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Autor: Martinoli, P. / Nsabimana, M. / Racine, G.A.

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Locked and unlocked phases of a two-dimensional lattice of superconducting vortices

By P. Martinoli, M. Nsabimana, G. A. Racine and H. Beck,
Institut de Physique, Université de Neuchâtel,
CH–2000 Neuchâtel, Switzerland
J. R. Clem, Ames Laboratory–USDOE and Department of
Physics, Iowa State University, Ames, Iowa 50011, USA

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Abstract. The flux lattice in a thin film of a type II superconductor, whose thickness is periodically modulated, allows for an investigation of various phase transitions typical of two-dimensional systems. We present a continuum approach in which the mismatch δ between the equilibrium lattice structure and the spacial period of the modulation plays a major role. In the ground state the lattice is locked by the modulation potential when δ is small, whereas for large enough δ the lattice is freely floating, its structure showing periodic discommensurations. A phase diagram in the (δ, T)-plane can be established by taking into account thermal fluctuations. Critical current data at various temperatures show good agreement with the theoretical predictions.

I. Introduction

Phase transitions in two-dimensional (2D) systems have received considerable attention recently. In several experiments the 2D crystal under consideration is exposed to the force field created by a periodic substrate. Among other situations, this is the case of a 2D lattice of superconducting vortices interacting with a periodic pinning potential. As pointed out by Martinoli and coworkers [1] some years ago, thin superconducting films, whose thickness is periodically modulated in one direction, provide such a system. In this paper we show how critical current measurements in thickness modulated layers can be used to probe a locking–unlocking phase transition of the 2D vortex lattice occurring when the conditions of flux line density and/or temperature are changed in this particular physical system. Some aspects of the locking–unlocking transition were reported in a recent letter [2]. Here we describe it in more detail.

The phase diagram of 2D crystals in a periodic potential has been studied by a number of authors [3]. Dealing with situations where the periodic substrate is, as in our case, anisotropic, a recent theory by Pokrovsky and Talapov [4] (PT) is particularly relevant to the understanding of our experiments, where the 2D vortex lattice experiences the 1D periodic force field created by the thickness modulation. At absolute zero ($T = 0$), PT predict the existence of stable locked (L)-phases when the degree of mismatch between the natural (undistorted) lattice and the underlying periodic pinning structure does not exceed some critical value.

In an L-phase the vortex lattice is a 2D epitaxial (or commensurate) solid in registry with the substrate periodicity. At the critical mismatch PT predict a second order transition from a registered L-phase to an incommensurate unlocked (U)-phase. In the U-phase the vortex lattice is a "floating" 2D solid characterized by the presence of a superstructure consisting in a 1D periodic sequence of domain wall dislocations. These and other interesting features of the LU-phase transition at $T = 0$ are discussed in Section II.A.

At finite temperatures thermal fluctuations of the vortices in the L-phase tend to unlock the vortex lattice from the periodic pinning structure, thereby driving the transition to the U-phase at a sufficiently high temperature. As a consequence of Brownian motion of the vortices, the critical degree of mismatch tolerated by an L-phase becomes smaller and smaller as the temperature rises and finally vanishes at a critical temperature T_{LU}, above which an L-phase can no longer exist. The corresponding LU-phase boundary has been calculated by PT using a renormalization-group technique [4]. In Section II.B we propose an alternative approach based on the more transparent Self-Consistent Harmonic Approximation (SCHA). The expression for T_{LU} deduced from this model agrees with that obtained by PT but the shape of our LU-phase boundary differs considerably from that of PT. For instance, our phase diagram does not show the rather surprising reentrant behaviour which one deduces by inspection of the PT-theory. It is further argued that, above T_{LU}, the vortex lattice is a 2D floating solid exhibiting topological order [5] or a liquid according to whether T_{LU} is lower or higher than T_M, the temperature at which the vortex lattice is expected to melt through thermal dissociation of bound pairs of dislocations [5–7].

Pinning phenomena in thickness modulated superconducting films prove to be a sensitive probe of the LU-phase transition. Since in an L-phase the vortex lattice is obviously pinned by the periodic film structure while in a U-phase it is free to slide under the influence of an arbitrarily small driving force, characteristic peaks reflecting the presence of various L-phases show up in the critical current curves $I_c(B)$ [1]. As the magnetic field B governs the vortex density, the width of the peaks is a measure of the critical mismatch at which the LU-phase transition takes place. With rising temperature the intensity of the I_c-maxima decreases and finally undergoes a relatively rapid degradation as one approaches a critical temperature which we identify with T_{LU}. For $T > T_{LU}$ the structures in $I_c(B)$ are completely washed out indicating that the vortex lattice is no longer locked to the periodic substrate. These and other features of our I_c-data are discussed in Section III in the light of the theoretical predictions of the previous section.

II. Theoretical considerations

(A) *Phase transition at zero temperature*

Let us first briefly recall some of the basic concepts and results of the PT-theory [4]. To this purpose we consider a 2D triangular lattice of superconducting vortices, with lattice parameter a, in static interaction with a 1D harmonic potential of amplitude Δ and wave vector $\vec{q}$ ($q = 2\pi/\lambda_g$). We shall focus our attention on situations where $\vec{q}$ is very close to one of the vectors, $\vec{g}$, of the reciprocal vortex lattice, the condition $\vec{q} = \vec{g}$ defining a configuration of perfect matching between the (undistorted) lattice and the sinusoidal pinning potential. It is assumed that the lattice of Abrikosov vortices is incompressible [8] and, further, that the pinning is weak when compared to the lattice stiffness, i.e. $\Delta < \mu$, where μ is the shear modulus of the vortex lattice [9]. Under these conditions only long wavelength shear deformations turn out to be relevant and, as a consequence, the vortex lattice can be treated as an elastic continuum. Then, the energy of the system can be written as the sum of an elastic contribution and of the potential energy due to the periodic pinning force

$$\mathscr{E} = \int \left[\frac{\mu}{2} \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 + \Delta(1 - \cos q\phi) \right] dx\, dy. \qquad (1)$$

