a 2D crystal. For an incompressible 2D crystal, as it is the case for a lattice of superconducting vortices, the melting temperature T_M is given by the following implicit relation [3] :

$$4\pi K_B T_M = \mu_R(T_M^-) a^2 \quad , \tag{1}$$

where a is the lattice parameter and $\mu_R(T_M^-)$ the effective shear modulus of the crystal at the phase transition as T_M is approached from the solid phase. In the static case ($\omega = 0$) μ_R jumps discontinuously to zero at T_M and vanishes in the liquid phase. Expressing μ_R in terms of superconducting parameters, Fisher [2] has shown that for a lattice of vortices Eq. (1) can be written in the form :

$$\frac{T_M}{T_c} = 1 - \frac{3.8}{A_1} \frac{R_{n\square}}{R_u} \quad , \tag{2}$$

an expression showing that the melting transition should occur always below T_c, the BCS superconducting transition temperature. In Eq. (2) $R_{n\square}$ is the normal-state sheet resistance of the superconducting film, R_u the universal resistance $\hbar/e^2$ and A_1 is a constant, which lies between 0.4 and 0.75, accounting for the renormalization of the shear modulus.

Recently, Fiory and Hebard [4] provided clear evidence for melting phenomena occurring in a 2D lattice of superconducting vortices. However, since vortex pinning was not explicitly included in their analysis, they were unable to ascertain whether the observed transition was driven by the unbinding of dislocation dipoles as predicted by detailed theories [5,6] of 2D melting.

In this paper, we report a study of the ac complex impedance of superconducting Al-films mounted in the so-called Corbino-disk geometry [7]. In this particular configuration the oscillating driving Lorentz force acting on the vortices, which results from an ac current flowing radially in the superconducting disk, couples only to shear deformations of the vortex medium. As a consequence, it was originally thought that this experiment was ideally suited to provide important insight into the unique dynamical aspects of dislocation-mediated melting.

The experimental results described in Section II are indeed consistent with the hypothesis of vortex-lattice melting. A detailed analysis of the data, however, shows that vortex pinning, as in Fiory-Hebard's experiments, plays an essential role in determining the dynamic response of the vortex medium in both the solid and fluid phase. In Section III, therefore, we develop a model which explicitly incorporates pinning phenomena in the dynamics of an elastic vortex continuum with dislocations. The model qualitatively explains the essential features of our data and brings new insight into the role of pinning in melting phenomena of a lattice of superconducting vortices.

II. EXPERIMENTAL RESULTS

To realize the desired radial current-density distribution characterizing the Corbino-disk geometry, granular Al-films of circular shape were mounted in a coaxial current-feeding configuration. Electrical contacts were obtained by pressing against the film surface the indium tip of the central electrode and an indium O-ring which acts as outer circular electrode. In order to allow free access of magnetic flux to the film region, the external superconducting In-electrode was interrupted over a very short portion of its circular path. The diameter, $2R_i$, of the central contact is of the order of 1 mm, whereas the corresponding dimension, $2R_0$, of the external electrode is 18 mm. The ac complex impedance Z of the superconducting film was inferred from $V = ZI$, where I is the constant rms value of the driving ac current and V the rms value of the ac potential difference between the central and the outer electrode measured with a conventional phase-sensitive detector. I never exceeded ~ 1 μA, a value resulting in a maximum current density of the order of ~ 1 A/cm^2 in the immediate vicinity of the central electrode. Typically, at these current levels the sensitivity of the detector allowed to measure impedances of the order of a few mΩ. The frequency range covered by the experiments reported in this paper extends from 100 Hz up to 100 kHz. At low frequencies (less than ~ 5 kHz) the correct phase setting was obtained by adjusting the phase shift of the detector to null the signal from the film well above its transition temperature (at T = 4.2 K). This corresponds to the 90^0 phase setting used to measure the quadrature or imaginary part of Z (Im[Z]). At high frequencies, where spurious inductive pick-up from the measuring circuit was not negligible, a more elaborated procedure was used.

According to Hebard and Fiory [8], the complex impedance Z of a 2D superconductor can be written in the form :

$$Z = i\omega L_K + Z_V \ , \tag{3}$$

an expression stating that Z is the series connection of the inductive contribution, ωL_K, due to the superfluid background, and of the impedance Z_V arising from superconducting vortices. Estimates of the kinetic inductance $L_K = (1/2)\mu_o \Lambda \ell n(R_o/R_i)$, where Λ is the effective penetration depth in thin superconducting films, using typical parameters for our Al-films show that ωL_K, in the temperature region of interest, is always well below the sensitivity of our detector. Thus, except very near T_c, where ωL_K, being inversely proportional to the superfluid density which diverges as $[1-(T/T_c)]^{-1}$, makes the dominant contribution to Z, what we actuelly measure in our experiments is the complex vortex impedance Z_V .

