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TIME OF FLIGHT EXPERIMENTS WITH CONDUCTION ELECTRONS  
- AN APPLICATION OF WEAK LOCALIZATION -

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Magneto-resistance measurements on a thin disordered (two-dimensional) film correspond to a time of flight experiment with conduction electrons. It allows the determination of the inelastic lifetime, the spin-orbit coupling time and the magnetic scattering time of the conduction electrons.

I. The echo of the scattered conduction electron

While the resistance in three-dimensional metals is rather well described by the (classical) Boltzmann theory one finds clear deviation from this description in two dimensions. The anomalous behaviour of the resistance in two dimensions was first pointed out by Abrahams et al. /1/. Anderson et al. /2/ and Gorkov et al. /3/ showed that at low but finite temperature the conductance is not constant but has a temperature dependent correction and Altshuler et al. /4/ calculated a magneto-resistance. This effect which is generally called weak localization is more than just a new anomaly in the resistance of low dimensional systems. Weak localization corresponds to a time of flight experiment with conduction electrons. As we will see below a magnetic field allows observation of the scattering of the electrons as a function of time.

The correction to the conductance is generally calculated in the Kubo-formalism and is given by the fan diagram (fig. 1a). The corresponding scattering process is described in fig. 1b in k-space. We consider at the time  $t=0$  an electron of momentum  $k$  which has the wave function  $\exp(ikr)$ . The electron  $k$  is scattered after the time  $\tau_0$  into state  $k'_1$ , after  $2\tau_0$  into the state  $k'_2$  etc. There is a finite probability that the electron will be scattered into the state  $-k$ ; for example after  $n$  scattering events. This scattering sequence

$$k \rightarrow k'_1 \rightarrow k'_2 \rightarrow \dots \rightarrow k'_{n-1} \rightarrow k'_n = -k$$

is drawn in fig. 1b in the k-space. The momentum transfer is  $g_1, g_2 \dots g_n$ .

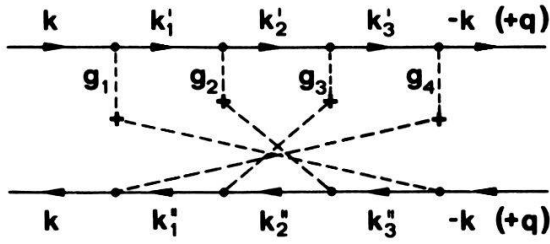
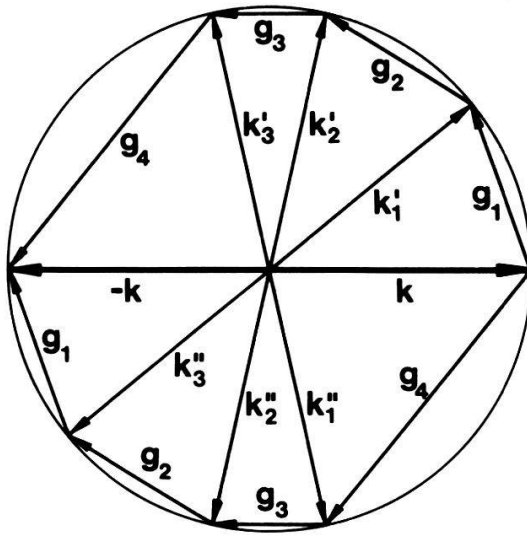


Fig. 1a: The fan-diagram, introduced by Langer and Neal, which allows calculations of quantum corrections to the resistance within the Kubo-formalism.

Fig. 1b: The physical interpretation of the fan-diagram in Fig. 2a.



There is an equal probability for the electron  $k$  to be scattered in  $n$  steps from the state  $k$  into  $-k$  via the sequence

$$k \rightarrow k''_1 \rightarrow k''_2 \rightarrow \dots \rightarrow k''_{n-1} \rightarrow k''_n = -k$$

where the momentum transfer is  $g_n, g_{n-1}, \dots, g_1$ . This complementary scattering series has the same changes of momentum in opposite sequence. If the final state is  $-k$ , then the intermediate states for both scattering processes lie symmetric to the origin. The important point is that the amplitude in the final state  $-k$  is the same for both scattering sequences.