In writing this expression we have jumped ahead to the conclusion of PT asserting that the lowest energy configuration of the vortex lattice is characterized by a quasi 1D deformation field $\vec{w}$ which, in an $x'-y'$ reference frame with x' pointing in the $\vec{q}$-direction, has components of the form

$$u' = -\delta x' + \phi(x), \qquad v' = \delta y' - \phi(x), \qquad (2)$$

where $\delta = 1 - (g/q)$ measures the degree of mismatch. These expressions clearly show that there are two distinct contributions to $\vec{w}$. The first one is an area conserving deformation resulting from a uniform compression ($\delta > 0$) or expansion ($\delta > 0$) $-\delta x'$ along x' combined with a uniform expansion ($\delta > 0$) or compression ($\delta < 0$) $\delta y'$ along y'. This deformation is chosen such that the potential energy contribution to $\mathscr{E}$ vanishes: the vortices are forced to lie in the valleys of the cosine-potential. Superposed to this uniform field is a 1D deformation $\phi(x)$ which, for an incompressible lattice, is found to propagate in a direction x forming an angle of 45° with $\vec{q}$ [4, 8]. Thus, in the x-y coordinate system rotated by 45° with respect to $x'-y'$ the deformation field $\vec{w}$ has the components

$$u = \delta y, \qquad v = \delta x - \sqrt{2}\, \phi(x). \qquad (3)$$

As it clearly emerges from these expressions, in the new reference system the uniform deformation described in connection with equation (2) results from the superposition of two uniform shear deformations, one along x and the other along y. By inserting u and v, as given by equation (3), into the general form of the elastic energy of an isotropic 2D continuum [4] one immediately obtains equation (1).

To determine $\phi(x)$, we simply minimize the functional $\mathscr{E}[\phi(x)]$ with respect to $\phi(x)$, thereby obtaining the following sine–Gordon equation [10] for the "phase" field $\Phi(x) = q\phi(x)$

$$\sin \Phi - l^2 \frac{\partial^2 \Phi}{\partial x^2} = 0, \qquad (4)$$

where $l^2 = 2\mu/\Delta q^2$. Its solution in terms of elliptic functions

$$\Phi(x) = \pi + 2am(x/kl) \qquad (5)$$

is a stair-shaped function representing a regular sequence of kinks (discommensurations), whose period L is related to k by

$$L = 2klK(k), \qquad (6)$$

where $K(k)$ is a complete elliptic integral of the first kind. Using equations (5) and (6), the potential energy (1) can be expressed as a function of the variational parameter k. Minimization of $\mathscr{E}(k)$ with respect to k leads to the condition

$$\delta = \frac{2}{\pi}\left(\frac{\Delta}{\mu}\right)^{1/2}\frac{E(k)}{k}, \tag{7}$$

where $E(k)$ is a complete elliptic integral of the second kind. From the properties of $E(k)$ it follows that there are solutions of equation (7) satisfying the condition $0 \leqslant k \leqslant 1$ only if δ is larger than a critical mismatch δ_c given by

$$\delta_c = \frac{2}{\pi}\left(\frac{\Delta}{\mu}\right)^{1/2}. \tag{8}$$

For $|\delta| \geqslant \delta_c$ $\Phi(x)$ is of the form (5) and, as a consequence, the vortex lattice is in the incommensurate U-phase. This is shown in Fig. 1, where we have assumed that the starting matching configuration is that corresponding to $\vec{q} = \vec{g}_1$, $\vec{g}_1$ being one of the six nearest-neighbour reciprocal lattice vectors ($g_1 = 4\pi/a\sqrt{3}$). Since $\delta > 0$ for the configuration shown in Fig. 1, large portions of the lattice appear to be uniformly compressed along x' and expanded along y' and are essentially

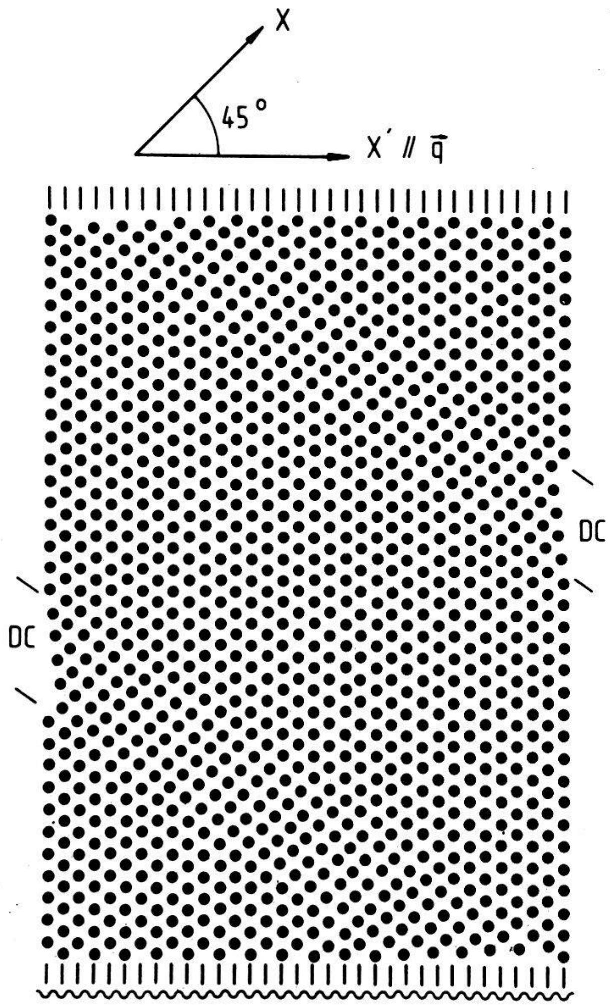

Figure 1
Incommensurate U-phase for $\delta = 0.13$ ($B/B_{10} = 0.76$). Discommensurations (DC) form a periodic 1D sequence propagating at 45° with respect to $\vec{q}$.