In Figs. 1 and 2 we show experimental results, at B = 5 Gauss, for an Al-film (Al*1*) having $R_{n\square} = 70\ \Omega$. Other parameters of Al*1* are $T_c \approx 2.04$ K and d ≈ 100 Å. We observe a rapid increase in dissipation (Fig. 1) which sets in at a temperature lying within the "melting range" predicted by Eq. (2) ($1.71 \lesssim T_M \lesssim 1.86$ K). Since our method is not sufficiently sensitive to detect the presumably very small dissipative component Re[Z] in what is believed to be a pinned solid vortex

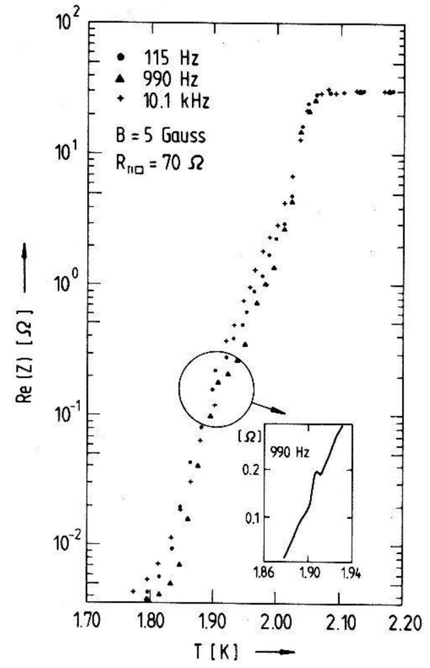

Fig. 1 : Temperature dependence of the real part of the ac complex impedance of Al1.

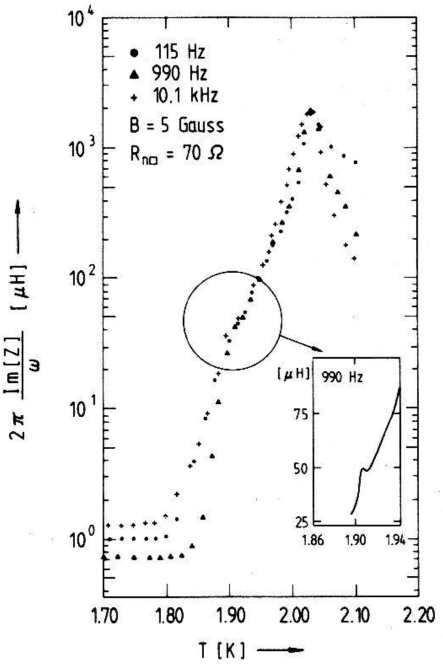

Fig. 2 : Temperature dependence of the imaginary part of the ac complex impedance of Al1.

phase, we are not able to ascertain whether Re[Z] exhibits, near T_M, the break in slope observed by Fiory and Hebard [4] using a different technique. Closer inspection of Fig. 1 shows an additional feature of the data, namely a slight change in slope around 1.9 K which for the results at 990 Hz is associated with the appearance of the small peak shown in the insert. Above 1.9 K Re[Z] is essentially independent of ω in the explored frequency range and exhibit a thermally activated behaviour. The inductive component Im[Z] of Al*1* (Fig. 2) shows a definite break in slope at about 1.8 K. Below this temperature, Im[Z] is certainly larger than Re[Z], an indication that pinning effects play a major role in determining the dynamic response of what is presumably the solid-like vortex phase. Above 1.8 K, however, there is a crossover to a régime where Im[Z] is less than Re[Z], a feature consistent with the motion of uncorrelated vortices interacting as individual "particles" with the structural inhomogeneities responsible for vortex pinning (see Section III c). Im[Z] also shows a little peak structure at about 1.9 K in the data taken at 990 Hz and a weak change in slope at the other frequencies. Above 1.9 K there is again evidence for thermally activated vortex motion and Im[Z] is proportional to ω at low frequencies. On the high temperature side the data of Fig. 2 culminate in a well defined peak at 2.03 K which is related to the smoothed divergence of L_K at the superconducting - normal state transition.

Peak structures and changes in slope above T_M are very pronounced in the data for Al*2* (Fig. 3, 4 and 5), a granular Al-film having the same $R_{n\square}$ as Al*1*, $T_c \approx 2.27$ K, d $\approx$ 200 Å and for which T_M lies in the 1.90 - 2.07 K range. As shown by Fig. 4 and 5, the position and strength of the peaks are practically independent of frequency. Their intensity, however, depends, even at the lowest excitation levels used in our experiments, on the amplitude of the driving ac current. Moreover, the dissipation associated with the peak in Re[Z] is much larger than what one calculates for free vortex motion from the Bardeen-Stephen theory [9].

