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Influence of the pressure and viscosity effects on the magneto-acoustic oscillations in bounded plasmas

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Abstract. Radial magneto-acoustic oscillations in cylindrical plasmas bounded by a solid wall are investigated accounting for the viscous forces and for the pressure gradients in the MHD-equations. It is shown that both these effects representing strong singular perturbation at a solid boundary can play an important role in theoretical analysis of the plasma diagnostic problems. (They allow, for instance, to explain discrepances, found in [3], between experimental and theoretical values of the transverse plasma conductivities). It is shown as well that the pressure and viscous effects can influence greatly the impedance of a cylindrical plasma column bounded by a solid wall and, thus, their proper evaluation could be of interest also in connection with problems of the plasma heating.

1. Introduction

As it was shown in [1], the presence of a solid wall S surrounding a plasma can strongly influence magneto-acoustic oscillations (MAO) excited there since nearby the wall the plasma velocity $\vec{v}$ drops rapidly to zero (boundary condition $\vec{v}(S)=0$!) and, as a consequence, the viscous force proportional to a second derivative of $\vec{v}$ over a distance counted normally to S, increases rapidly.

Another effect of this kind, which was not accounted for in [1], is due to the fact that the pressure gradients in the MHD-equations (usually neglected in case $p \ll B_0^2/8\pi$, B_0 is an external magnetic field) can be, nevertheless, important, for the same reason, nearby the wall. It is indeed, well known (see below), that the pressure gradients are expressed through $\vec{v}$ (via the continuity equation) as follows:

$$-\nabla p = \frac{i\rho c_s^2}{\omega} \operatorname{graddiv} \vec{v}$$

(c_s is the sound velocity, ρ is the mass density, ω is the oscillation frequency) and, hence, at the wall the pressure gradient ∇p, similarly to the viscous force, increases rapidly.

From a purely mathematical standpoint, the neglect of the viscous forces and of the pressure gradients in the MHD-equations represents itself, in case of plasmas bounded by a solid wall, a singular perturbation in a sense, that it leads to the lowering of the order of the differential equations considered. (In case of radial MAO in a cylindrical plasma column this reduction is equivalent to the replacement of a fourth order differential equation by a second order equation). As a result, the "reduced" equations obtained in this way have solutions which do not satisfy the constraints to be imposed on the plasma mass velocity $\vec{v}$ at the wall S (e.g. $\vec{v}(S)=0$). (See a general mathematical discussion of such problems in [2]). Thus, a solid wall must be considered as a strong singular perturbation, which manifests itself in the appearance of the "boundary layer" adjoining to S, where the distributions of the physical quantities pertaining to MAO can differ drastically from those resulting from the reduced equations obtained by neglecting the pressure and viscous effects. What is more important, these effects can influence greatly, as it was found in [1], even the "bulk" plasma parameters, e.g. the impedance of the plasma column or the ratio of the magnetic fields at its axis and at the boundary.

In this paper the theory developed in [1] is modified by inclusion of the pressure gradients in the MHD-equations of the one-fluid approximation. This generalization, developed in the cylindrical case (Section 3) both for homogeneous and non-homogeneous plasmas, proves to be especially important for the fully ionised plasmas when the viscosity coefficients are relatively small and when the pressure effects play a predominant role. Application of general formulas obtained in this way to plasma diagnostic problems studied in [3, 4] allows to eliminate some discrepances between experimental and theoretical results (calculated from the reduced MHD-equations), found in these papers (Section 4). Besides, as it is shown in Section 5, these boundary effects can influence greatly the impedance of a plasma column and, hence, their proper evaluation could be of interest also in problems of supplementary heating of the fusion plasmas.

It must be indicated here, that the influence of the pressure gradients was considered in an earlier paper [5] devoted to an analysis of MAO in a current carrying plasma column. On the other hand, in [6] the same problem was studied, accounting for the viscous forces and with the pressure gradients neglected. However, in both these papers some additional assumptions were made (transition from a system of equations (9)–(14) to a second order equation (18) in [5] or transition from a system of equation (10), (12) to a second order equation (29) in [6]), which are equivalent to disregarding the singular nature of the perturbation caused by a solid boundary. Thus, results found in [5], [6] are not always applicable to MAO in plasmas bounded by a solid wall.