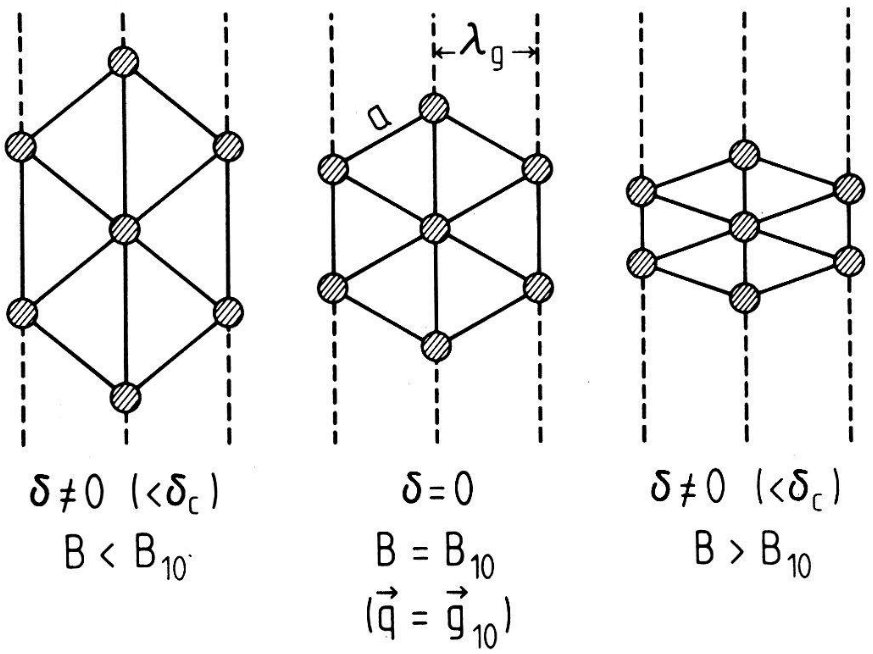

Figure 2
Three different deformation states of the fundamental commensurate L_{10}-phase.

commensurate to the underlying 1D periodic pinning structure. These regions are separated from each other by a 1D periodic sequence of domain wall discommensurations propagating at 45° with respect to $\vec{q}$. At the discommensuration sites the phase field $\Phi(x)$, which is essentially constant in the nearly commensurate regions between successive kinks, changes by 2π over distances of the order of $\sim kl$. The period L of the superstructure diverges logarithmically (see equation 6) as δ approaches δ_c^* $(k \to 1)$.

For $|\delta| < \delta_c$ there are no solutions of equation (7) and, consequently, $\Phi(x)$ is no longer given by equation (5). In this case $\mathscr{E}$ has its minimum value when the potential energy term associated with the 1D pinning field on the right-hand side of equation (1) vanishes, i.e. when $\Phi(x) = 0$ everywhere. Obviously, this corresponds to the commensurate L-phase shown schematically in Fig. 2 for $\delta = 0$ (matching configuration $\vec{q} = \vec{g}_1$) and for vortex densities lower and higher than that corresponding to $\vec{q} = \vec{g}_1$.

The areal potential energy density, $F_\square$, of the vortex lattice can be written in the form

$$F_\square = 2\mu\delta^2 - 2\Delta\{[\delta/\delta_c E(k)]^2 - 1\}\theta(|\delta| - \delta_c), \tag{9}$$

where $\theta(z)$ is the Heaviside function: $\theta(z) = 1$ for $z > 0$, $\theta(z) = 0$ for $z < 0$. The first term on the right-hand side of Eq. (9) is the elastic energy density associated with the uniform deformations characterizing both the L- and the U-phase, whereas the second one arises from the phase field $\Phi(x)$ and, consequently, contributes to $F_\square$ only in the U-phase. $F_\square(\delta)$ is plotted in Fig. 3 together with the result of a calculation based on a discrete lattice model [8] where, however, only harmonic shear deformations of the vortex lattice were considered. With this important restriction the LU-phase transition occurs for $\delta = 0$, but other features turn out to be identical to those predicted by the PT-model. In particular, the U-phase is characterized by the presence of a sinusoidal transverse deformation of the lattice propagating in a direction at 45° with respect to $\vec{q}$. A more detailed