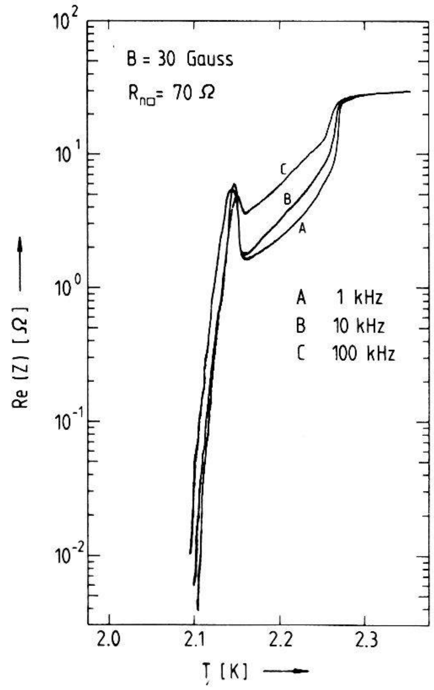

Fig. 3 : Temperature dependence of the real part of the ac complex impedance of Al2. The amplitude of the driving ac current is 1 μA .

Both features provide a clear indication for a non-linear dynamic response of the vortex medium, presumably due to the presence of strong vortex pinning in Al*2*. This conjecture is consistent with the observation of a remarkable difference in the shape of the current-voltage characteristics of Al*1* and Al*2*.

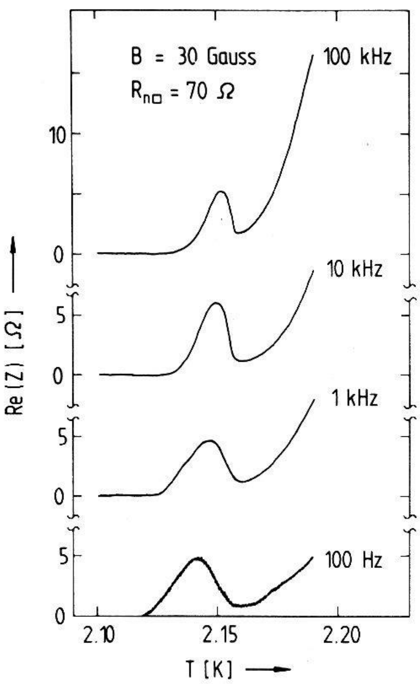

Fig. 4 : *Detail of the peak structures of Fig. 3 shown on a linear plot.*

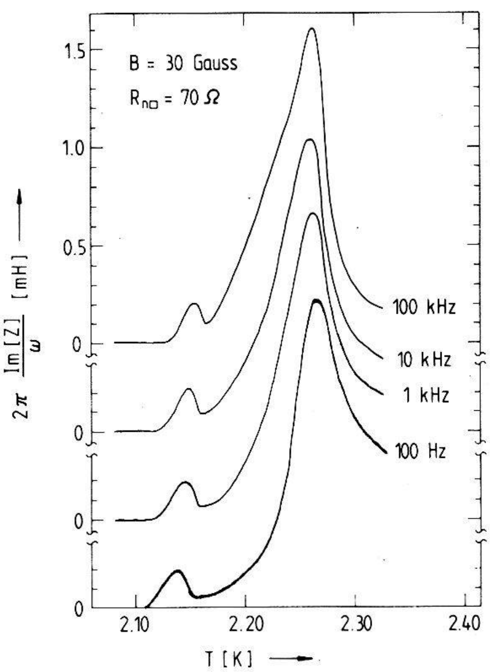

Fig. 5 : *Temperature dependance of the imaginary part of the ac complex impedance of Al2 shown on a linear plot. The driving current is the same as in Fig. 3.*

Well above T_M, the I-V-curves of both films show a thermally activated behaviour with vanishing critical current at low currents and a linear flux-flow régime at high currents. Below T_M both films have a finite critical current but, while Al*1* enters the flux-flow régime with a gradual transition, Al*2* shows, at $I = I_c$, a sudden voltage jump after which the film is in the flux-flow state. This feature is interpreted as a manifestation of the existence of very strong pinning centers whose effect is superposed to the usual weak vortex pinning due to the granular nature of the Al-films. Inspection of the structure of our films has, in fact, revealed the presence of holes, presumably resulting from imperfect nucleation, and whose concentration is particularly high for Al*2*. Holes are known to act as very efficient pinning centers [10]. It is clear, therefore, that a careful analysis of the experimental results reported in this

Section must also take into account phenomena associated with strong vortex pinning.

III. THEORETICAL CONSIDERATIONS

A. 2D Vortex Lattice Without Pinning

In the following discussion the lattice of quantized superconducting vortices is considered as a 2D elastic continuum where thermally excited dislocations form a system of "vector charges" with properties similar to those of a 2D Coulomb gas [3,11]. Since it is energetically more favourable to create pairs of dislocations ($\vec{b}$, $-\vec{b}$) with opposite Burgers vectors, the total Burgers vector charge of the dislocations vanishes. As a consequence, there is no macroscopic bending in the 2D crystal, a condition equivalent to that of overall electroneutrality in the 2D Coulomb gas analogue.