Since the final amplitudes  $A'$  and  $A''$  are phase coherent and equal,  $A'=A''=A$ , the total intensity is  $|A'+A''|^2 = |A'|^2 + |A''|^2 + A'^* + A'A''^* = 4|A|^2$ . If the two amplitudes were not coherent then the total scattering intensity of the two complementary sequences would only be  $2|A|^2$ . This means that the scattering intensity into the state  $-k$  is by  $2|A|^2$  larger than in the case of incoherent scattering. This additional scattering intensity exists only in the back-scattering direction.

At finite temperature the scattering processes are partially inelastic. As a consequence the amplitudes  $A'$  and  $A''$  lose their phase coherence (after the time  $\tau_i$ ) and the coherent backscattering disappears after  $\tau_i$ . The integrated momentum of the electron  $k$  decreases with increasing  $\tau_i$ .

The coherent back-scattering is not restricted to the exact state  $-k$ ; one has a small spot around the state  $-k$  which contributes. Its radius is the inverse of the diffusion length in real space  $\sqrt{Dt}$  ( $D = v_F^2 \tau_0 / 2$  = diffusion constant in two dimensions). The spot of coherent final state i.e. its area shrinks with increasing time as  $\pi/(Dt)$ . Therefore the portion of coherent back-scattering is proportional to  $1/t$ . In the presence of an electrical field the coherent back-scattering reduces the contribution of the electron  $k$  to the current and the conductance is decreased by

$$\Delta L_{||} = - e^2 / (2\pi^2 \hbar) * \ln(\tau_i / \tau_0) \quad (1)$$

The important consequence of the above consideration is that the conduction electrons perform a typical interference experiment. The (incoming) wave  $k$  is split into two complementary waves  $k'_1$  and  $k''_1$ . The two waves propagate individually, experience changes in phase, spin orientation, etc. and are finally unified in the state  $-k$  where they interfere. The intensity of the interference is simply measured by the resistance. In the situation which has been discussed above the interference is constructive in the time interval from  $\tau_0$  to  $\tau_i$ .

## II. Time of flight experiment by a magnetic field

One of the interesting possibilities for an interference experiment is to shift the relative phase of the two interfering waves. For charged particles this can be easily done by an external magnetic field. In a magnetic field the phase coherence of the two partial waves is weakened or destroyed. When the two partial waves surround an area  $F$  containing the magnetic flux  $\Phi$ , then the relative change of the two phases is  $(2e/\hbar)\Phi$ . The factor 2 arises because the two partial waves surround the area twice. Altshuler et al. /5/ suggested performing such an "interference experiment" with an cylindrical film in a magnetic field parallel to the cylinder axis. Then the magnetic phase shift between the complementary waves is always a multiple of  $2e\Phi/\hbar$  ( $\Phi$  = flux in the area of the cylinder). Sharvin and Sharvin /6/ showed in a beautiful experiment that then the resistance oscillates

with a flux period of  $\phi_0 = h/(2e)$ . However, for a thin film in a perpendicular magnetic field the pairs of partial waves enclose areas between  $-2Dt$  and  $2Dt$ . When the largest phase shift exceeds 1, the interference is constructive and destructive as well and the average cancels. This happens roughly after the time  $t_H = \hbar/(4eDH)$ . This means essentially that the conductance correction in the field  $H$  i.e.  $\Delta L(H)$  yields the coherent back-scattering intensity integrated from  $\tau_0$  to  $t_H$

$$L(H) \sim \int_{\tau}^{t_H} I_{\text{coh}} dt \sim -L_{00} \log(t_H / \tau_0) \quad (2)$$