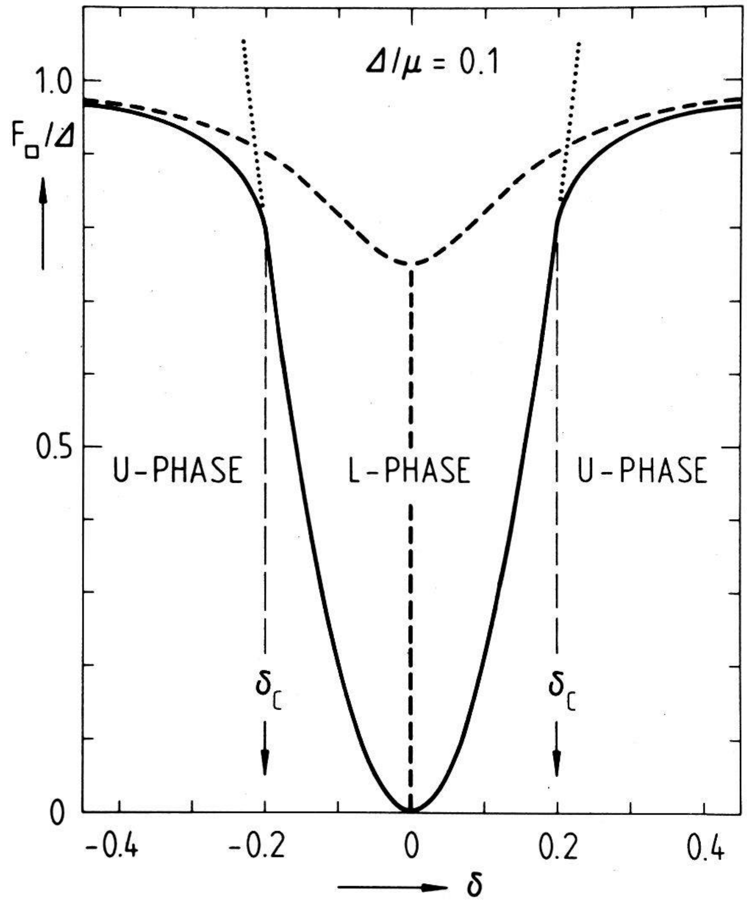

Figure 3
Areal energy density of a 2D lattice in a 1D periodic force field as a function of the mismatch δ. The full curve is based on equation (9). The dashed curve follows from a discrete lattice model allowing only for harmonic deformations. In this case the LU-transition occurs at $\delta = 0$.

account of the discrete lattice model, which proves to be very useful in the description of vortex lattice dynamics at high frequencies, will be published elsewhere.

(B) *Phase transition at finite temperatures*

To study the LU-phase transition at finite temperatures, we first consider the case of perfect matching ($\delta = 0$), which is particularly simple. For $\vec{q} = \vec{g}$ the vortices execute a Brownian motion around the equilibrium positions they would assume at the bottom of the potential wells at $T = 0$. Accordingly, the Langevin equation of motion for a vortex at the lattice site $\underline{l}$ can be written as

$$\eta \dot{\vec{u}}_{\underline{l}} = -\sum_{\underline{l}'}{}' \tilde{G}(\underline{l} - \underline{l}') \vec{u}_{\underline{l}'} - \vec{q}\Delta' \sin(\vec{q} \cdot \vec{u}_{\underline{l}}) + \vec{F}_{\underline{l}}(t), \tag{10}$$

where the four terms represent, successively, the viscous damping force, the lattice restoring force, the harmonic pinning force and the fluctuating Langevin force acting on the vortex at $\underline{l}$. $\eta^{-1} = R_\square / B\phi_0$, where $R_\square$ is the sheet flux-flow resistance of the superconducting film, is the mobility of a free vortex, $\tilde{G}(\underline{l} - \underline{l}')$ the elastic matrix and Δ' is related to Δ by $\Delta = (B/\phi_0)\Delta'$. $\vec{F}_{\underline{l}}(t)$ is assumed to have a white noise spectrum defined by the correlation function

$$<F_{\underline{l}}^{\alpha}(t)F_{\underline{l}'}^{\beta}(t') = 2\eta k_B T\delta_{\alpha\beta}\delta_{\underline{l}\underline{l}'}\delta(t-t') \tag{11}$$

stating that the Langevin force is uncorrelated in direction, space and time. To solve equation (10) for the mean square fluctuation $\langle u^2\rangle$ of the vortices it is convenient to expand $\vec{u}_{\underline{l}}(t)$ in normal modes of the vortex lattice

$$\vec{u}_{\underline{l}}(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\sum_{\underline{k},p} u_{\underline{k}}(\omega)\hat{e}_{\underline{k}p}e^{i(\vec{k}\cdot\vec{l}-\omega t)}\,d\omega, \tag{12}$$

where the $u_{\underline{k}}(\omega)$ are the normal mode amplitudes and the $\hat{e}_{\underline{k}p}$ are polarization vectors for longitudinal ($p=l$) and transverse ($p=t$) deformations. Linearizing the equation of motion (10) in the so-called Self-Consistent Harmonic Approximation (SCHA) and considering, as in Section II.A, only transverse modes of the vortex lattice, from equations (10) and (12) the following expression for the t-component, $u_{\underline{k}t}(\omega)$, of $\vec{u}_{\underline{k}}(\omega)$ is deduced

$$u_{\underline{k}t}(\omega) = \frac{F_{\underline{k}t}(\omega)}{D_{\underline{k}t} + \Delta_R(\vec{q}\cdot\hat{e}_{\underline{k}t})^2 - i\eta\omega} \tag{13}$$

where $D_{\underline{k}t}$ is the matrix element of the (diagonal) dynamical matrix associated with transverse modes and $F_{\underline{k}t}(\omega)$ is the transverse Fourier component of the Langevin force. Within the framework of SCHA the effective strength, Δ_R, of the pinning potential experienced by the vortices is given by

$$\Delta_R = \Delta e^{-\frac{1}{2}q^2\langle u_{tx}^2\rangle} \tag{14}$$

where $\langle u_{tx}^2\rangle$ is the mean square transverse fluctuation along the direction x parallel to $\vec{q}$. Equation (14) shows very clearly how the renormalization effect of the thermal fluctuations, which is the essential feature leading to the LU-phase transition, enters our problem: through a Debye–Waller factor which reduces the amplitude of the periodic force field acting on the vortices. To calculate the mean square transverse fluctuation

$$\langle u_t^2\rangle = \frac{1}{\pi}\lim_{T\to\infty}\frac{1}{T}\int_0^{\infty}\sum_{\underline{k}}|u_{\underline{k}}(\omega)|^2\,d\omega, \tag{15}$$