The equation of motion for the total displacement field $\vec{u}$ of the vortex medium can be written in the form [12] :

$$\eta \frac{\partial u_i}{\partial t} = \frac{\partial \sigma_{ik}}{\partial x_k} + F_i \ , \tag{4}$$

where the three terms represent, successively, the viscous damping force, the elastic restoring force in presence of dislocations and the external driving force acting (per unit surface) on the 2D vortex medium. The viscosity coefficient η is given by $\eta = B^2/R_{f\square}$, where $R_{f\square}$ is the flux-flow sheet resistance. Since there is no macroscopic bending in the 2D vortex crystal, the stress tensor σ_{ik}, which is related by Hooke's law to the elastic part of the total strain tensor, can be expressed as :

$$\sigma_{ik} = C_{ik\ell m}\left(\frac{\partial u_m}{\partial x_\ell} - P_{\ell m}\right) \ , \tag{5}$$

where $P_{\ell m}$ is the plastic part of the total strain tensor, the so-called dislocation polarization tensor, describing the strain field due to the dislocations. For an isotropic continuum the tensor of the elastic moduli $C_{ik\ell m}$ is given by :

$$C_{ik\ell m} = \lambda \delta_{ik}\delta_{\ell m} + \mu(\delta_{i\ell}\delta_{km} + \delta_{im}\delta_{k\ell}) , \tag{6}$$

where λ and μ are Lamé's coefficients. In particular, μ is the shear modulus of the vortex continuum without dislocations.

In order to discuss effects arising from the presence of thermally excited dislocations, it is convenient to rely again on the 2D Coulomb gas analogue. With this in mind it is quite natural to describe the response of the dislocations to the stress σ_{ik} by means of a susceptibility $\chi_{ik\ell m}(\omega)$ defined by the relation :

$$\tilde{P}_{ik}(\omega) = \chi_{ik\ell m}(\omega)\tilde{\sigma}_{\ell m}(\omega) , \tag{7}$$

where the sign "~" denotes the Fourier transform of the corresponding physical quantity. In writing Eq. (7) we have implicitly assumed that the stress field varies slowly over distances of the order of the diffusion length traveled by a dislocation during one cycle [6]. Accordingly, the stress field generated in the vortex medium in response to the external driving force is effectively perceived as a uniform stress by the dislocations. Thus, one can approximate $\chi_{ik\ell m}(\vec{q};\omega)$ by its value, $\chi_{ik\ell m}(\omega)$, at $\vec{q} = 0$.

Considering the vortex lattice as an incompressible continuum, from Eqs. (4), (5), (6) and (7), we obtain :

$$- i\eta\omega\vec{\underset{\sim}{u}} = \mu_R(\omega)\Delta\vec{\underset{\sim}{u}} + \vec{\underset{\sim}{F}} . \tag{8}$$

This is the conventional equation of motion for a dissipative elastic medium driven by an oscillating external force. In our approach effects arising from the thermally generated dislocations are incorporated in an effective (or renormalized) shear modulus

$$\mu_R(\omega) = \frac{\mu}{\varepsilon(\omega)} , \tag{9}$$

where $\varepsilon(\omega) = 1 + 2\mu\chi(\omega)$ is a dielectric constant accounting for screening of the stress field by the dislocations. $\varepsilon(\omega)$ has been calculated by Ambegoakar et al. [11] in connection with the dynamical response of the 2D Coulomb gas and more recently by Zippelius et al. [6] in a detailed dynamical theory of dislocation-mediated 2D melting. It can be written in the form :

$$\varepsilon(\omega) = \varepsilon_b(\omega) + i2\mu\frac{\sigma}{\omega}, \tag{10}$$

showing that $\varepsilon(\omega)$ is the sum of two contributions. The first one, $\varepsilon_b(\omega)$, which is complex, is due to the motion of bound pairs of dislocations (dislocation dipoles) in response to the oscillating stress field. The second one is associated with free dislocation charges. It vanishes below T_M but makes the dominant contribution to $\varepsilon(\omega)$ above T_M. The analogy with the 2D Coulomb gas allows us to write down immediately the expression for the conductivity σ entering this second term :

$$\sigma = b^2\frac{D}{k_BT}n_f, \tag{11}$$

where D/k_BT and n_f are, respectively, the mobility and the density of the free dislocations. n_f, in turn, is approximately given by $n_f \approx \xi_+^{-2}(T)$, where $\xi_+(T)$ is the correlation length in the fluid vortex phase ($T > T_M$) [5] :

$$\xi_+(T) \approx a\exp\left[\frac{2\pi}{s}\left(\frac{T}{T_M}-1\right)^{-\nu}\right], \tag{12}$$

where $\nu \approx 0,37$ for a triangular lattice and s is a non-universal constant.

