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Size effects on electron transport in multifilament superconducting wires

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Abstract. Numerical results are presented for the longitudinal and transverse resistivity of multifilamentary superconducting wires in the normal state, as a function of the bulk mean free path in the matrix material, the radius of the filaments and the distance between them. Size effects are seen to play an important rôle in the range of parameters characteristic of technical superconductors.

A typical superconducting wire consists of several tens up to a thousand of Nb_3Sn, NbTi or Nb filaments, 5 to 50 micrometers in diameter, in a Cu or Cu–CuNi matrix, with an axis to axis distance d between adjacent filaments of 2.2 to 2.6 times their radius r_0. Among the many parameters which determine the quality of the wire are ρ_L and ρ_T, the longitudinal (i.e. parallel to the filaments), and transverse resistivity of the matrix. The former should be as small as possible, so as to minimize the power dissipation and concomitant temperature rise if some region of the superconductor should suddenly go normal, in which case practically all the current will be carried by the matrix. The latter, on the other hand, should be as large as possible in order to suppress the eddy currents responsible for a sizeable fraction of the AC losses. Although a large body of experimental and theoretical work on superconducting wires has been accumulated over the last ten years (see e.g. the article by Kwasnitza [1] and references therein), it is only very recently that the importance of size effects on the two parameters ρ_L and ρ_T was fully recognized [2].

In this paper we present numerical calculations of the longitudinal and transverse resistivity for an ideal wire consisting of a hexagonal array of superconducting filaments whose resistivity in the normal state is much larger than that of the matrix (and will therefore be taken as infinite), as a function of the ratio of the radius of the filaments to their separation. The lateral dimensions of the wire do not enter the problem since in all practical cases they are several orders of magnitude larger than the electronic mean free path λ_B in the bulk matrix material.

Our treatment is based on Chamber's kinetic formulation [3, 4]. In view of the pronounced small-scale roughness observed at the boundary of the filaments by electron-microscopy [5], we consider the scattering at the matrix-superconductor interface as purely diffuse. In the absence of magnetic field and

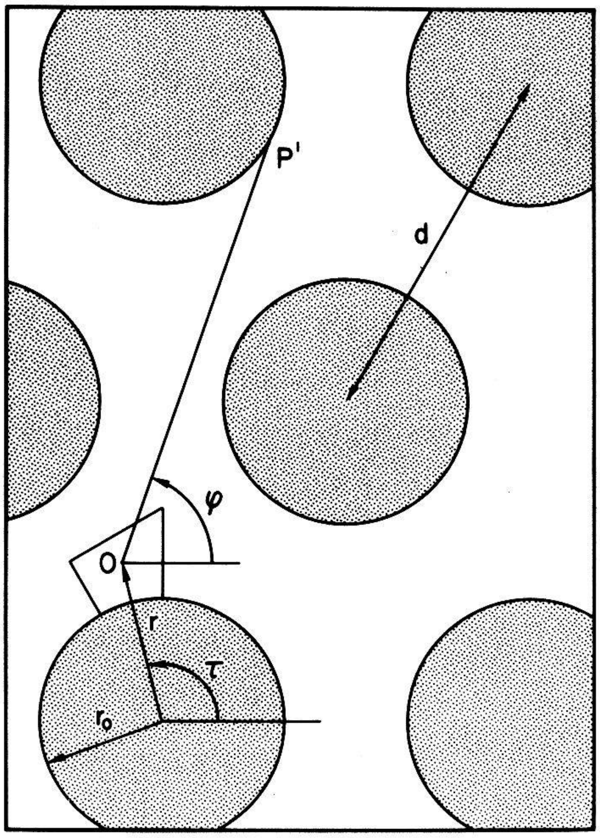

Figure 1
Geometry for the calculation of the longitudinal resistivity of a multifilamentary superconducting wire. The filaments form a hexagonal array. The current-density is computed for 30 points O, with polar coordinates (r, τ), in an irreducible zone (A in the text) of the cross section.

under the assumption of an isotropic bulk mean free path, the current density at a point O of the matrix is then given by [6]

$$\vec{\jmath}\,(O)=\frac{3\sigma_B}{4\pi\lambda_B}\int \vec{r}(\vec{r}\cdot\vec{E})r^{-4}e^{-r/\lambda_B}\,d^3r \tag{1}$$

where $\vec{E}$ is the (static) electric field at the point r, σ_B is the bulk DC conductivity, and the integral is over the volume of the matrix.