we assume a Debye model, for which $D_{\underline{k}t} = \mu k^2$, and replace the sum over $\underline{k}$ in equation (15) by an integral over a smooth density of states. In the weak pinning limit $\Delta \ll \mu$ considered here we then deduce from equations (11), (13) and (15)

$$\langle u_t^2\rangle = \frac{k_B T}{4\pi\mu}\ln(\mu/\Delta_R). \tag{16}$$

This expression shows quite clearly that the L-phase, which for the case of perfect matching ($\delta = 0$) under consideration is stable as long as Δ_R is finite, is a 2D solid with conventional long range order. As expected for 2D systems, $\langle u_t^2\rangle$ diverges logarithmically as Δ_R vanishes. Since, by equipartition, $\langle u_{tx}^2\rangle \approx \frac{1}{2}\langle u_t^2\rangle$ in the limit $\Delta \ll \mu$, from equations (14) and (16) one obtains

$$\Delta_R/\Delta = (\Delta/\mu)^{T/(T_{LU}-T)}, \tag{17}$$

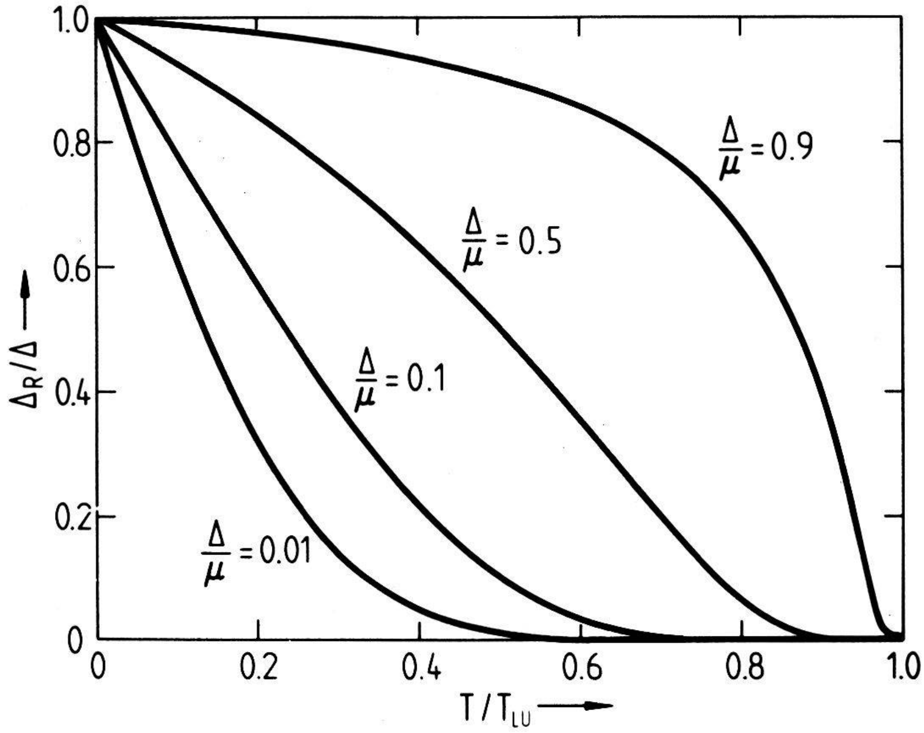

Figure 4
Temperature dependence of the effective pinning potential amplitude.

where T_{LU} is implicitly defined by

$$k_B T_{\mathrm{LU}} = (4/\pi)\mu(T_{\mathrm{LU}})\lambda_g^2. \tag{18}$$

From Fig. 4, where Δ_R/Δ is plotted as a function of T/T_{LU} for a set of (Δ/μ)-values, it clearly results that at T_{LU} the vortex lattice undergoes a transition from a perfectly matched L-phase ($\Delta_R \neq 0$) to a "floating" U-phase ($\Delta_R = 0$). It should be noticed that the expression (18) for T_{LU}, as deduced in our SCHA-scheme, is the same as that obtained with a renormalization-group technique by PT.

For a moderately dense lattice of vortices in dirty superconducting films μ can be written in the form [9]

$$\mu = \tfrac{1}{2} n_\square (\phi_0/4\pi)^2 \frac{1}{\Lambda}, \tag{19}$$

where $n_\square = B/\phi_0$ is the areal vortex density and $\Lambda = 2\lambda^2/d$ an effective penetration depth for 2D superconducting layers (d is the film thickness), whose temperature dependence is given by [11]

$$\Lambda^{-1}(T) = 2.17(4\pi/\phi_0)^2 (R_u/R_{n\square}) k_B T_c \left[\frac{\Delta(T)}{\Delta(0)}\, Tgh\left(\frac{\Delta(T)}{2k_B T}\right)\right]. \tag{20}$$

In Eq. (20) $\Delta(T)$ is the BCS-energy gap, $R_{n\square}$ the normal state sheet resistance of the film and R_u the universal resistance $\hbar/e^2$. Since μ is a function of $n_\square$, equation (18) shows that T_{LU} depends upon the matching configuration under consideration. For a triangular lattice such configurations are defined by [8]

$$B_{n_1 n_2} = \frac{\sqrt{3}}{2} \frac{\phi_0}{\lambda_g^2} (n_1^2 + n_2^2 + n_1 n_2)^{-1}, \tag{21}$$

where n_1 and n_2 are integers. Then, in the limit of low sheet resistances $R_{n\square} \ll R_u$ from equations (18) – (21) one obtains for the transition temperature T_{LU} of the fundamental matching configuration $\vec{q} = \vec{g}_1$ $(n_1 = 1,\ n_2 = 0)$

$$\frac{T_{LU}}{T_c} = 1 - 0.31 \frac{R_{n\square}}{R_u}, \tag{22}$$

where T_c is the BCS-transition temperature of the film. The LU-transition temperatures corresponding to configurations defined by higher values of n_1 and n_2 lie below that given by equation (22).

