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EXPERIMENTAL TESTS OF LOCALIZATION IN SEMICONDUCTORS

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ABSTRACT

Our experimental results indicate that both randomness and electron-electron interactions are important driving mechanisms of the metal-insulator transition in noncrystalline systems. Although previous studies are consistent with Mott's minimum metallic conductivity, our measurements at much lower temperatures and higher resolution in density argue for a continuous transition. A review of these experiments compared to the latest theoretical developments indicates that our understanding is still incomplete.

Because of their device applications, doped semiconductors are the best characterized of all random systems where the Metal-Insulator (MI) transition has been studied. Very nearly pure, dislocation free, single crystals of Si or Ge provide an ideal host for impurity doping. The doping concentration can be controlled and impurities such as P are known to substitute for host atoms randomly. Not only can the density of dopant atoms be changed at will, but their electrical occupation can be altered by means of compensation; that is, by counterdoping with impurities of the opposite type. We argue here that, in uncompensated samples, both single particle randomness and many body electron correlation effects are important; other work suggests that, by adding compensation, the correlation effects can be suppressed relative to the localization effects. Finally, we take special note of the fact that the region particularly close to the critical density  $n_c$  of the MI transition in Si and Ge can be continuously investigated by varying uniaxial stress (or high magnetic field).

Shallow donors and acceptors act as hydrogen-like centers imbedded in the host material. The outer electron of a donor such as P moves with large Bohr radius  $a_B$  (16 Å in Si) over many lattice sites, making the discreteness of the lattice relatively unimportant. At low concentrations, there is negligible overlap of the wave functions of neighboring electrons and the material is an insulator at  $T = 0$  K. At higher concentrations overlap occurs and the material becomes a metal. This MI transition takes place when the Bohr radius is of the order of the average spacing between the impurities.

The Metal-Insulator transition has been observed in a variety of semiconductors with greatly different values of  $n_c$ .<sup>1,2</sup> Mott<sup>1</sup> has proposed, and the data support, the following universal scaling form

$$n_c^{1/3} a_B \simeq 1/4 \quad (1)$$

Furthermore, all of the many conductivity studies prior to 1980<sup>1,2</sup> seemed to support the 1971 prediction by Mott<sup>3</sup> that, at the transition, the  $T = 0$  K conductivity  $\sigma(0)$  jumps discontinuously from 0 to a minimum metallic value given by

$$\sigma_{\min} \simeq 0.026 e^2 / \hbar n_c^{-1/3} \quad (2)$$

Mott's calculation is based in part on the Ioffe-Regel criterion that the electron mean free path cannot be less than the inverse of the Fermi wave vector

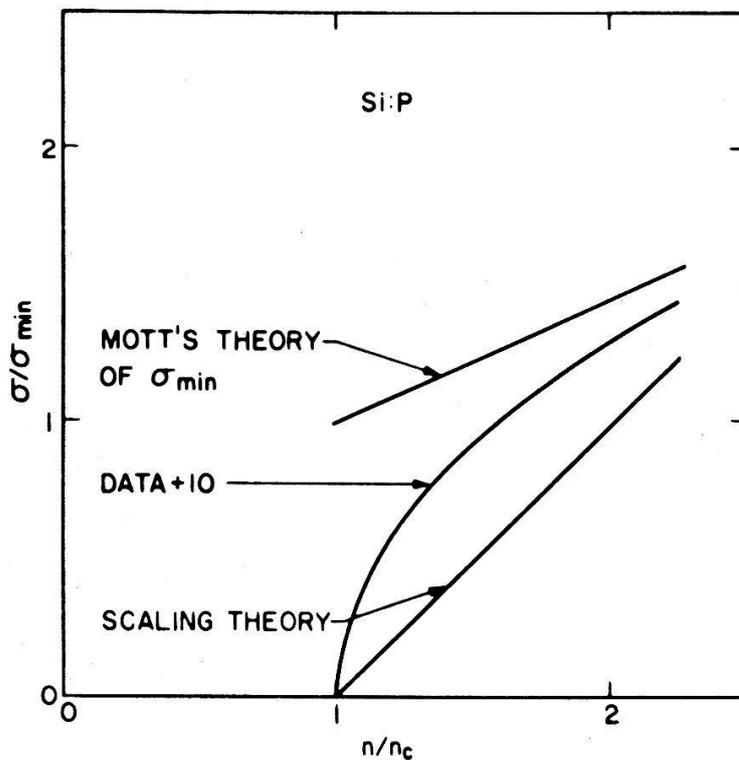
$k_F^{-1}$ , as well as on considerations of Anderson localization<sup>4</sup> of noninteracting electrons at the presence of random potential wells.