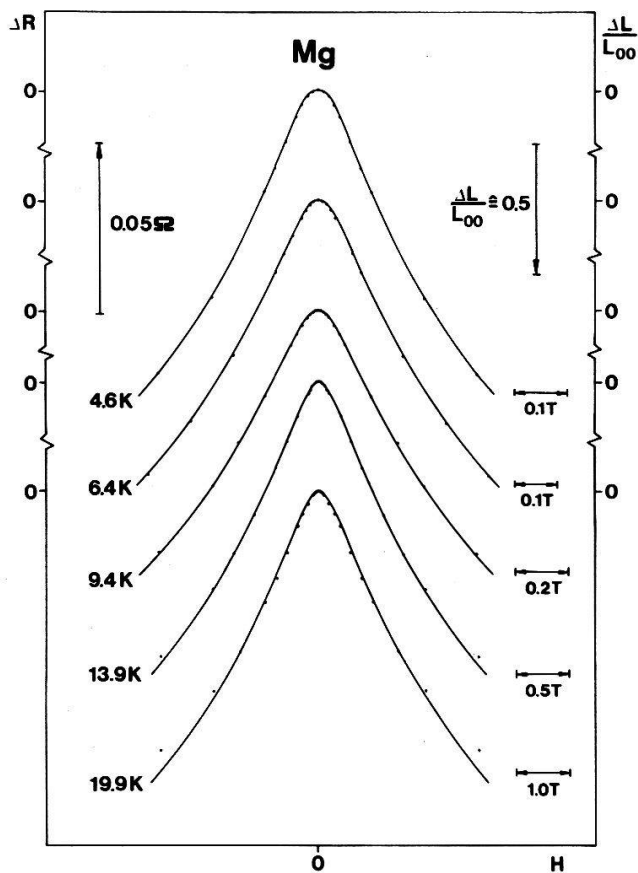
This means that the magnetic field allows a time of flight experiment. If a magnetic field  $H$  is applied the contribution of coherent back-scattering is integrated in the time interval between  $\tau_0$  and  $t_H = \hbar/(4eDH)$ . If one reduces the field from the value  $H'$  to the value  $H''$  and measures the change of resistance this yields the contribution of the coherent back-scattering in the time interval  $t_{H'}$  and  $t_{H''}$ . Since the magnetic field introduces a time  $t_H$  into the electron system all characteristic times  $\tau_n$  of the electrons can be expressed in terms of magnetic fields  $H_n$ .

$$\tau_n \Leftrightarrow H_n$$

where  $\tau_n H_n = \hbar/(4eD)$ . In a thin film this is given by  $\hbar e N/4$  which is of the order of  $10^{-12}$  to  $10^{-13}$  Ts ( $\rho$ =resistivity of the film and  $N$ -density of electron states for both spin directions).

Magneto-resistance measurements on thin films have been performed by several groups /7/, /8/, /9/, /10/, /11/, /12/, /13/, /14/, /15/, /16/, /17/. For avoiding the influence of spin-orbit coupling the magneto-resistance experiment must be performed with a very light metal because spin orbit coupling causes severe complications. In fig. 2 the magneto-resistance of a Mg film is plotted as a function of the applied magnetic field /8/. The units of the field are shown on the right side of the curves. The Mg is quench-condensed at helium temperature, because the quench condensation yields homogeneous films with high resistance. The agreement between the experimental points and the theory is very good (the spin-orbit coupling of Mg is already incorporated; see below). The experimental result proves the destructive influence of a magnetic field on weak localization. It measures the area in which the coherent electronic state exists as a function of temperature and allows the quantitative determination of the coherent scattering time  $\tau_i$ .

