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# Confining potential based on local wave equations<sup>1)</sup>

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*Abstract.* We investigate to which extent the recently proposed local relativistic wave equations can account for the nonperturbative part of the  $q\bar{q}$ -interaction. The wave equations lead to linear trajectories and a strong singlet-triplet splitting that survives in the limit of massless quarks as required by chiral symmetry. We discuss the flavour dependence of slopes and intercepts and analyze perturbative *QCD* contributions in ladder approximation. The main features of the spectrum turn out to be stable with respect to these perturbative corrections. Recent data on the mass of  $\eta_c$  indicate that our scheme overestimates the actual value of the singlet-triplet splitting in the  $c\bar{c}$ -system by a factor two. We conclude that a local relativistic potential is unable to account for all nonperturbative effects – sizeable contributions on the level of mass terms are missing.

The perturbative treatment of *QCD* – based on freely propagating quarks as a zero order approximation – is adequate as long as the characteristic distances involved are small in comparison with the scale  $\Lambda^{-1}$  of the strong interaction. For bound states of very heavy quarks of mass  $m$  perturbation theory is a self-consistent approximation scheme because the  $1/r$  potential due to perturbative gluon exchange provides for binding within a small radius  $a = (\frac{1}{2}m^4_3\alpha_s)^{-1}$  where  $\alpha_s = g^2/4\pi$  is the effective coupling strength at this scale.

If the quarks are not sufficiently heavy, a quantitative analysis of the bound state problem requires an improved zero order approximation that includes the main collective effects of the soft gluon cloud surrounding the quarks. Clearly, the splitting of the total Hamiltonian  $H_{QCD} = H_0 + V$  into a zero order term  $H_0$  and a residual interaction  $V$  is to some extent a matter of convenience. Yet, if a perturbative expansion with respect to  $V$  is to make sense then  $H_0$  should account for the basic properties that are believed to characterize *QCD*: freedom at short

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distances, confinement at large distances, spontaneously broken chiral symmetry, Lorentz invariance, a decent qualitative account of the hadron spectrum etc. Some time ago we have proposed a zero order approximation  $H_0$  in the  $q\bar{q}$  sector that does satisfy these requirements [1]. The proposal is based on a very simple input: Locality of the effective long range  $q\bar{q}$  interaction. The purpose of this note is to discuss an interesting picture for the spin- and flavour-dependence of the long range potential that is suggested by this model and to calculate the perturbative *QCD* corrections.

The relevant quantity characterizing the  $q\bar{q}$  bound states is the properly renormalized gauge invariant wave function of a meson state  $|p\rangle$

$$\psi_{ab}^{ij}(x, y) = \langle 0 | T\bar{q}^j(y) \exp -ig \int_x^y dl^\mu B_\mu q_a^i(x) | p \rangle = e^{-ip(x+y)/2} \psi_{ab}^{ij}(z, p) \quad (1)$$

where  $z = x - y$ ;  $a, b$  are Dirac indices,  $ij$  denote fixed quark flavours, a sum over color is understood and the integral is taken along the straight line connecting  $x$  and  $y$ . Perturbation theory derives from an approximation in which  $\psi$  satisfies the pair of free Dirac equations. The interaction modifies the free equations by terms that can be computed order by order in perturbation theory. (The Bethe-Salpeter equation is the product of the two wave equations.) Our modified zero order approximation  $H_0$  is defined by approximating the pair of exact wave equations for  $\psi(x, y)$  by a pair of *local first order differential equations*.

There are at least two reasons to investigate local effective interactions as a starting point rather than nonlocal interactions represented by an integral operator: i) *Initial value problem*. A pair of first order differential equations uniquely determines the wave function  $\psi(x, y)$  all over Minkowski space from given initial data at  $x^0 = y^0 = 0$ . This guarantees that the relative time variable  $z^0 = x^0 - y^0$  does not carry an independent degree of freedom and that the spectrum contains exactly the same states as the nonrelativistic quark model. A pair of covariant integral equations in general does not admit an initial value problem of this type.