From Eq. (8) we can now deduce the complex vortex impedance Z_v of the Corbino-disk geometry. In this configuration one is dealing with an azimuthal driving force of the form $\underset{\sim}{\vec{F}} = (BI/2\pi r)\hat{e}_\phi$. As a consequence, only shear deformations propagating in the radial direction are excited. Then, a simple calculation shows that :

$$Z_v = \frac{R_{f\square}}{2\pi} \int_0^\infty C(q)\left[1 + \frac{i}{\omega \tau_q}\right]^{-1} dq \tag{13}$$

where $C(q) \approx q^{-1}$ for $R_o^{-1} < q < R_i^{-1}$ and $C(q) \approx 0$ otherwise. In Eq. (13) τ^{-1} is the (complex) relaxation rate of the transverse mode $\vec{q}$ in the dissipative vortex medium :

$$\tau_q^{-1} = \frac{\mu_R}{\eta} q^2 = \frac{\mu}{\eta \varepsilon(\omega)} q^2 \; . \tag{14}$$

Since $\mathrm{Re}[\varepsilon^{-1}(\omega)]$ and $\mathrm{Im}[\varepsilon^{-1}(\omega)]$ show, respectively, a shoulder and a peak at a temperature $T(\omega) \gtrsim T_M$ determined by the condition $\xi^2[T(\omega)] \approx D/\omega$ [11,13], one would expect characteristic structures in Z_v associated with the melting transition of the vortex lattice. Closer inspection of Eq. (13) shows, however, that, at T = 0, the relaxation times τ_q of the relevant shear modes consistent with the above form of C(q) lie between $\sim 5 \times 10^{-3}$ s and ~ 1 s for a typical choice of parameters and are even much longer in the vicinity of T_M. In the temperature region of interest $\omega\tau_q$ is therefore much larger than unity at the frequencies used in our experiments. As it clearly results from Eq. (13), in this régime vortex motion in ideal, i.e. pinning-free, superconducting films is controlled by viscous forces only and consequently all information about a possible melting transition of the vortex lattice is lost in this case. As it will be shown in the subsequent discussion, vortex pinning, unavoidable in real films, provides the essential mechanism allowing the detection of melting phenomena of the vortex lattice.

B. Solid Vortex Phase With Pinning

If one assumes that vortex pinning does not seriously affect the dynamic response of the dislocations described by the dielectric constant (10), the equation of motion for the vortex continuum is simply Eq. (8) with an additional force $-\nabla U(\vec{r})$ arising from the interaction of the vortices with a random pinning potential $U(\vec{r})$. It is well-known that Al-films prepared, as in our experiment, by evaporating the metal in a controlled oxygen atmosphere consist of a random distribution of metallic grains. The resulting structural inhomogeneities act to pin vortices by interaction with their normal cores. Since the grain size is usually much less than the core diameter, which is of the order of the coherence length, the pinning potential $U(\vec{r})$ is weak and can be treated as a perturbation [14]. In the experiments described in Section II, however, we found evidence also for strong pinning effects, possibly associated with a dilute random distribution of holes, which are known to be very efficient pinning centers [10]. We think, therefore, that in our films $U(\vec{r})$ will behave in a way somewhat similar to that sketched in Fig. 6, where a dilute random distribution of deep potential wells (holes) is superposed to the weak small-scale random potential due to the grains. In order to perform an explicit calculation of Z_v, the strong pinning component of $U(\vec{r})$ is approximated by a random distribution of deep parabolic wells of equal strength and, on the average, a distance L apart. In the dilute limit considered here we assume $L \gg a$. The weak pinning part of $U(\vec{r})$ is represented by randomly distributed shallow parabolic wells. In the equilibrium configuration each well is occupied by a vortex sitting at the bottom of the well. Using this model, a calculation within the framework of the so-called Coherent Potential Approximation (CPA) shows [15] that Z_v is still given by Eq. (13) where, however, the relaxation rate τ_q^{-1} is replaced by :

$$\tau^{-1} = \tau_q^{-1} + \tau_L^{-1} + \tau_o^{-1} \tag{15}$$

In this expression τ_L^{-1} is the relaxation rate of lattice modes with a wavelength of the order of ~ L which are induced by the strong component of $U(\vec{r})$:

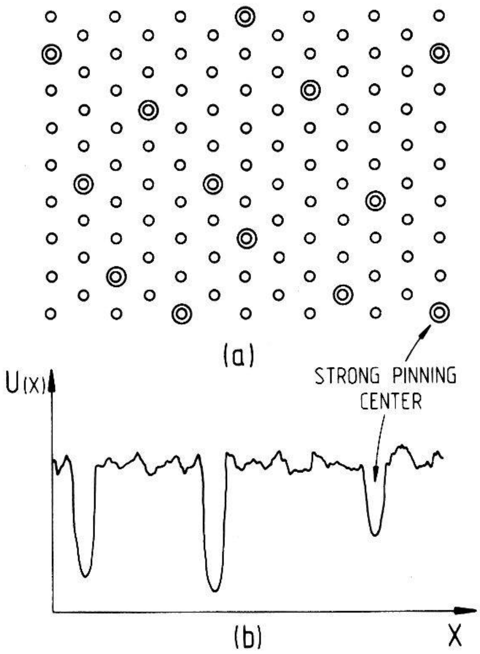

Fig. 6 : (a) *Vortex lattice with randomly distributed strong pinning centers (holes).*

(b) *Profile of the random pinning potential U(x) along a selected direction x. The weak random potential between the strong wells (holes) represents vortex pinning arising from the granular nature of the Al-films.*

$$\tau_L^{-1} \approx \frac{\mu}{\eta\varepsilon(\omega)} \frac{1}{L^2} . \qquad (16)$$

τ_0, on the other hand, is the relaxation time of a single vortex in a shallow parabolic well and, therefore, does not depend on $\mu_R = \mu/\varepsilon(\omega)$.