To calculate ρ_L, we assume a homogeneous and longitudinal electric field. Attaching a system of polar coordinates to the point O, with the z-axis parallel to the wire, we can write for the longitudinal component of the current density at O

$$j_L(O)=\sigma_B E\left[1-\frac{3}{4\pi}\int_0^{2\pi}d\phi\int_0^{\pi}d\theta\,\sin\theta\cos^2\theta\, e^{-OP/\lambda_B}\right] \tag{2}$$

where P is a point at the surface of a filament such that the vector OP lies entirely in the matrix. By averaging $j_L(O)$ over a suitably chosen unit cell A in the xy-plane (see Fig. 1), and dividing by E, we obtain the longitudinal conductivity $\sigma_L = 1/\rho_L$. The length OP for a given pair of angles (ϕ, θ) is equal to $OP'/\sin\theta$, where P' is the projection of P on the xy-plane, and the determination of OP' is a problem in analytical geometry which can easily be solved on a computer.[1] With

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[1]) Given a point O, the search for the points P' was done explicitly up to the sixth nearest neighbour shell of filaments. For those directions for which P' lay beyond the sixth shell, the corresponding open channels were approximated by two parallel plates.

the change of variable $OP/\lambda_B = u/\sin\theta$, the ratio of the longitudinal to the bulk conductivity takes the form

$$\frac{\sigma_L}{\sigma_B} = 1 - \frac{3}{4\pi A}\int_A dA \int_0^{2\pi} d\phi \int_0^{\pi} d\theta \sin\theta \cos^2\theta\, e^{-u/\sin\theta} \tag{3}$$

The integral over θ appearing in (3) has been discussed in detail by Dingle [7] and is equal to $2S_4(u)$ in his notation. For u smaller than 2 we have expressed it as a series expansion in u, including terms up to order 16, and with a remainder smaller than 10^{-5}. For larger values of u, we used a Romberg integration routine. For the integral over ϕ, we applied a simple trapezoidal rule, with 360 points in the interval $[0, 2\pi]$. Finally, the average over A was performed with help of a bicubic spline quadrature, using 30 calculated points in the unit cell. We have computed ρ_L/ρ_B for $0.1 \leqq \lambda_B/d \leqq 10$ and $0.2 \leqq r_0/d \leqq 0.5$, and we estimate the accuracy of our results to better than 0.5 percent for the worst possible case of a large bulk mean free path ($\lambda_B/d = 10$) and small filament radius ($r_0/d = 0.2$). A plot of the (ρ_L/ρ_B) surface as a function of the two variables λ_B/d and r_0/d is shown in Fig. 2 and a few selected numerical values are given in Table 1. Projected curves on the λ_B/d plane can be found in reference [2], which contains a brief report of our work and a comparison with experiments.

For the computation of the transverse resistivity ρ_T, we consider the situation, sketched in Fig. 3, of a DC voltage applied perpendicularly, i.e. along the x-axis,