2. Basic equations

We will consider MAO in quasineutral, monoatomic plasmas consisting of electrons, ions A^+ (mass m_i) and neutral atoms A. The temperatures of the electrons, T_e, and of the heavy particles, T_i, will be assumed to be constant, but the densities of charged, n, and of neutral, n_a, particles will be in general characterised by some non-homogeneous distributions $n(\vec{r})$, $n_a(\vec{r})$. Basic linearised MHD-equations of the one-fluid approximation can be written in this case as follows

$$-i\rho\omega\vec{v} = -\nabla p + \frac{1}{c}[\vec{j}\vec{B}_0] + \vec{f} \tag{1}$$

$$\vec{j} = \sigma\cdot\left(\vec{E} + \frac{1}{c}[\vec{v}\vec{B}_0] + \frac{1}{ne}\nabla p_e\right) \tag{2}$$

where $\vec{f} = \eta(\Delta\vec{v} + \frac{1}{3}\nabla \operatorname{div} \vec{v})$ is the viscous force, η is a viscosity coefficient and σ is a conductivity tensor for the cold plasma. The pressure p in (1) is a sum of partial pressures p_e, p_i, p_a of electrons, ions and neutrals. Assuming for all of them the adiabatic law $p_\alpha \sim n_\alpha^{\gamma}\alpha$ i.e.

$$\frac{\nabla p_\alpha}{p_\alpha} = \gamma_\alpha \frac{\nabla n_\alpha}{n_\alpha} \quad \text{or, since} \quad p_\alpha = n_\alpha T_\alpha, \qquad \nabla p_\alpha = \gamma_\alpha T_\alpha \nabla n_\alpha \tag{3}$$

(n_α is a particle density, index α stands for electrons e, ions i, or atoms a, the temperature T_α is expressed in energetic units) we get from the continuity equation $i\omega n_\alpha = \operatorname{div}(n_{\alpha 0}\vec{v}_\alpha)$ a relation $-\nabla p_\alpha = (i\gamma_\alpha T_\alpha/\omega)\nabla \operatorname{div}(n_{\alpha 0}\vec{v}_\alpha)$ where $n_{\alpha 0}$ are the unperturbed particle densities. As the electron velocity v_e can be represented in the form $\vec{v}_e = \vec{v} - (\vec{j}/en)$ where the current density $\vec{j}$ satisfies the condition of quasineutrality $\operatorname{div} \vec{j} = 0$, one can represent ∇p_e by the formula

$$-\nabla p_e = \frac{i\gamma_e T_e}{\omega} \nabla \operatorname{div}(n\vec{v}). \tag{4}$$

The expression for ∇p_a depends upon the relation between the ion-atom collision frequency ν_{in} and the oscillation frequency ω. Assuming $\nu_{\text{in}} \gg \omega$ one can take $\vec{v}_i = \vec{v}_a = \vec{v}$ (atoms A are completely carried away by the ions A^+) and we obtain then $-\nabla p_a = (i\gamma_a T_i/\omega)\nabla \operatorname{div}(n_{a0}\vec{v})$. Using also an expression $-\nabla p_i = (i\gamma_i T_i/\omega)\nabla \operatorname{div}(n\vec{v})$, introducing the mass density $\rho = m_i(n + n_a)$ and an effective sound velocity $c_s = [(\gamma/m_i)(T_i + T_e x)]^{1/2}$, $x = n/(n + n_a)$ (for $\gamma_e = \gamma_a = \gamma_i = \gamma$) we have finally

$$-\nabla p = -\nabla(p_e + p_i + p_a) = \frac{i}{\omega} \nabla c_s^2 \operatorname{div}(\rho\vec{v}). \tag{5}$$

On the other hand, for $\nu_{\text{in}} \ll \omega$ the neutrals are not carried away by the ions (see e.g. a discussion of the problem in [1]) and we can take now $\vec{v}_a \ll \vec{v}_i$ or $p_a \ll p_i$. We find accordingly in this case

$$-\nabla p \approx -\nabla(p_e + p_1) = \frac{i}{\omega} \nabla c_s^2 \operatorname{div}(\rho\vec{v}), \tag{6}$$

where

$$\rho = m_i n, \qquad c_s = \left[\frac{\gamma}{m_i}(T_e + T_i)\right]^{1/2}.$$

Formulas (4), (6) are valid also for a fully ionised plasma.