Eq. (15) has a simple physical interpretation. For long-wavelength Fourier components of the driving force, such that $qL << 1$, the presence of strong pinning centers at an average distance L from each other causes the excitation of lattice deformations of much shorter wavelengths, of the order of ~ L. For this case, the response of the vortex medium is strongly influenced by pinning, and the lattice relaxation rate is much faster than in a pinning-free situation ($\tau_L^{-1} >> \tau_q^{-1}$). In the opposite limit ($qL >> 1$) the strong pinning centers have little effect on the lattice response.

Typically, for our Al-films we expect $a << L << R_i$. Then, since $\tau_q^{-1} << \tau_L^{-1}$ for the relevant modes of the Corbino-disk geometry, from Eq. (13) one obtains :

$$Z_v = R_f[1 + \frac{i}{\omega\tau}]^{-1} , \tag{17}$$

where $R_f = (R_{f\square}/2\pi)\ln(R_o/R_i)$ is the flux-flow resistance of the Corbino disk and $\tau^{-1} \simeq \tau_L^{-1} + \tau_o^{-1}$ in this case. In the low-temperature solid vortex phase ($T << T_M$), where only a few bound pairs of dislocations are thermally excited ($\varepsilon \simeq 1$), estimates of τ_L using reasonable values of L ($L \simeq 10 - 50$ μm) and of τ_o [2] show that $\omega\tau << 1$ at our frequencies. Then, from Eq. (17) it follows that $\mathrm{Re}[Z_v] = R_f(\omega\tau)^2$ and $\mathrm{Im}[Z_v] = R_f(\omega\tau)$ and, consequently, $\mathrm{Re}[Z_v] << \mathrm{Im}[Z_v]$, a result which clearly shows the importance of pinning in reducing the dissipation and in enhancing the inductive response of the solid vortex phase. As discussed in Section II, this prediction of the model agrees with our low-temperature experimental data (Figs. 2 and 3). Under certain conditions, both $\mathrm{Re}[Z_v]$ and $\mathrm{Im}[Z_v]$ exhibit, for $T \gtrsim T_M$, frequency-dependent structures, which reflect the peculiar behaviour of $\varepsilon^{-1}(\omega)$ in this intermediate temperature region (see Section III A). These features will be discussed in more detail later on in this paper (Section III D). In the high temperature fluid vortex phase ($T >> T_M$) the presence of a large number of free dislocations leads to a vanishingly small $\varepsilon^{-1}(\omega)$ and thus $\tau \simeq \tau_o$. In this case, Z_v reduces to the expression for a single vortex, as one actually expects for a liquid in which particles are uncorrelated. However, our model is a typical "solid" model in which the displacement field $\vec{u}$

describes deformations with respect to a fixed equilibrium structure. Thus, Z_V, which is proportional to the vortex mobility, vanishes in the limit $\omega = 0$. Since this is clearly not the case for a liquid, where the dc mobility is finite, in the next subsection we discuss the response of the fluid vortex phase in presence of thermal fluctuations.

C. Fluid Vortex Phase With Pinning

Since the potential wells associated with the strong pinning centers are assumed to be very deep (with activation energy much larger than $k_B T$) and dilute ($L \gg a$), the contribution of vortices sitting in these wells to the overall vortex impedance of the fluid phase well above T_M is expected to be very small. For this reason we consider a model in which all vortices interact only with the weak component of the pinning potential, namely that associated with the granular structure of the Al-films. For $T \gg T_M$ vortex motion is uncorrelated, so that it is sufficient to consider the Brownian motion of an individual vortex in the random pinning force field. To our knowledge, the problem of finding the frequency-dependent mobility of such a particle has not been studied in detail. However, a solution containing all the essential physical features can be easily obtained from a 1D model in which the random potential $U(\vec{r})$ is replaced by a 1D sinusoidal field $U(x) = U \cos qx$. In this case, the Langevin equation of motion for a vortex of mass m can be written as :

$$m\ddot{x} = - \eta'\dot{x} + U_0 q \sin qx + f(t), \tag{18}$$

where $\eta' = \eta/n_\square$ ($n_\square = B/\phi_0$ is the areal vortex density) and $f(t)$ is the fluctuating Langevin force with a white-noise spectrum defined by the correlation function

$$\langle f(t)f(t')\rangle = 2\eta' k_B T \, \delta(t - t') . \tag{19}$$

The frequency-dependent vortex mobility, which is the Fourier transform of the velocity-velocity correlation function, and, consequently, Z_v can now be deduced from Eq. (18) using the continued-fraction method first applied by Fulde et al. [16] to a similar problem. In the so-called Smoluchowski limit, where the viscous force dominates over the inertial force in Eq. (18), a two-pole continued fraction expansion leads to :