The case of finite mismatch $(\delta \neq 0)$ is more delicate. It has been recently studied in a slightly different context (the 2D classical sine–Gordon system) by Puga et al. [12] using a renormalization-group approach, where, for the first time, δ is considered as a new renormalizable parameter. Although several aspects of the LU-transition emerging from their calculation turn out to be quite different from those following from the much simpler SCHA-scheme, the shape of the phase boundary $\delta_c(T)$ resulting from their approach is very similar to that predicted by SCHA. In the latter approximation $\delta_c(T)$ simply follows from equation (8) by replacing Δ with its renormalized value Δ_R given by equation (17). The resulting phase diagram is shown in Fig. 5, where, instead of $\delta_c(T)$, we have

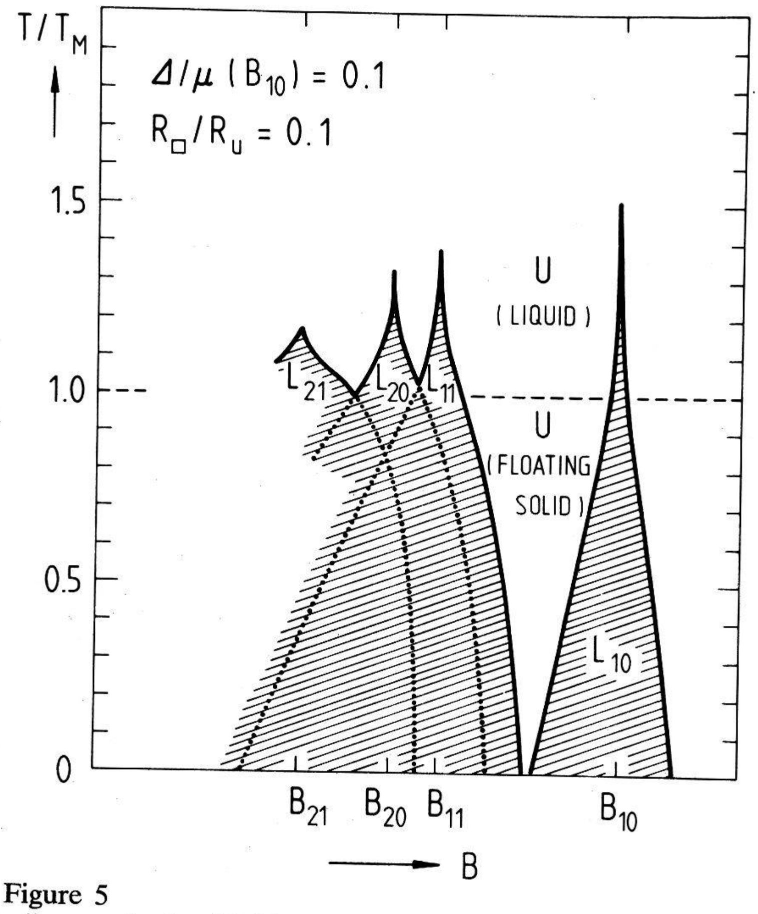

Figure 5
Phase diagram in the (B, T)-plane of a 2D vortex lattice interacting with a 1D periodic potential.

plotted the related quantity $B_c(T) = B_{n_1n_2}[1-\delta_c(T)]^2$. Since, for convenience, B_c is reported on a logarithmic scale, the phase boundary delimiting a given L-phase, which would be symmetric about $B_{n_1n_2}$ on a linear plot, appear asymmetrical. Assuming Δ independent of B, in constructing Fig. 5 Δ/μ was kept constant, for simplicity, within each of the different L-phases, but was scaled according to equations (19) and (21) from L-phase to L-phase. In Fig. 5 temperatures are conveniently measured in units of T_M, the melting temperature of the 2D vortex lattice [5–7], which, as shown by the following equation, is independent of B at moderate vortex densities

$$k_B T_M = \frac{1}{4\pi}\mu(T_M)a^2. \tag{23}$$

With this additional aspect of the problem in mind, it is argued that, if the LU-transition takes place for $T > T_M$, it is a transition from a solid L-phase to a fluid-like U-phase. This is the case for the L-phases of lower order (n_1 and n_2 small) of Fig. 5, where T_{LU} is larger than T_M. With a straightforward calculation based on equations (18), (21) and (23) it can be shown, however, that there is a particular commensurate phase, the L_{22}-phase, for which T_{LU} becomes equal to T_M. For L-phases of higher order T_{LU} is always lower than T_M and, consequently, the LU-transition is from a solid epitaxial L-phase to a solid floating U-phase.

III. Critical currents

To test some of the theoretical ideas put forward in the previous section, critical current (I_c) measurements were performed on thickness modulated granular Al-films as a function of magnetic field and temperature. A combined holographic photolithographic technique was used to fabricate grating-like film profiles with $\lambda_g \lesssim 1\ \mu m$. To meet the condition, $\Delta < \mu$, for weak pinning, the relative thickness modulation $\Delta d/d$ was kept below ~20–25%. The most relevant superconducting and normal state properties of the two Al-films studied in this paper are summarized in Table 1.

Since a registered L-phase is pinned by the periodic film structure, a finite force is required to depin the vortex lattice and, subsequently, to sustain vortex motion in the dissipative flux-flow régime. In our experiments such a force is provided by a uniform transport current flowing parallel to the grooves of the grating-like film profile. A U-phase, on the other hand, is not pinned by the periodic substrate, its energy being independent, at least within the framework of

Table 1

Film	d[Å]	$\Delta d/d$[(a)]	$\lambda_g[\mu m]$	$R_{n\square}[\Omega]$	T_c[K]	$(\xi_0 l)^{1/2}$[Å][(b)]	$\lambda_L(0)\left(\frac{\xi_0}{l}\right)^{1/2}$[Å][(b)]
Al1	200	~0.2	0.79	15	1.89	365	4300
Al2	200	~0.2	0.77	35	2.16	223	6140

a Determined by combined optical and electrical methods.
b Calculated using $\rho l = 4\times10^{-12}\ \Omega\,cm^2$ and $\lambda_L(0) = 157$ Å for Al. ξ_0 was scaled from the bulk Al value (1, 6 μm) according to our T_c.