ii) *Causality*. If the time evolution is described by local differential equations whose driving terms are the free Dirac operators then the quarks propagate in a causal manner: the wave function at time  $x^0 = y^0 > 0$  depends only on those initial data at  $x^0 = y^0 = 0$  that are in the backward light cone of the points in question. Causality of the underlying field theory does of course not imply that the equations of motion for the two quark wave function are causal in this sense: perturbation theory leads to nonlocal interaction kernels. These nonlocalities reflect the fact that the gluon degrees of freedom are not described explicitly, but are included in the effective  $q\bar{q}$  interaction. It is however crucial that the zero order approximation of perturbation theory, free quarks, is causal; the perturbation expansion of course insures causality on the Fock space of field theory order by order. If we were to start with a zero order approximation to the equations of motion for  $\psi$  that fails to be causal we would immediately be forced to go beyond the  $q\bar{q}$  sector to establish that the scheme does not violate causality on Fock space. We consider it significant that there are local effective interactions that do retain the causality properties of free quark propagation.

The price for having a pair of local first order differential equations as a zero order approximation is amazingly high: In order to be compatible the two

equations have to satisfy a set of integrability conditions, whose solution determines the form of the potential essentially uniquely (a complete list of solutions of the integrability conditions is given in Ref. 3).

In the case of *quarks with the same mass*, the resulting wave equations describe two spin  $\frac{1}{2}$  particles confined by a harmonic force [1]. They read

$$\begin{aligned} & \left\{ \Sigma a^+ + \frac{i}{2} \Delta p + \mu p z P_+ - i \kappa P_- \right\} \psi(z | p) = 0 \\ & \left\{ \Delta a - \frac{i}{2} \Sigma p + i(m - \mu \Delta \Sigma) P_+ \right\} \psi(z | p) = 0 \\ & \Sigma_\mu \psi = \frac{1}{2} (\gamma_\mu \psi + \gamma_5 \psi \gamma_5 \gamma_\mu), \quad \Delta_\mu \psi = \frac{1}{2} (\gamma_\mu \psi - \gamma_5 \psi \gamma_5 \gamma_\mu), \\ & p_\pm \psi = \frac{1}{2} (\psi \pm \gamma_5 \psi \gamma_5), \quad a_\mu = \frac{\partial}{\partial z^\mu} - \lambda z_\mu, \quad a_\mu^+ = -\frac{\partial}{\partial z^\mu} - \lambda z_\mu \end{aligned} \quad (2)$$

where  $a_\mu$ ,  $a_\mu^+$  are the annihilation and creation operators of the relativistic oscillator. The spring constant  $\lambda$  of the oscillator is related to the parameters  $\kappa$  and  $\mu$  by  $\lambda = \kappa \mu$ . The model involves three independent constants  $\kappa$ ,  $m$  and  $\mu$  which should be fixed by the scale  $\Lambda$  of *QCD* and by the quark mass. The free Dirac equations are recovered if one sets  $\mu = 0$  and  $\kappa = m$  (the free limit corresponds to  $\Lambda = 0$ ).

The wave equations (2) have two striking consequences: i) The leading long range part of the interaction (viz. the terms  $\lambda \Sigma z$  and  $\lambda \Delta z$ ) forces all states to lie on linear trajectories with a common slope,  $M^2 = 8\lambda n + 4\kappa m$ , ( $n = 0, 1, \dots$ ). ii) The structure of the constant mass terms in equations (2) leads to a nontrivial spin dependence of the intercepts:  $n = l + s + 2k$ , where  $l$ ,  $s$ ,  $k$  are the orbital angular momenta, the total quark spin and the radial quantum number respectively. In particular, equations (2) lead to a strong hyperfine splitting related to the spring constant  $\lambda$ :

$$M_\rho^2 - M_\pi^2 = 8\lambda_{\bar{u}d} = \frac{1}{2}(M_{p'}^2 - M_\rho^2), \quad M_\psi^2 - M_{\eta_c}^2 = 8\lambda_{\bar{c}c} = \frac{1}{2}(M_{\psi'}^2 - M_\psi^2) \dots \quad (3)$$

and likewise for higher orbital and radial excitations.

The strong hyperfine splitting is intimately related with spontaneously broken chiral symmetry: the parameter  $m$  is the current quark mass that measures the strength of the divergence of the axial current. In the chiral limit  $m \rightarrow 0$ ,  $M_\pi^2 = 4\kappa m \rightarrow 0$  while  $\lambda$ , and, consequently, the  $\rho$  mass stay finite.