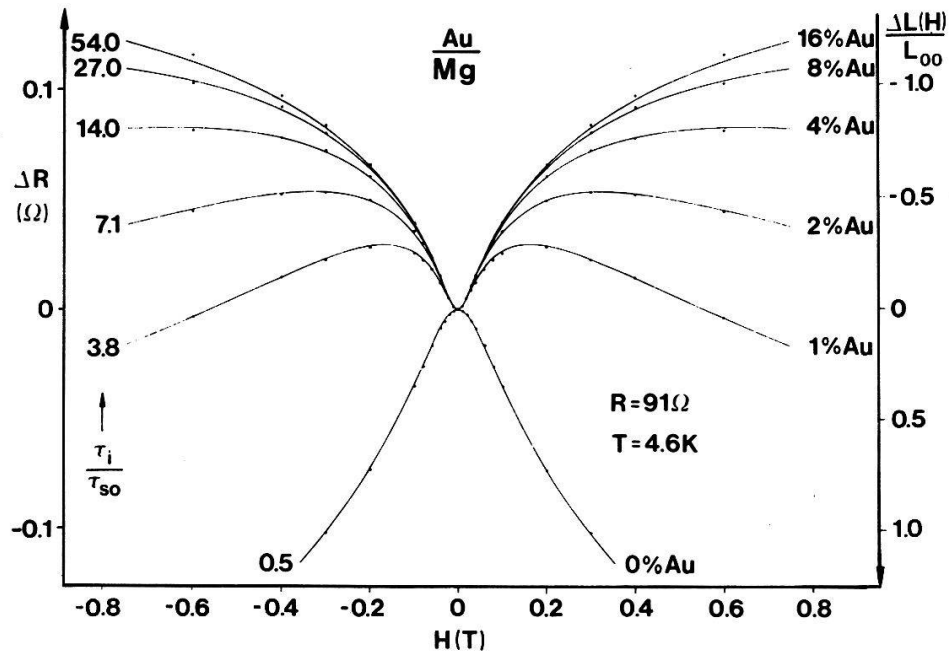
The temperature dependence follows a  $T^{-2}$  law for Mg.



**Fig. 2:** The magneto-resistance of a thin Mg-film for different temperatures as a function of the applied field. The units of the field are given on the right of the curves. The points represent the experimental results. The full curves are calculated with the theory. The small spin-orbit coupling is taken into account.

### III. Spin-orbit coupling

One of the most interesting questions in weak localization is the influence of spin-orbit coupling. Hikami et al. /18/ predicted that in the presence of strong spin-orbit coupling a logarithmic decrease of the resistance with decreasing temperature (weak antilocalization). As a consequence the magneto-resistance should change sign as well. The prediction by Hikami et al. could be experimentally confirmed by the author /19/. In fig. 3 the magneto-resistance of a Mg film at 4.5 K is plotted for increasing coverage of Au. The points represent the experimental results, whereas the full curves are calculated with the theory of Hikami et al. The adjustable parameter is the spin-orbit coupling time which decreases with increasing Au coverage (this experiment also yields the spin-orbit coupling of the pure Mg film). Obviously weak localization provides a new and very



**Fig. 3:** The magneto-resistance of a thin Mg-film at 4.5 K (lower curve). A superposition of 1/100 atomic layer of Au (statistically) changes the behaviour completely. Further superposition with Au (the thickness is given in % of an atomic layer on the right side of the curves) increases the spin-orbit coupling. The points are measured. The full curves are obtained with the theory by Hikami et al. The ratio  $\tau_i/\tau_{so}$  on the left side gives the strength of the adjusted spin-orbit coupling. It is essentially proportional to the Au-thickness.

sensitive method to measure the spin-orbit coupling directly, i.e. with a substructure and not only by a broadening of a resonance.

Thin films of Cu possess already a considerable spin-orbit coupling and show therefore even without an Au coverage a similar structure in the magneto-resistance as fig. 4 demonstrates. Pure Au films on the other hand are almost in the limit of strong spin orbit coupling and possess therefore a positive magneto-resistance [20].