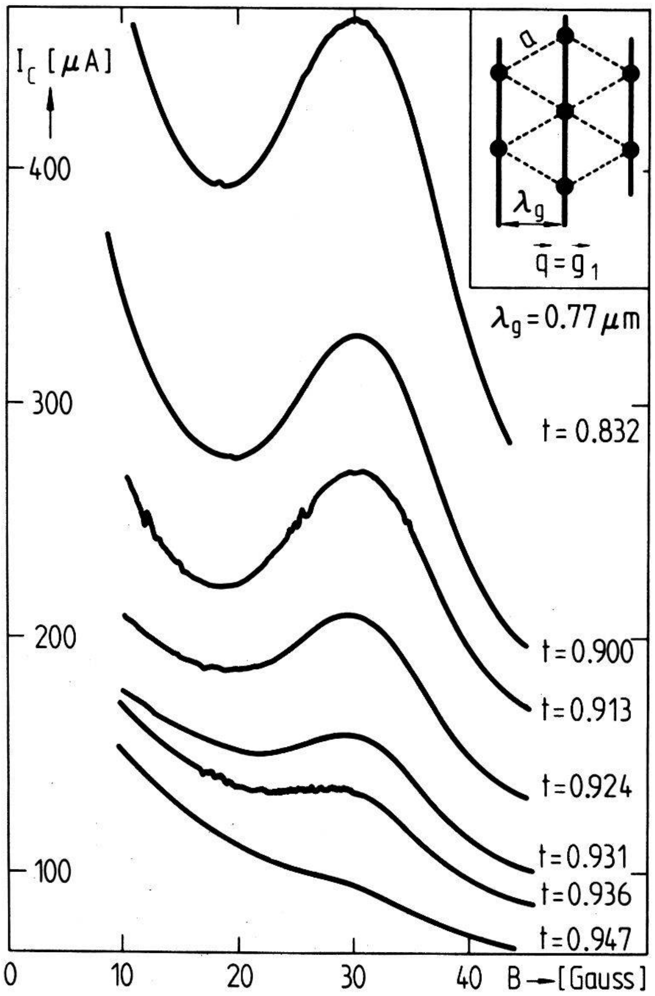

Figure 6
Critical current vs. magnetic field curves of a thickness modulated film (Al2) for different reduced temperatures $t = T/T_c$.

the continuum approximation, of the relative position of the vortex lattice with respect to the pinning potential. Therefore, the critical current for entering the flux-flow régime vanishes in this case.

In Fig. 6 $I_c(B)$-curves of the film Al2 are shown for different temperatures. One can easily verify, using equation (21), that the peak at $B \approx 30$ gauss is the signature of the fundamental L_{10}-configuration shown in the insert of Fig. 6. According to our previous discussion, the width of this peak, taken in the limit of vanishing critical current, is a measure of the critical mismatch $\delta_c(T)$ and, consequently, could in principle be used to determine the L_{10}U-phase boundary. For two reasons, however, this appears to be, in practice, a problem of difficult solution. The first and most important one is that in our films, as we shall show later on, Δ is of the order of μ, typically $\Delta/\mu \approx 0.9$. Under this condition considerable overlapping of the L_{10}-phase with the L_{11}-phase is expected (in Fig. 5 overlapping of the different L-phases is enhanced as Δ/μ increases). This is at the origin of the relatively high shoulder on the low field side of the I_c-peak in Fig. 6. Additional evodence for substantial overlapping effects is also provided by the fact that the L_{11} and L_{20}-configurations were hard to resolve in our experiments. The second reason is that in real films one is dealing with unavoidable pinning effects due to randomly distributed inhomogeneities, which result in a finite contribution to I_c even in the U-phase. Clearly, both overlapping and random pinning effects render a determination of the peak width quite uncertain. In the rest of the paper, therefore, we shall concentrate on a much more accessible experimental quantity: the temperature dependent strength $I_{cM}(T)$ of a critical current peak.

For perfect matching the equilibrium position of a vortex is determined [13, 14] by balancing the Lorentz driving force $\vec{F}_L = d(\vec{j} \times \vec{\phi}_0)$ against the pinning force experienced by the vortex in the effective cosine-potential $\Delta'_R(1-\cos q\phi)$, where $\Delta'_R = \Delta_R/n_\square$. This results in the following expression for the transport current density

$$j = \frac{q\Delta'_R}{\phi_0 d} \sin \Phi \tag{24}$$

Obviously, the critical current density j_{cM} is reached for $\Phi = \pi/2$, a condition corresponding to vortices located halfway between the bottom and the top of the potential wells. Thus, using equation (17), j_{cM} can be written as

$$j_{cM} = \frac{q\Delta'}{\phi_0 d}\left(\frac{\Delta}{\mu}\right)^{T/(T_{LU}-T)}. \tag{25}$$