To study the dependence of the model parameters  $\kappa$  and  $\mu$  on the scale  $\Lambda$  of *QCD* and on the quark mass we note that in the chiral limit  $m = 0$  the quantities  $\kappa$  and  $\mu$  have some finite values  $\kappa_0$  and  $\mu_0$  related to  $\Lambda$  by yet unknown numerical constants. To increase the quark mass by  $\delta m$  one adds a mass term to the Hamiltonian which amounts to the change  $m \rightarrow m + \delta m$ ,  $\kappa \rightarrow \kappa + \delta m$  (no change in  $\mu$  or  $\lambda$ ). This modification of the parameters is however inconsistent with locality: either  $\mu$  or  $\lambda$  have to be renormalized if one wants to avoid a clash with the integrability condition that requires  $\lambda = \kappa \mu$ . The simplest choice is to keep  $\mu$  fixed (i.e. flavour independent) and to renormalize  $\lambda$ :

$$\kappa = \kappa_0 + m, \quad \mu = \mu_0, \quad \lambda = \mu_0(\kappa_0 + m) \quad (4)$$

Once  $\kappa_0$  and  $\mu_0$  are known this information fixes the value of all parameters of the model as a function of the quark mass which of course remains arbitrary: In

particular the slope  $\alpha' = (8\lambda)^{-1}$  becomes a known function of the quark mass: it decreases with increasing quark mass in agreement with the experimental fact (note that for the oscillator the spacing of the daughters is given by the slope of the leading trajectory)

$$(M_{\rho'}^2 - M_{\rho}^2) : (M_{\psi'}^2 - M_{\psi}^2) : (M_{\gamma'}^2 - M_{\gamma}^2) \simeq 1 : 2 : 6 \quad (5)$$

Neglecting the effects of the residual interactions for the moment we obtain a qualitative test of the relations (4). From  $\lambda_{\bar{c}c} + \lambda_{\bar{u}d} \simeq 2$  one finds  $\kappa_0 + m_{\bar{c}c} \simeq 2(\kappa_0 + m_{\bar{u}d})$ . Since the light quark mass is negligibly small we obtain  $\kappa_0 \simeq m_c = 1.2 \text{ GeV}$ . From the slope we then get  $\mu_0 = 120 \text{ MeV}$  and from  $M_{\pi}^2 = 4\kappa m_{ud}$  one obtains  $m_{ud} = \frac{1}{2}(m_u + m_d) \simeq 4 \text{ MeV}$ , in reasonable agreement with the current estimates in the literature [4]. Once the universal constants  $\mu_0$  and  $\kappa_0$  are determined, one can calculate the mass of the  $b$  quark from the position of  $Y$  and predict the magnitude of the radial splitting  $M_{\gamma'}^2 - M_{\gamma}^2$ . One obtains  $m_b \simeq 4.03 \text{ GeV}$  and  $M_{\gamma'}^2 - M_{\gamma}^2 = 10.04 \text{ GeV}^2$  to be compared with the experimental value 10.83.

The relations (4) imply that the frequency of the oscillator  $\omega = 2\lambda/m$  rather than the spring constant  $\frac{1}{2}m\omega^2$  tends to a universal constant for sufficiently large quark mass. (Note that for  $m \ll \kappa_0$  it is rather  $\lambda$  that will appear universal). Had we instead assumed  $\lambda$  to be flavour independent (necessitating an awkward renormalization of  $\mu$ ) we would have to conclude that the pattern (5) cannot be understood in terms of our zero order approximation. (It may of course be the case that Regge trajectories are in general curved at their low energy end such that the experimental values (5) cannot be taken as indicative of their true slope; the masses of the states  $\psi''$  and  $Y''$  indeed indicate some curvature).

We now turn to the *residual interaction*  $V$ . According to asymptotic freedom the short distance behaviour of  $V$  is determined by the one gluon exchange diagram (note that the oscillator Hamiltonian  $H_0$  exhibits free short distance behaviour). We separate this leading short distance contribution  $V_{sd}$  from  $V$  and write  $V = V_{sd} + R$ . If the zero order approximation  $H_0$  indeed describes the leading long range behaviour of the Hamiltonian, the remainder  $R$  is negligible both at short and at large distances – it behaves like an effective mass term. In the following we neglect  $R$  all together; this amounts to the assumption that the local mass terms in the oscillator wave equations approximately describe the nonperturbative effects of intermediary range.