For homogeneous plasmas formulas (5), (6) can be evidently simplified as follows:

$$-\nabla p = i\frac{\rho c_s^2}{\omega} \nabla \operatorname{div} \vec{v}.$$

Equations (1)–(3), together with definitions (4)–(6), can be reduced to a set of two equations with mass velocity $\vec{v}$ and electric vector $\vec{E}$ considered to be the unknown quantities. This set of equations forms a closed system provided some boundary conditions for $\vec{v}$ and $\vec{E}$ at the boundary S are specified.

3. Radial MAO

If MAO are excited by a long coil in a long cylinder of radius a filled with a plasma whose densities n, n_a depend upon the radius r counted from the cylinder axis, one can assume that all the vectors $\vec{E}$, $\vec{H}$, $\vec{v}$, $\vec{j}$ are functions of the radius r only. In this case the non-zero components of these vectors are: azimuthal component E_ϕ, axial component H_Z, radial component v_r, azimuthal component j_ϕ. The basic equations (1)–(3) can be now reduced to a system

$$-i\rho\omega v_r = \frac{B_0\sigma_\perp}{c}\left(E_\phi - \frac{1}{c}v_rB_0\right) + \frac{4}{3}\eta\Delta_1 v_r + \frac{\partial}{\partial r}\frac{\partial \zeta r v_r}{r\,\partial r} \tag{7}$$

$$\Delta_1 E_\phi = \frac{4\pi i\omega\sigma_\perp}{c^2}\left(E_\phi - \frac{1}{c}v_rB_0\right), \qquad \Delta_1 = \frac{\partial}{\partial r}\frac{1}{r}\frac{\partial}{\partial r}r \tag{8}$$

$$\frac{1}{r}\frac{\partial}{\partial r}rE_\phi = i\frac{\omega}{c}H_Z \tag{9}$$

where $\zeta = i\rho c_s^2/\omega$ and $\sigma_\perp$ is the transverse component of the conductivity tensor σ. The closed system (7)–(8) is to be implemented by the boundary conditions

$$v_r(a) = 0, \qquad \left.\frac{1}{r}\frac{\partial}{\partial r}rE_\phi\right|_a = i\frac{\omega}{c}H_z(a) = i\frac{\omega}{c}H_{ex} \tag{10}$$

where H_{ex} is the field generated by the coil. (We have made use in (7) of the fact that the viscosity coefficient η depends only slightly upon the densities n, n_a and hence it can be considered to a good approximation as a constant quantity). The quantity $\zeta = (i\rho c_s^2/\omega)$ in (7) plays formally the role of a second viscosity coefficient ζ. The formulas (18)–(21) derived in [1] for homogeneous plasmas characterised by two viscosity coefficients η, ζ can be therefore used immediately as solutions of system (7)–(8) in case $n = \text{const}$, $n_a = \text{const}$. We rewrite here the expression (20) of [1], which corresponds to the assumptions $|\varepsilon_1| \ll 1$,

$$|\varepsilon_2| \ll |\varepsilon_1|, \qquad \left(\varepsilon_1 = \frac{\omega\nu_m}{c_A^2}, \qquad \varepsilon_2 = \frac{\omega\nu_h}{c_A^2},\right.$$

$$\left.\nu_m = \frac{c^2}{4\pi\sigma_\perp}, \qquad \nu_h = \left(\zeta + \frac{4}{3}\eta\right)\Big/\rho, \qquad c_A = \frac{B_0}{\sqrt{4\pi\rho}}\right):$$

$$N = \left|\frac{H_Z(0)}{H_{ex}}\right| = \frac{1}{|J_0(\kappa) - i\lambda J_1(\kappa)|}, \qquad \kappa = \frac{\omega a}{c_A}\left(1 + \frac{1}{2}i\varepsilon_1\right),$$

$$\lambda = \sqrt{\frac{\nu_h}{\nu_m}}, \tag{11}$$

J_0, J_1 are the Bessel functions. Near the first magneto-acoustic resonance (MAR) $\omega \approx \omega_0 = (q_0c_A/a)$, $q_0 = 2.4$, $J_0(q_0) = 0$, (11) reduces to (see the calculations leading to the formula (20) in [1]):

$$N = \frac{0.8}{\left(\left(\frac{\omega - \omega_0'}{\omega_0}\right)^2 + \gamma^2\right)^{1/2}}, \qquad \omega_0' = \omega_0\left(1 + \frac{1}{q_0}\,Im\lambda\right),$$

$$\gamma = \mathrm{Re}\left(\frac{q_0\nu_m}{2c_Aa} + \frac{\lambda}{q_0}\right). \tag{12}$$