In order to analyse our I_c-data with equation (25) we need a model for Δ' (or Δ), the characteristic energy scale of the pinning mechanism operating in our thickness modulated films. In the thin film limit ($d \ll \lambda$) the potential energy $\varepsilon(\vec{r})$ of a vortex located at $\vec{r}$ can be expressed by the convolution [15]

$$\varepsilon(\vec{r}) = \int f(\vec{r}\,' - \vec{r})\, d(\vec{r}\,')\, d^2r', \tag{26}$$

where $d(\vec{r}\,') = d + \Delta d \cos qx$ is the thickness modulation and $f(\vec{r}\,' - \vec{r})$ the free energy density distribution within the flux line. There are three major contributions to f: an electromagnetic contribution f_{em} arising from the field and supercurrent distributions in the vortex, a contribution f_k representing the kinetic energy cost to produce the vortex and a contribution f_c due to the condensation energy paid in creating its normal core. In our case f_{em} is expected to contribute very little to the integral in equation (26), its characteristic scale of variation, the effective penetration depth Λ, being much larger than λ_g ($q\Lambda \gg 1$). Varying over distances of the order of the coherence length ξ, which is much smaller than λ_g in the temperature region of interest here, f_k and f_c provide the dominant contributions to ε. Using Clem's model [16] for f_k and f_c, from equation (26) one obtains in the limit $q\xi < 1$ and of large GL-parameters $\kappa = \lambda/\xi$

$$\varepsilon(x) \approx 2(\Delta d/d)(\phi_0/4\pi)^2 \frac{1}{\Lambda}(1+\cos qx). \tag{27}$$

Accordingly, Δ is identified as

$$\Delta \approx 2n_\square(\Delta d/d)(\phi_0/4\pi)^2 \frac{1}{\Lambda}. \tag{28}$$

This expression shows that Δ has the same temperature dependence as μ, a considerable simplification in the analysis of the I_c-data. By combining equations (25) and (28), I_{cM} can finally be written in the form

$$\frac{I_{cM}(T)}{I_{cM}(0)}=\frac{\Lambda(0)}{\Lambda(T)}\left(\frac{\Delta}{\mu}\right)^{T/T_{LU}-T)} \tag{29}$$

where $\Delta/\mu \approx 4(\Delta d/d)$. This result shows very clearly that with rising temperature thermal fluctuations further reduce $I_{cM}(T)$ with respect to the "BCS"-value $I_{cM}(T)=I_{cM}(0)\,[\Lambda(0)/\Lambda(T)]$. After substraction of the background due to random pinning, which was deduced from a flat but otherwise identical reference film, the critical currents $I_{cM}(T)$ of Al1 and Al2 were fitted to equation (29) using $I_{cM}(0)$, Δ/μ and T_{LU}/T_c as fitting parameters. The result of this analysis is shown in Fig. 7 where, for comparison, theoretical curves calculated by neglecting the effect of thermal fluctuations are also shown. Good agreement with equation (29) is found for a reasonable choice of the parameters. T_{LU}/T_c, in fact, scales with $R_{n\square}$ approximately as predicted by equation (22) where, however, the numerical coefficient (0.31) is found to be about an order of magnitude too small to account for the experimental values of Fig. 7. At the present stage of our investigations, however, it is not possible to attribute this discrepancy to an intrinsic weakness of

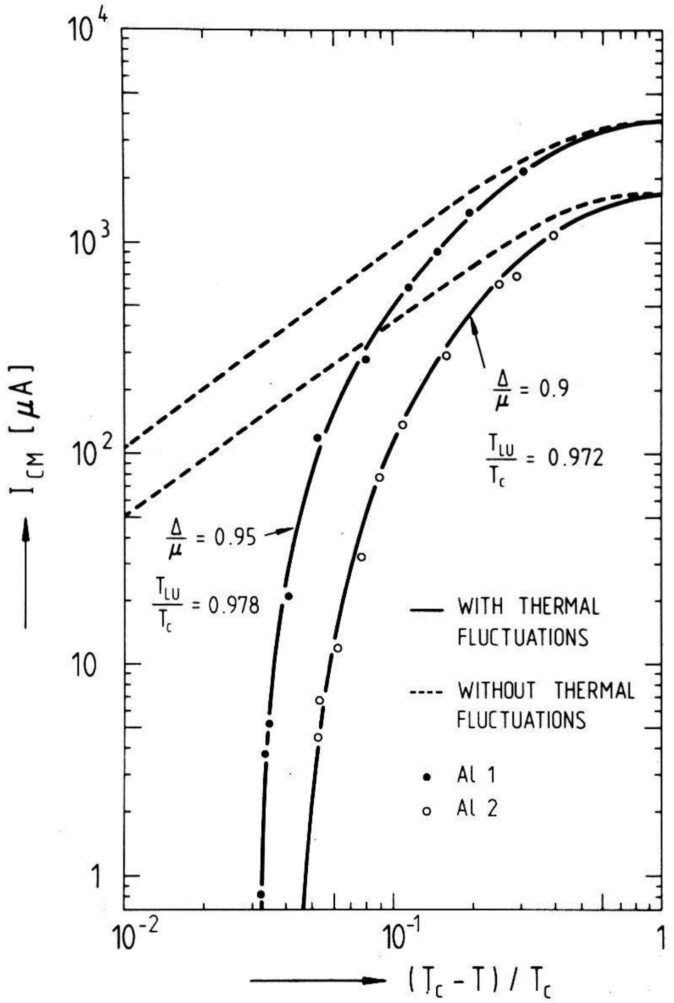

Figure 7
Temperature dependence of the critical currents of thickness modulated films for the fundamental matching configuration $\vec{q}=\vec{g}_1$. Theoretical curves are calculated from equation (29) with (full lines) and without (dashed lines) the effect of thermal fluctuations.

the model discussed in Section II.B. As for Δ/μ, there is good agreement between the values deduced from the fit and those estimated with $\Delta/\mu \approx 4(\Delta d/d)$ using values of $\Delta d/d$ (see Table 1) determined by combined optical and electrical methods. On the basis of these results, we conclude that the concept of a pinning force field renormalized by thermal fluctuations provides a good description of our experiments.

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