In contrast to Mott's  $\sigma_{\min}$ , the recently developed scaling theories of localization incorporate as a fundamental assumption that  $\sigma(0)$  varies smoothly at the transition:

$$\sigma(0) = \sigma_0 \times (n/n_c - 1)^\nu, \quad (3)$$

where the exponent  $\nu$  is of the order of 1 for noninteracting electrons.<sup>5</sup>

The long history of transport measurements in Si:P has been reviewed recently by Fritzsche.<sup>6</sup> In this paper we limit ourselves to our recent stress tuning experiments<sup>7,8</sup> on Si:P. To a greater extent than in earlier work, we



1. Rough comparison of zero temperature conductivity  $\sigma(0)$  with theoretical predictions close to the MI transition. In Mott's (1981) theory of  $\sigma_{\min}$  the conductivity jumps to  $\sigma_{\min}$  at  $n_c$  and is proportional to  $\gamma^2$  (from the electronic specific heat) above the transition.<sup>3</sup> Various scaling theories<sup>5</sup> which are strictly valid for  $\sigma < \sigma_{\text{scale}} \sim \sigma_{\min}$  predict a continuous conductivity behavior  $\sigma = \sigma_{\min} (n/n_c - 1)^\nu$ , where exponent  $\nu \cong 1$ . The measured conductivity<sup>7,9</sup>  $\sigma(0) = 260(n/n_c - 1)^{0.5}$  (scaled down by a factor of 10 in this picture!) differs from these theories both in the magnitude and in the exponent.

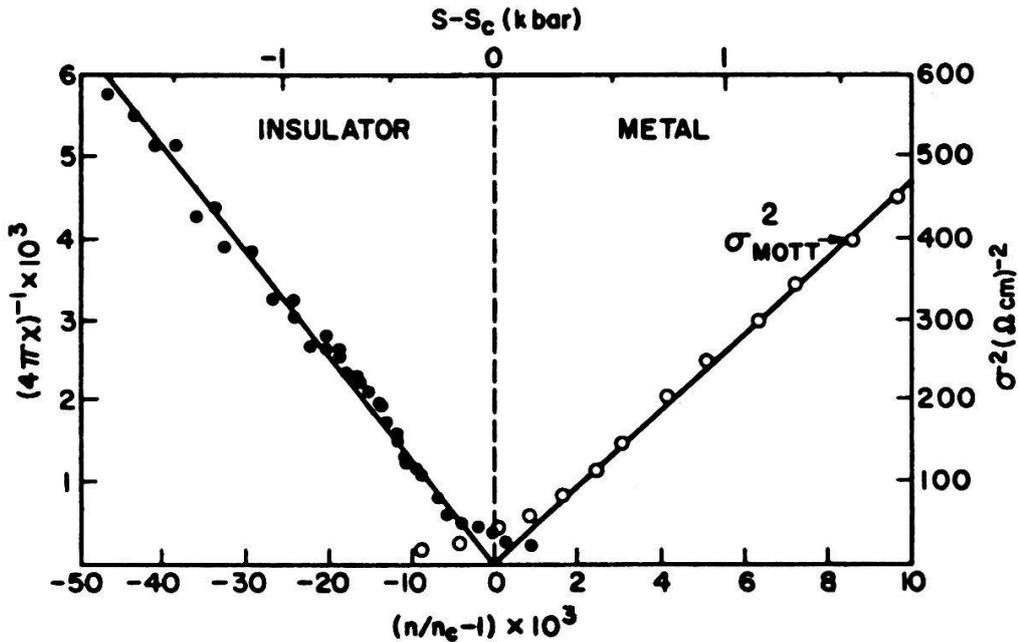
approach the zero temperature limit as we tune with  $S$  through the transition. This change in the experimental technique has produced results that require changes in our understanding of the MI transition in disordered systems.