The maximal value of N corresponding to $\omega = \omega_0'$ is given according to (12) by

$$N_0 = \frac{0.8}{\mathrm{Re}\left(\dfrac{q_0 \nu_m}{2c_A a} + \dfrac{\lambda}{q_0}\right)}. \tag{13}$$

Expressions (11)–(13) can be readily generalised to the case $v_{\mathrm{in}} \gg \omega$ (i.e. with the mass density ρ defined as $\rho = m_i(n + n_a)$) for non-homogeneous plasmas. For this aim we reduce system (7)–(8), introducing the dimensionless variables $v = v_r/c$, $E = E_\phi/B_0$, $r \to (r/a)$ and the functions $\mu = \delta/a$, $\delta = (\sqrt{\nu_h \nu_m}/c_A)$, $q = \omega a/c_A$, $f = (d\zeta/dr)/(\zeta + \frac{4}{3}\eta)$, to the form

$$-\mu^2\left(\Delta_1 v + f2\frac{dv}{dr}\right) = E - (1 - i\varepsilon_1)v \tag{14}$$

$$\varepsilon_1 \Delta_1 E = -iq^2(E - v). \tag{15}$$

The quantity μ can always be considered as small under conditions used in plasma diagnostic experiments, as shown by the estimations in [1]. We can therefore treat the left side in (14) as a small perturbation everywhere, except in the immediate vicinity of the wall S. Dropping the left side in (14), we get the reduced equations corresponding to the neglect of the pressure and viscous effects: $v = E/(1 - i\varepsilon_1)$ and

$$\Delta_1 E + \frac{q^2}{1 - i\varepsilon_1} E = 0. \tag{15}$$

(15) allows a solution

$$E = Ae(r) \tag{16}$$

where the function $e(r)$ is regular at $r = 0$, A is a constant. (For a homogeneous plasma $e = J_1(\kappa r)$ with κ defined in (11)).

On the other hand, at the boundary $r = 1$ the coefficients in (15), (14) can be replaced by the constant values corresponding to $r = 1$: $\mu_0 = \mu(1)$, $\varepsilon_{10} = \varepsilon_1(1)$, $f_0 = f(1)$, $q_0 = q(1)$. Performing further a stretching transformation $x = (1 - r)/\mu_0$ we reduce (15), (14) for $r \to 1$ to the system:

$$-\frac{d^2 v}{dx^2} + \mu_0 \frac{2dv}{dx} = E - (1 - i\varepsilon_{10})v \tag{17}$$

$$\varepsilon_{10}\frac{d^2 E}{dx^2} = -iq_0^2 \mu_0^2 (E - v).$$

Dropping in (17) the terms proportional to small parameters μ_0, μ_0^2 we get the boundary layer equation

$$\frac{d^2}{dx^2}\left(\frac{d^2 v}{dx^2} - (1 - i\varepsilon_{10})v\right) = 0 \tag{18}$$

having a solution $v = B + C \exp(-\sqrt{1 - i\varepsilon_{10}}\, x)$. This means that

$$E = -\frac{d^2 v}{dx^2} + (1 - i\varepsilon_{10})v = D + C' \exp(-\sqrt{1 - i\varepsilon_{10}\, x}) \tag{19}$$

(B, C, D, C' are some constants)

A solution, which coincides with (19) nearby the boundary and with (16) elsewhere can be evidently constructed as follows

$$E = A'e(r) + C' \exp\left(-\sqrt{1-i\varepsilon_{10}}\,\frac{1-r}{\mu_0}\right) \tag{20}$$

(A', C' are some constants).