The first order contribution of the gluon exchange diagram to the mass of the bound state was analyzed in [2]. The corresponding covariant formula can be integrated over relative time and written in the rest frame as

$$M^2 = \frac{4}{3}\alpha_s \int d^3z \phi^+(\vec{z}) V \psi(\vec{z}) \quad (6)$$

where  $\phi(\vec{z})$  is the adjoint of  $\psi(z)$  (see [2]). In the “gauge”  $\gamma = 1$  the gluon exchange potential reads

$$V = V_1 \equiv -\frac{1}{r} + \frac{1}{r^3} z^i z^j \gamma_{00} \gamma_{ij} \quad (7)$$

where  $r = |\vec{z}|$ ,  $\gamma_{\mu\nu}\psi = \gamma_\mu\psi\gamma_\nu$ . For  $\alpha_s = g^2/4\pi = 0.55$ , the first order formula fits the known charmonium triplet states [2]. The strong splitting (3) between the singlet and triplet states however results in a very low mass for  $\eta_c$  (2840 MeV), in

disagreement with the recent experiment [5] (2976 MeV). Also the masses of all other singlet states lie about 250 MeV below the triplet partners.

The relatively large value of  $\alpha_s$  necessitates a check of the stability of this result against higher order contributions. As a further motivation to investigate the exchange of more than one gluon we note that for very heavy quarks the bound states should to a good approximation be described by the ladder graphs – for heavy quarks it is of course not permissible to treat gluon exchange only to first order. A proper treatment of the ladder graphs in the presence of the harmonic long range force should provide us with an interpolation between the pure oscillator ( $\alpha_s = 0$ ) and positronium ( $\lambda = 0$ ).

We have suggested a simple prescription to iterate the gluon exchange graphs in [2]: solve the eigenvalue equation

$$H_{\text{eff}}\psi = E\psi \quad (8)$$

where  $H_{\text{eff}} = H_0 + \frac{4}{3}\alpha_s V$  and  $H_0$  denotes the rest frame Hamiltonian associated with our model (obtained from the wave equations at  $\vec{p} = z^0 = 0$  by eliminating  $\partial/\partial z^0$ ):

$$H_0 = 2i(\Sigma_0 \Sigma_i + \Delta_0 \Delta_i) \partial_i - 2\lambda i(\Sigma_0 \Sigma_i - \Delta_0 \Delta_i) z^i + 2\kappa \Sigma_0 P_- + 2(m + \mu \vec{\Delta} \vec{\Sigma}) \Delta_0 P_+. \quad (9)$$

In fact, this simple recipe does not work: the gluon exchange term is too singular at  $r \rightarrow 0$ . The problem is not particular to our model, but occurs even in the positronium limit ( $\mu \rightarrow 0, \kappa \rightarrow m$ ). For positronium a proper analysis of the *BS* equation [6] also shows how to solve this problem: one has to project the effective Hamiltonian on the positive energy eigenstates of  $H_{\text{free}}$ . Accordingly we write the effective Hamiltonian in the form