It is a consequence of quantum theory and proved by a rather sophisticated neutron experiment that spin 1/2 particles have to be rotated by  $4\pi$  to transferring the spin function into itself. A rotation by  $2\pi$  reverses the sign of the spin state. Weak antilocalization gives another experimental proof of this fact. In the presence of spin-orbit coupling

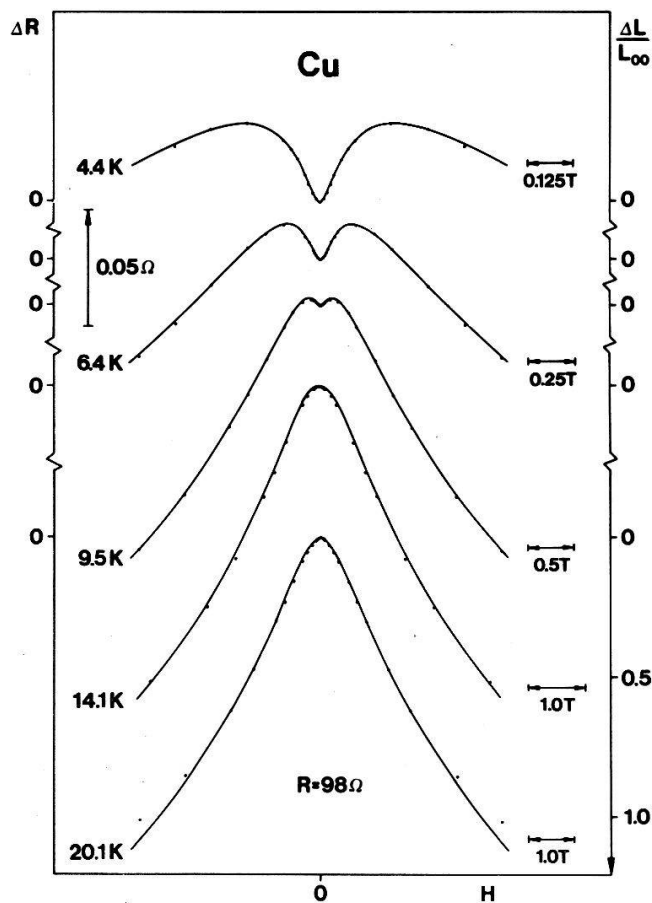


Fig. 4: The magneto-resistance of a Cu-film for different temperatures. The Cu possesses a natural spin-orbit coupling and therefore the pure metal shows the destructive interference of rotated spins. Again the points are experimentally measured whereas the full curves are calculated with the theory, adjusting the inelastic lifetime and the spin-orbit coupling time.

the conduction electrons slightly rotate their spin during each scattering process. During the whole scattering series (') the spin orientation diffuses into a final state  $s'$  which can be obtained by a rotation  $T$  of the original spin state  $s$ . ( $s' = Ts$ ). It is straight forward to show, that the final spin state of the complementary scattering series (") is  $s'' = T^{-1}s$ . Without the spin rotation the interference of the two partial waves is constructive (in the absence of an external field). In the presence of spin-orbit coupling the interference becomes destructive if the relative rotation of  $s'$  and  $s''$  is  $2\pi$ . It can be shown that for strong spin-orbit coupling the destructive part exceeds the constructive one [21]. This means that the back-scattering is reduced below the statistical one. This corresponds to an echo in the forward direction and an decrease of the resistance.

#### IV. Magnetic scattering

Another interesting application of weak localization is the determination of magnetic scattering by magnetic ions. The magnetic ion introduces

an interaction with a conduction electron  $J^*Ss$ , where  $S$  and  $s$  are the ion and electron spins. The magnetic ions scatter the two complementary waves differently and destroy their coherence after the magnetic scattering time  $\tau_s$ . Therefore weak localization is blocked at low temperature (where  $\tau_i > \tau_s$ ) and the magneto-resistance curves remain broad (because only for  $t_H < \tau_s$  or  $H > \hbar/(4eD \tau_s)$  the magnetic field can overcome the destructive influence of the magnetic scattering). Such measurements yield the temperature dependence of  $\tau_s$ . For 1/1000 atomic layer of Fe on Mg the magnetic scattering time was determined and found to be temperature independent /22/.

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