$$Z_v = R_f \left[1 + \frac{\tau_D/\tau_o'}{1 - i\omega\tau_D}\right]^{-1} , \tag{20}$$

where $\tau_D^{-1} = D_v q^2$ is the vortex diffusion rate over distances of the order of the wavelength of the periodic pinning potential ($D_v = k_B T/\eta'$ is the vortex diffusion constant). The relaxation time ratio τ_D/τ_o' is given by :

$$\frac{\tau_D}{\tau_o'} = [I_o^2(y) - 1] , \tag{21}$$

where $y = U_o/k_B T$ and τ_o' is related to τ_o [Eq. (15)] by $\tau_o' = \tau_o \, I_o(y)/I_1(y)$. $I_o(y)$ and $I_1(y)$ are modified Bessel functions. In the low-frequency limit ($\omega\tau_D << 1$) from Eq. (20) one deduces :

$$\mathrm{Re}[Z_v] = \frac{R_f}{I_o^2(y)} , \quad \mathrm{Im}[Z_v] = R_f \, \omega\tau_D \, \frac{I_o^2(y) - 1}{I_o^4(y)} . \tag{22}$$

These expressions show that, at low frequencies, the dissipative component of Z_v is independent of ω, whereas the dispersive component scales linearly with ω. Moreover, as one easily deduces from the properties of $I_o(y)$, both $\mathrm{Re}[Z_v]$ and $\mathrm{Im}[Z_v]$ exhibit a thermally activated behaviour and $\mathrm{Re}[Z_v] > \mathrm{Im}[Z_v]$. These are precisely the features shown by our high-temperature experimental results (Fig. 2 and 3), which can indeed be fitted to the above theoretical expressions. Assuming that, near T_c, $R_f \approx R_n B/H_{c2}(T) \sim [1 - (T/T_c)]^{-1}$ and that U_o varies with temperature as the superfluid density, i.e. $U_o(T) = U_{oo}[1 - (T/T_c)]$ near T_c, one obtains an acceptable fit of the data for Al1 using $U_{oo} \approx 50$ K and $\tau_D \approx 2 \times 10^{-5}$ s. This value of U_{oo} is consistent with an estimate of Fisher [2] for granular films, who finds $U_o(T_M)$ of the order of $k_B T_M$ in films whose T_M is relatively close to T_c. Moreover, using the Bardeen-Stephen theory [9] to calculate η', from τ_D we infer $\lambda_p = 2\pi/q \sim 35$ μm. Because of the random nature of the pinning potential operating in our films, it is difficult to assess the significance of this figure.

D. The Transition Region

A description of the pinned vortex medium at intermediate temperatures ($T \gtrsim T_M$) is difficult and certainly requires a more detailed theoretical treatment. From a phenomenological point of view, however, it is possible to describe its dynamic response in the vicinity of the melting transition using a simple interpolation scheme. With this in mind we write for the vortex impedance Z_v :

$$Z_v = R_f \left[1 + \frac{\tau_D^*/\tau}{1 - i\omega\tau_D^*} \right]^{-1} , \tag{23}$$

where τ is still given by $\tau^{-1} \approx \tau_L^{-1} + \tau_o^{-1}$. Well above T_M ($T \gg T_M$), in the fluid vortex phase, $\varepsilon^{-1}(\omega)$ vanishes and, consequently, $\tau \approx \tau_o$. If τ_D^* is identified as the vortex diffusion time τ_D defined in III C., then Eq. (23) becomes identical to Eq. (21), the dynamic response of the vortex fluid. On the other hand, if τ_D^* becomes very large ($\tau_D^* \to \infty$), Eq. (23) transforms into the vortex impedance of the pinned solid vortex phase Eq. (17) . These considerations show that the dynamic response of the vortex medium in the transition region ($T \gtrsim T_M$) can be described in simple terms if one assumes that in Eq. (23) τ_D^* diverges as one approaches the melting temperature T_M from above, i.e. $\tau_D^* \to \infty$ as $T \approx T_M^+$. It seems therefore natural to identify τ_D^*, for $T \gtrsim T_M$, as the time free dislocations need to diffuse over distances of the order of the correlation length $\xi_+(T)$ [see