$$H_{\text{eff}} = H_0 + \frac{4\alpha_s}{3} \Pi_0 V \Pi_0 \quad (10)$$

where  $\Pi_0$  projects onto the positive energy timelike eigenstates of  $H_0$ . Explicitly,  $\Pi_0$  is given by  $\sum_n (2M_n)^{-1} \psi_n(\vec{z}') \phi_n^+(\vec{z})$ . The eigenvalue equation (8) with  $H_{\text{eff}}$  given by (10) simply requires us to diagonalize the matrix  $\int d^3z \phi_n^+ V \psi_m$  where  $n$  and  $m$  run over a complete set of positive energy time-like solutions of the model. (Note that matrix diagonalization of  $H_{\text{eff}}$  accounts for the spin dependent piece of the gluon exchange force nonperturbatively, in distinction to the standard non-relativistic procedure which treats this piece only to lowest order in  $v^2/c^2$ . For excited states the difference between the two procedures can be substantial [7]. we have worked out the eigenvalues of  $H_{\text{eff}}$  numerically. The results for the spectrum [8] of charmonium and upsilon are given in the table. Column a) contains the results of the first order calculation taken from Ref. 2. (In this column the mean square deviation  $\Delta M$  includes the 8 states  $\eta_c, \psi, \chi_0, \chi_1, \chi_2, \eta'_c, \psi'$  and  $\psi^*$ ). Column b) is based on the Hamiltonian (10) with  $V$  given by equation (7). The parameters  $\lambda, M_0$  and  $\alpha_s$  are fitted to the 6 states  $\psi, \chi_0, \chi_1, \chi_2, \psi'$  and  $\psi^*$  with a mean square deviation  $\Delta M = 23$  MeV. The masses of the singlet states remain low as the multigluon exchange does not change the large singlet-triplet splitting substantially. The spectrum of  $b\bar{b}$  states is shown in column d). The quark mass was adjusted to reproduce  $Y(9460)$ . The values of the model parameters are fixed in accordance with the discussion given above with  $\kappa_0 = m_c$  taken from b).

Charmonium					Upsilon				
	Exp	a)	b)	c)	Exp	d)	e)		
$1^1S_0$	$\eta_c$	2976	2840	2832	2833		9.20	9.20	
$1^3S_1$	$\psi$	3097	3105	3127	3133	Y	9.46	9.46	
$1^1P_1$			3272	3270	3271		9.61	9.60	
$1^3P_0$	$\chi_0$	3413	3399	3381	3382		9.80	9.79	
$1^3P_1$	$\chi_1$	3508	3502	3497	3497		9.84	9.82	
$1^3P_2$	$\chi_2$	3554	3534	3532	3532		9.85	9.84	
$2^1S_0$	$\eta'_c$		3430	3412	3415		9.77	9.76	
$1^1D_2$			3608	3586	3584		9.90	9.89	
$2^3S_1$	$\psi'$	3686	3704	3707	3707	Y'	10.018	10.02	10.00
$1^3D_1$	$\psi^*$	3773	3795	3782	3779		10.12	10.10	
$3^3S_1$	$\psi''$	4030		4202	4197	Y''	10.41	10.52	10.48
$2^3D_1$	$\psi^{**}$			4256	4249		10.59	10.55	
$4^3S_1$	$\psi'''$			4645	4636		10.98	10.93	
$3^3D_1$	$\psi^{***}$			4689	4676		11.04	10.99	
$\lambda[\text{GeV}^2]$		.267	.252	.248 ± .020		.616	.593		
$M_0[\text{GeV}]$		3.27	3.19	3.182 ± .090		9.582	9.57		
$m[\text{GeV}]$		1.16	1.13	1.13 ± .04		4.26	4.25		
$\alpha_s$		.55	.39	.36 ± .06		.26	.25		
$\Delta M[\text{MeV}]$		17	23	24					

Numerical results for the spectrum of  $c\bar{c}$  and  $b\bar{b}$  bound states.

a) First order formula. b), d) Spectrum of the Hamiltonian (10) with  $V$  given by (7). c), e) Same as in b) and d) respectively, however with  $V$  given by (13).

The values of the strong coupling constant for the  $b\bar{b}$  system may be scaled up either by renormalizing with the factor  $\ln M_\psi^2/M_Y^2$  or with  $\ln \lambda_c/\lambda_b$ . In the first case we get  $\alpha_s = .26$  (which is the value which we used in column d), in the second case we instead have  $\alpha_s = .32$ . The prediction for  $Y' - Y$  is stable within 40 MeV.