In Fig. 1 we compare our experimental results<sup>7,9</sup> with Eqs. (2) and (3) on a coarse density scale. First, the scale of  $\sigma(0)$  exceeds by one order of magnitude, the estimate that  $\sigma_o \sim \sigma_{\min}$ . Secondly the functional form of the data

$$\sigma(0) = 260(n/n_c - 1)^{0.50 \pm 0.05} (\Omega\text{cm})^{-1} \quad (4)$$

differs from both theories. Finally, Eq. (4) describes the data when  $10^{-3} < (n/n_c - 1) < 1$  suggesting a wider critical region than expected. Neither theory takes into account the correlation effects that we believe to be important in our uncompensated samples.

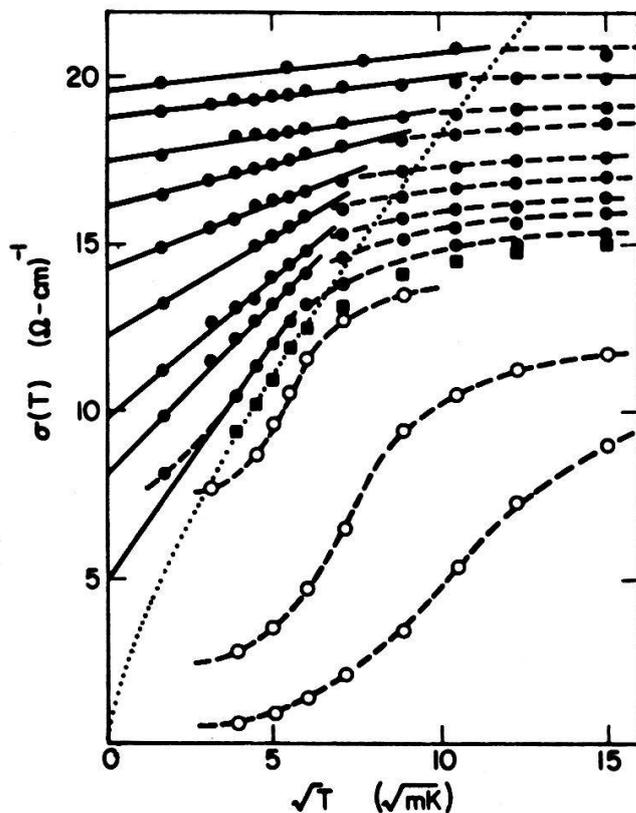
The sharp decrease in the conductivity near  $n_c$  (Fig. 1) might appear



2. Uniaxial stress  $S$  is applied to slightly insulating samples to tune them continuously through the MI transition. Above the transition,  $\sigma(0)$  decreases smoothly at least down to  $\frac{1}{4}\sigma_{\min}$ , near  $(n/n_c - 1) = 10^{-3}$ , obeying the power law  $\sigma(0) \propto (S - S_c)^{1/2} \propto (n/n_c - 1)^{1/2}$ . Similarly, on the insulating side of the transition, the dielectric susceptibility  $4\pi\chi$  diverges continuously for  $(n_c/n - 1) > 10^{-2}$ . The apparent rounding of  $4\pi\chi$  near the transition arises from Schottky barrier effects. The conductivity is measured using a 4-probe technique<sup>7</sup> and  $4\pi\chi$  is measured by a 2-probe capacitance technique on a different sample.<sup>8</sup>