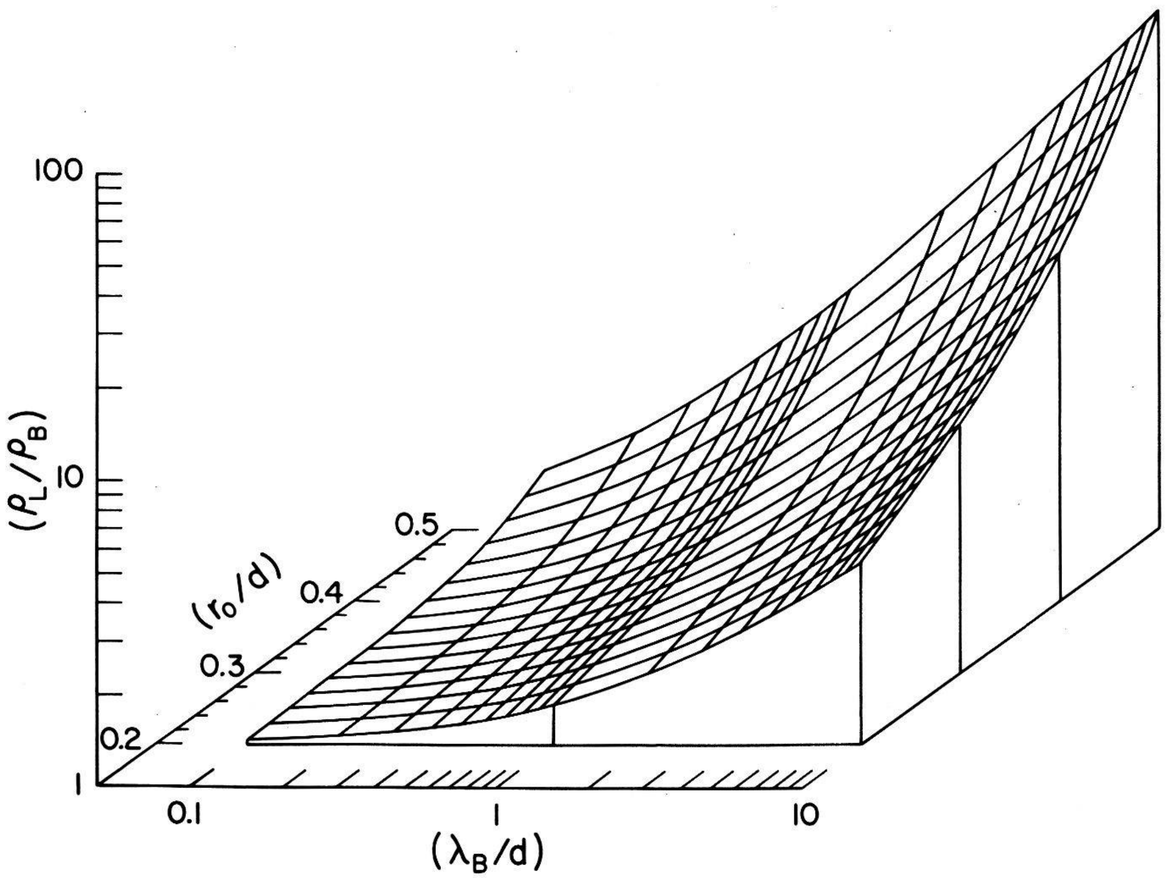

Figure 2
Longitudinal resistivity ρ_L of a multifilamentary superconducting wire in the normal state, as a function of the ratio of the bulk mean free path in the matrix material λ_B and the filament radius r_0 to the interfilamentary distance d.

Table 1
Size effect enhanced longitudinal resistivity of a multifilamentary superconducting wire in the normal state for a hexagonal configuration of filaments and selected values of the ratios of the bulk mean free path λ_B in the matrix material and filament radius r_0 to the interfilamentary distance.

λ_B/d	r_0/d	ρ_L/ρ_B
0.1	0.3.	1.07
0.5	0.3	1.35
1.0	0.3	1.69
5.0	0.3	3.99
0.1	0.4	1.15
0.5	0.4	1.76
1.0	0.4	2.48
5.0	0.4	7.62
0.1	0.5	1.57
0.5	0.5	3.65
1.0	0.5	6.13
5.0	0.5	25.1

between two flattened sides of the wire. In contrast to the longitudinal case, where it could be taken as constant, the electric field is now a complicated function of position determined by the condition of local charge neutrality. The evaluation of the field distribution throughout the sample amounts to a self-consistent solution of the Boltzmann equation, a rather untractable problem for our geometry. We therefore resort to an approximation which, for the range of parameters (λ_B, r_0, d) encountered in commercial wires, should produce a close upper bound to the true value of ρ_T. Namely, we replace the actual sample by an array of undulated thin films, as illustrated in Fig. 3, and assume the electric field to be parallel to the films and constant in magnitude. Each row of filaments along the x-axis contributes one such channel, the resistivity of which can be interpolated from the