Eq. (12)] in the fluid vortex phase, i.e. $\tau_D^* \simeq \xi_+^2(T)/D$. Relying on this phenomenological approach, we have determined the vortex impedance near the solid-liquid phase transition from Eq. (23) using Eq. (10) for $\varepsilon(\omega)$. To this purpose the contribution $\varepsilon_b(\omega)$ associated with thermally excited bound pairs of dislocations has been calculated using the procedure described in Ref. [13]. Three parameters enter this calculation, namely $\ell \simeq \ell n[(D/\omega)^{1/2}/a]$, $\mu/\mu_R(T_M^-)$ and s. Since estimates of D are not available so far, we have set $D \simeq D_v$ obtaining $\ell \sim 7$ for a frequency of 1 kHz and $a \approx 1$ µm, which correspond to $B \simeq 20$ gauss. To be consistent with Fisher's estimate [2], we have chosen $\mu/\mu_R(T_M^-) = 2$. Then, setting $s = 2\pi$, $\omega\tau_o = 10^{-3}$ and $\omega\tau_L = 10^{-4}$ at low temperatures, where $\varepsilon \approx 1$, we obtain the results shown in Fig. 7. A peak in dissipation and a shoulder in the inductive component of Z_v show up approximately at the temperature T_ω defined in Section III A [$\xi_+^2(T_\omega) \simeq D/\omega$]. These

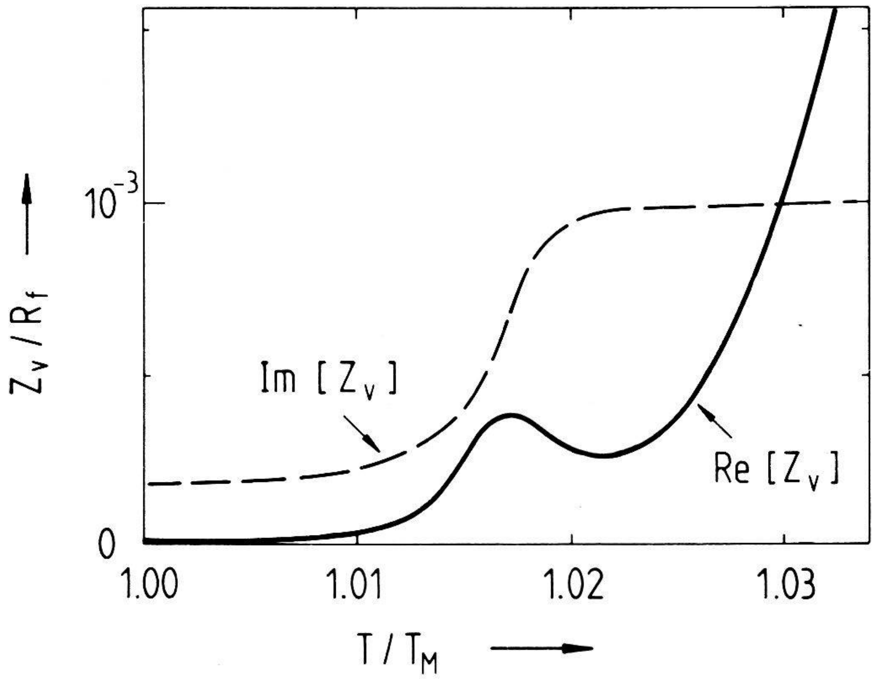

Fig. 7 : *Real and imaginary part of the complex vortex impedance as a function of temperature in the vicinity of the melting transitions as deduced from Eq. (23).*

structures are a manifestation of the unique behaviour of $\varepsilon^{-1}(\omega)$ in the vicinity of T_M. If one lowers the frequency, while keeping all other parameters fixed, the characteristic structures of Fig. 7 are still well resolved but their intensity decreases approximately linearly with ω. At higher frequencies, the structures at T_ω are washed out. Notice that the results shown in Fig. 7 confirm the already discussed and experimentally observed crossover from $Re[Z_V] < Im[Z_V]$ in the low-temperature solid vortex phase to $Re[Z_V] > Im[Z_V]$ in the high-temperature fluid vortex phase.

The model discussed above shows that vortex pinning plays an essential role in experiments probing characteristic features of dislocation-mediated melting. Whether the structures emerging in our experimental data just above the predicted melting temperature can be understood on the basis of the present model remains, however, an open question. It is possible that the small peak in the 990 Hz-data of Al*1* results from the dislocation unbinding mechanism. As predicted by the model, it disappears, in fact, at higher frequencies and is probably blurred by noise at lower frequencies. Further experimental work, however, is necessary in order to ascertain this conjecture. Unexplained by the present treatment are the well-resolved peak structures observed for Al*2*, which do not shift with ω, and whose intensity is by far too large to be accounted for by our model. As already mentioned in Section II, their explanation will probably require the development of a more elaborate non-linear theory.

As a final point, we would like to stress an intrinsic difficulty one faces in experiments probing melting phenomena of a lattice of superconducting vortices. Unlike the case of the superfluid transition in 2D superfluid He-films [13] and in 2D superconducting films [17], where it is possible to couple the driving field directly to the vortex excitations, in experiments probing the shear modulus of a lattice of vortices the external force couples primarily to the total displacement field of an elastic continuum with thermally excited dislocations and not directly to the dislocation field. As a consequence, characteristic features arising from the dislocation-unbinding transition are difficult to observe. In this paper we have shown that vortex pinning is an essential tool to overcome this difficulty. We think that experiments dealing with well-controlled pinning structures [10,18] will prove to be very useful in ascertaining the nature of the melting transition.

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