to resemble the discontinuous drop suggested by Mott. However, in Fig. 2 using data under stress, we can expand the density scale near the transition and plot both  $\sigma(0)$  above and  $(4\pi\chi)^{-1} \propto [\text{Im}(\sigma)]^{-1}$  below the transition.<sup>7,8</sup> We find that Eq. (4) is at least valid down to  $(n/n_c - 1) = 10^{-3}$  and a conductivity value  $\sim \frac{1}{4} \sigma_{\text{min}}$ . The data in Fig. 1 do not require the existence of  $\sigma_{\text{min}}$ . We can understand the divergence of the dielectric susceptibility below the transition qualitatively in terms of delocalization of the electronic wave functions. We find a simple power law divergence of  $4\pi\chi$  of the form

$$(4\pi\chi)^{-1} \propto (n_c/n-1)^\xi \quad (5)$$



3. Electrical conductivity as a function of temperature  $T$  for  $3 \text{ mK} < T < 225 \text{ mK}$ , and for a series of values of uniaxial stress  $S$ . At values of  $\sigma(T)$  above the dotted line we find  $\sqrt{T}$  temperature dependence for  $S > S_c$  (solid circles). In this region the data can be readily extrapolated to  $T = 0 \text{ K}$ . The open circles ( $S < S_c$ ) are affected by rounding due to finite frequency ( $\omega/2\pi=11 \text{ Hz}$ ) (compare Figs. 2 & 4). The values of  $S$  for the circles, from the top, are approximately 8.03, 7.82, 7.56, 7.36, 7.15, 7.00, 6.83, 6.71, 6.59, 6.33, 5.73, 5.12 kbar. The squares are interpolated  $\sigma$  values at  $S = S_c = 6.40 \text{ kbar}$ .

where the exponent<sup>10</sup>  $\xi = (1.1 \pm 0.1) \simeq 2\nu$ .

In Fig. 3 we present the temperature dependence of  $\sigma$  close to the MI transition. In earlier measurements,<sup>1,2</sup> which appeared to support the existence of  $\sigma_{\min}$ , the temperature range was considerably above the range in Fig. 3, and samples with  $d\sigma/dT < 0$  were assumed to be temperature independent closer to  $T = 0$  K and those with  $d\sigma/dT > 0$  were assumed to be insulating. However, we find at our lowest temperatures, in the metallic regime, ( $S > S_c$ ) significant temperature corrections of the form<sup>7,11</sup>

$$\sigma(T) = \sigma(0) + m \sqrt{T} \quad (6)$$

We have used Eq. (6) to extrapolate  $\sigma(T)$  to  $T = 0$  K for  $\sigma(0)$  above the dotted line in Fig. 3. We also note that this temperature region (where Eq. (6) is valid) tends to vanish as  $n \rightarrow n_c$  as, simultaneously,  $m$  tends to diverge.