The treatment of the residual interaction given above has the following shortcomings:

i) The matrix  $\int d^3z \phi_n^+ H_{\text{eff}} \psi_m$  is hermitean with respect to a positive norm only if the quarks are sufficiently heavy ( $m > 2\mu$ ). A suitable norm is given by  $\phi = N\psi$  with

$$N\psi^{\text{sing}} = \left( P_- + \frac{m}{\kappa} P_+ \right) \psi^{\text{sing}}; \quad N\psi^{\text{trip}} = \left( P_- + \frac{m-2\mu}{\kappa} P_+ \right) \psi^{\text{trip}} \quad (11)$$

For light quarks ( $m < 2\mu$ ), in particular, in the chiral limit  $m \rightarrow 0$ , it is inconsistent to assume that the residual interaction is dominated by the leading short distance contribution (7). A proper solution of this problem presumably requires an analysis of quark self energy effects which are ignored here.

ii) In the limit of very heavy quarks the effective Hamiltonian does not fully reproduce the instantaneous limit of positronium theory, described by the Fermi-Breit potential

$$V_2 = -\frac{1}{r} + \frac{1}{2r^3} (r^2 \delta^{ij} + z^i z^j) \gamma_{00} \gamma_{ij} \quad (12)$$

It is however not difficult to cure this defect by adding a suitable term to the gluon exchange potential,  $V = V_1 + \Delta V$ . We have investigated two solutions to this problem. a) The simplest prescription is to take  $V = V_2$  instead of  $V = V_1$ . We have shown in [2] that this modification has only a very small effect on the first order mass formula – the same applies to the higher order corrections. b) Alternatively, as advocated in [2] one may insist that the first order mass formula is given by the diagonal matrix elements of  $V_1$  i.e. that the diagonal matrix elements of  $\Delta V$  vanish. This condition by no means contradicts the requirement that  $V_1 + \Delta V$  reduces to  $V_2$  at short distances: the wave equations imply that the diagonal matrix elements of  $V_1 - V_2$  are the same as those of the “mass term”  $i\mu\gamma_{00}\Sigma_i z^i P_+/r$ . This guarantees that the potential

$$V = V_2 + \left\{ \frac{i\mu}{2r} \gamma_{00} \Sigma_i z^i P_+ + \text{h.c.} \right\} \quad (13)$$

(hermite conjugation with respect to the norm  $N$ ) not only leads to the same first order mass formula as  $V_1$  but also has the same leading short distance behaviour as  $V_2$ . Columns c) and e) contain the numerical results for the eigenvalues of the effective Hamiltonian corresponding to the potential (13). The eigenvalues are barely affected; the main effect of the modification is to renormalize the parameters  $M_0$ ,  $\lambda$  and  $\alpha_s$ . The errors quoted on the values of  $\lambda$ ,  $M_0$ ,  $\alpha_s$  are an educated guess of the stability of the fit. (The errors amount to allowing  $\Delta M$  to increase by 3 MeV.)

We conclude with the following remarks:

- i) Within a covariant framework the oscillator equations define the *simplest* (because local) zero order approximation to the effective  $q\bar{q}$  interaction. If the leading long range force should turn out not to be harmonic the idea of a local zero order approximation would have to be abandoned.
- ii) In this paper we have investigated the hypothesis that the wave equations not only describe the leading long range behaviour of the confining potential, but account for all nonperturbative effects involving the scale of the strong interactions, including mass terms. In this case the residual interaction  $V$  is fixed by asymptotic freedom and we are able to calculate the mass spectrum in terms of  $\alpha_s$  and the parameters involved in the wave equations. We obtain a decent description of the splittings within the triplet states of charmonium. The mass values are stable with respect to higher orders in  $\alpha_s$  within about 40 MeV.
- iii) The large singlet-triplet splitting is contradicted by a recent experiment [5]. From a small splitting we conclude that the wave equations do not properly describe the nonperturbative effects on the level of mass terms. Even if such terms could of course be tolerated in the residual interaction  $V$  we see no simple way to pin them down; within a covariant framework a small hyperfine splitting requires a more complicated structure of the interaction violating the strong locality requirement underlying our model.
- iv) there are two theoretical arguments for the presence of mass terms in  $V$ :  
 a) It is not possible to introduce *mass asymmetries* into  $H_0$  without spoiling its locality. If one insists on a local zero order approximation, the flavour asymmetries caused by *QFD* via quark masses have to be incorporated into the residual interaction.  
 b) Our treatment of the residual interaction spoils the *chiral symmetry* of the wave equations (see remarks above). On the phenomenological side, the

fact that our description of the  $c\bar{c}$  and  $b\bar{b}$  spectra does not seem to correctly reproduce the *high radial excitations* ( $\psi''$ ,  $Y''$ ) may also indicate that we are missing an important contribution to the residual interaction.

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