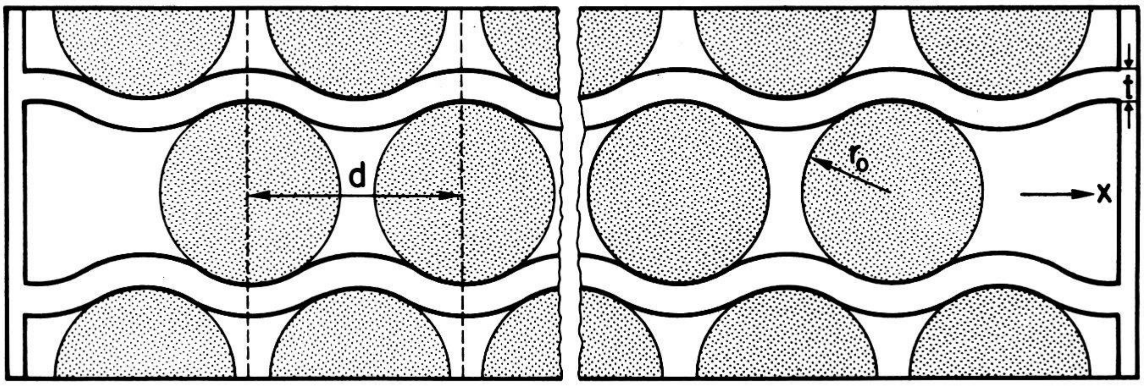

Figure 3
Geometry for the determination of the transverse resistivity of a multifilamentary superconducting wire. The full wire consists of a juxtaposition of strips like the one shown. The wiggly channels are the equivalent thin films used in the calculation, and the vertical dashed lines limit the unit cell in the x-direction, whose resistance is computed in the text. A potential difference V is applied along x between the two flattened sides of the wire.

tabulated values of Brändli and Cotti [8], or computed with help of the above described numerical techniques. The conversion of the results to those for the real system proceeds as follows: first, the sample is divided into unit cells of length d in the x-direction, $d\sqrt{3}$ in the y-direction, and unity in the z-direction, forming a rectangular lattice in the xy-plane. The resistance R_T of such a cell is equal to that of two segments of undulating film of length $(\pi/3)d$, connected in parallel

$$R_T = \tfrac{1}{2}\rho_{\text{Film}}\left(\frac{\lambda_B}{t}\right)\frac{(\pi/3)d}{t\cdot 1} \tag{4}$$

where $t = (d - 2r_0)$ is the thickness of the film. R_T is a well defined quantity readily comparable with experiment. The resistivity ρ_T is then obtained from the usual formula

$$\rho_T = R_T\frac{\langle S\rangle}{d} \tag{5}$$

where $\langle S\rangle$ is a suitable average of the cross section over the length d of the unit cell. This average should go to zero in the limit of touching filaments, since then no current can flow through the sample even in the absence of size effects. Furthermore, for vanishing bulk mean free path the normalized resistivity (ρ_T/ρ_B) should equal unity. This last condition, together with equations (4) and (5), leads to

$$\frac{\langle S\rangle}{d} = \frac{S_{\text{eff}}}{d} = \frac{6}{\pi}\left(1 - 2\frac{r_0}{d}\right)\cdot 1 \tag{6}$$

in centimeters, and

$$\rho_T\left(\frac{\lambda_B}{d}, \frac{r_0}{d}\right) = \rho_{\text{Film}}\left(\frac{\lambda_B}{t}\right) \tag{7}$$

The *effective mean cross-section for current flow*, S_{eff}, has the required limiting value at $r_0 = d/2$ and is systematically smaller than the average over d of the cross-section of the matrix material given by

$$\frac{S_{\text{mat}}}{d} = \left(\sqrt{3} - 2\pi\left(\frac{r_0}{d}\right)^2\right)\cdot 1 \tag{8}$$

Curves of (ρ_T/ρ_B) as a function of (λ_B/d) derived from equation (7) for a range of experimentally relevant values of (r_0/d) are shown in Fig. 4. They should be taken with a grain of salt since, strictly speaking, $\langle S\rangle$ could be a function of (λ_B/d) with the value S_{eff} for zero argument. We therefore suggest that the *resistance* R_T be used in comparisons with experiment.