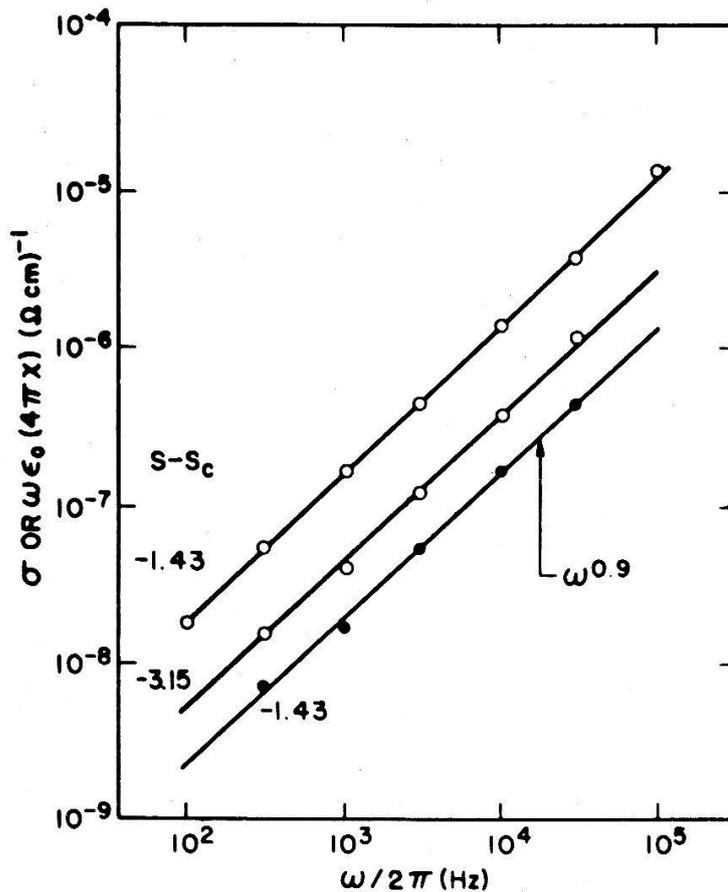
The details of the temperature dependence of  $\sigma$  (Fig. 3) and also the magnetoresistance<sup>12</sup> can be explained by including electron-electron interaction effects in the conductivity calculations.<sup>13</sup> These correlation effects have also recently been added<sup>14,15</sup> to the calculation of  $\sigma(0)$  by Grest and Lee<sup>15</sup> who find

$$\sigma(0) \propto (n/n_c - 1)^{0.6} \quad (7)$$

$$(4\pi\chi)^{-1} \propto (n_c/n - 1)^{1.4} \quad (8)$$

These exponents of  $\sigma(0)$  and  $\chi$  are closer to our experimental results [Eq. (4) & (5)], than those of the noninteracting theories. Finally, in Fig. 4 we present the frequency dependence of the conductivity in the insulating phase.<sup>8</sup> The ac conductivity has both real and imaginary parts, which tend to diverge as  $n \rightarrow n_c$ , as shown in Fig. 2. In Fig. 2, we showed a few finite values of  $\sigma(0)$  (measured at a frequency  $\omega/2\pi = 11$  Hz) for  $n$  less than the value we interpreted as  $n_c$ . We now see in Fig. 4, consistent with our interpretation of Fig. 2, that  $\sigma(0)$  does go to zero below  $n_c$  provided that the limit  $\omega \rightarrow 0$  is properly taken. The insulating phase has interesting properties similar to a glass close to the transition, which we do not understand completely at the moment.<sup>8</sup>

In conclusion, the recent studies of Si:P conductivity do not require the existence of a  $\sigma_{\min}$ .<sup>16</sup> The electron-electron correlation effects appear to be at least as important as single particle localization effects in uncompensated Si:P. Future developments in this field may come from the studies of



4. The frequency dependence of  $\text{Re}(\sigma) \equiv \sigma$  (solid circles) and  $\text{Im}(\sigma) \equiv 4\pi\chi\omega\epsilon_0$  (open circles) slightly below the MI transition and at  $T = 13$  mK. This measurement (using 2 capacitor plate contacts) yields  $\sigma(0) = \sigma(\omega \rightarrow 0) = 0$  for  $n < n_c$ , showing that the rounded conductivity results of Fig. 2 & 3 were affected by the 11 Hz frequency for  $n < n_c$ . Both parts of  $\sigma$  depend on frequency according to a simple power law  $\sigma \propto \omega^{0.9}$ , indicating a glass-type behavior of the localized electrons in the insulator.

compensated samples. Also, further theoretical and experimental efforts are needed for a better understanding of the insulating phase in the vicinity of the transition.

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