The alert reader may wonder about the relevance of our results for ρ_T to the cases of practical interest, where the filaments are in the superconducting state. The answer to that question is that there exists an important class of conductors to which our theory still applies, namely all the ones consisting of NbTi filaments. These are well known to form a very high resistivity diffusion layer at the superconductor-matrix interface, which will effectively prevent the normal electrons from entering the superconducting region.

In conclusion we believe that our calculations provide enough information on

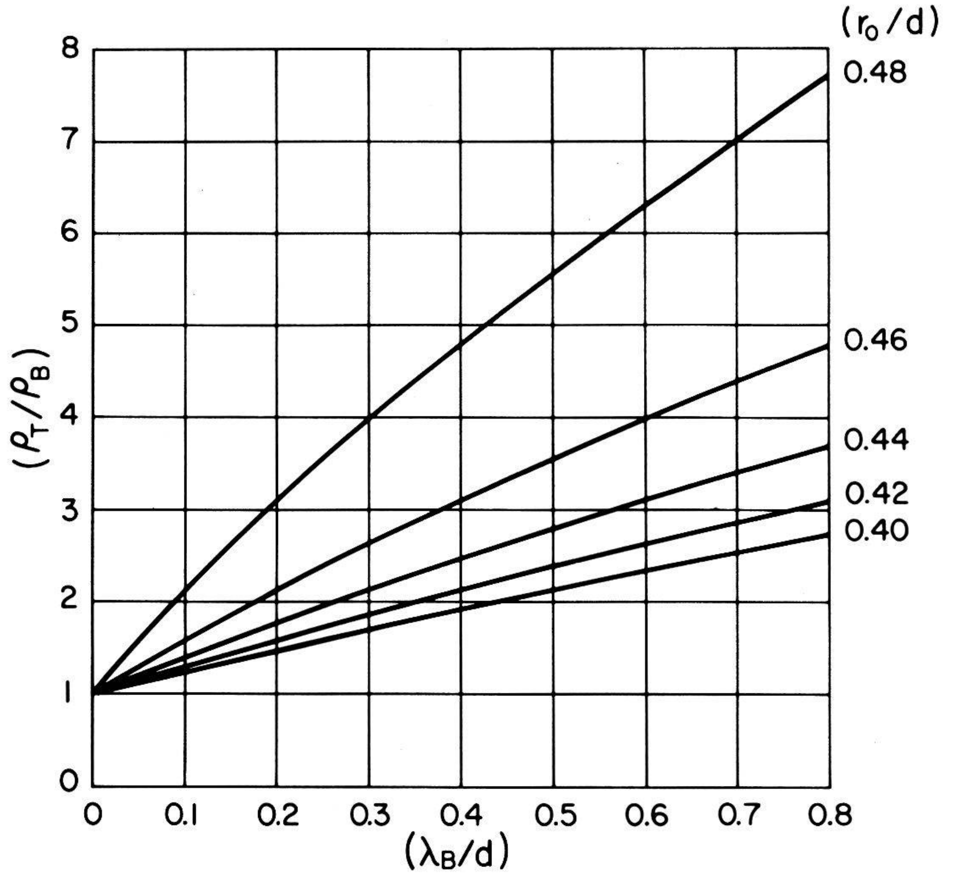

Figure 4
Transverse resistivity ρ_T of a multifilamentary superconducting wire in the normal state, as a function of the ratio of the bulk mean free path λ_B to the interfilamentary distance d for a few experimentally relevant values of the filament radius r_0.

size effects in multifilamentary superconducting wires to be useful to the practitioner. The action of a transverse magnetic field, present in virtually all applications, on these effects will depend on the actual geometry used. As long as the corresponding orbit diameter is larger than a typical linear dimension, say t, of the space between the filaments, our results should not be affected. As an example, for an electron on the Fermi surface of Cu, a filament diameter of 10 μm and a ratio r_0/d of 0.4, the critical value of the field is of the order of 7 Tesla.

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