It must satisfy the boundary conditions at $r=1$ (following from (9), (17) and from the requirements $v(1)=0$, $H_Z(a)=H_{\text{ex}}$):

$$\frac{1}{r}\frac{\partial}{\partial r} rE\bigg|_1 = i\frac{\omega}{c}a\frac{H_{\text{ex}}}{B_0}, \qquad \Delta_1 E = -\frac{iq_0^2}{\varepsilon_{10}}E. \tag{21}$$

Inserting (20) into (21), we get two equations defining two constants A', C' (in the case $|\varepsilon_{10}| \ll 1$):

$$-A'q_0^2 e(1) + C'\frac{1}{\mu_0^2} = -\frac{iq_0^2}{\varepsilon_{10}}(A'e(1)+C') \tag{22}$$

$$A'h(1) + C'\frac{1}{\mu_0} = i\frac{\omega}{c}a\frac{H_{\text{ex}}}{B_0},$$

where $h(r) = (1/r)(d(re)/dr)$ (i.e. $h = \kappa J_0(\kappa r)$ for a homogeneous plasma with κ defined in (11)). As follows from (22), the coefficient A' is given, within our approximations ($|\varepsilon_1| \ll 1$, $|\varepsilon_2| \ll |\varepsilon_1|$, by the formula

$$A' = \frac{(i\omega c_A a H_{\text{ex}}/cB_0)}{h(1) - i\lambda_0 e(1) q_0}, \qquad \lambda_0 = \sqrt{\frac{\nu_h(1)}{\nu_m(1)}}. \tag{23}$$

This means that the magnetic field distribution outside of the boundary layer can be defined as

$$H_Z = \left(B_0 A' \frac{1}{r}\frac{dre}{dr}\Big/ i\frac{\omega}{c}\right) = \frac{H_{\text{ex}}}{h(1) - i\lambda_0 e(1) q_0} h(r) \tag{24}$$

The result (24) can be also interpreted in another way. One can state, that the second order equation

$$\frac{1}{r}\frac{d}{dr} r\frac{dH_Z}{dr} + \left(\frac{\omega}{c_A}\right)^2 \frac{1}{1-i\varepsilon_1} H_Z = 0 \tag{25}$$

employed usually in plasma diagnostic research (see e.g. [3, 4]) must be implemented by a modified boundary condition

$$H_Z(a) = \frac{H_{\text{ex}}}{1 - i\lambda_0 e(1) q_0 / h(1)} \tag{26}$$

which is to be used instead of $H_Z(a) = H_{\text{ex}}$ and which accounts for the pressure and viscuous effects, neglected altogether in (25).

4. Comparison with experiment

The measurements of transverse conductivities $\sigma_\perp$ performed in [3] for the relatively dense, fully ionised and, to a good approximation, homogeneous Argon plasma revealed some systematic deviations from the theory [7] (see Fig. 18, 17 in [3] or Fig. 1 in this paper). Though these deviations lie within the limits of the experimental errors in [3], it seems desirable to try to explain them, owing to their systematic character, by some physical effect not accounted for in [3].

Under conditions used in [3] the viscous effects are negligible, but the pressure effects can lead, according to the present theory, to appreciable corrections. (The parameter $|\lambda|$ in (11), (12) is equal to about 0, 1 under conditions $T = 1.5$ eV, $n = 10^{16}$ cm^{-3}, $B = 10^4$ G typical for the experiments [3]). According to (13) the value of $\sigma_\perp^0$, as defined by the experimental ratio N_0 for the first MAR, is given by

$$\nu_m^0 = \frac{1.6}{q_0}\frac{c_A a}{N_0} \quad \text{or} \quad \sigma_\perp^0 = N_0 \frac{3c^2}{8c_A a\pi} \tag{27}$$

if the pressure and viscous effects are neglected ($\lambda = 0$). On the other hand, as follows from (13) in the low-frequency case investigated in [3] (ν_m is real) $\sigma_\perp$ is defined by

$$\sigma_\perp = \sigma_\perp^0 \frac{1}{1 - 1.25\, N_0 \sqrt{\dfrac{\gamma T}{\omega \nu_m^0 m_i}}} = \sigma_\perp^0 \frac{1}{1 - 0.068 \dfrac{N_0^{3/2} T_{\text{ev}}^{\;1/2}}{f}} \tag{28}$$

(we are using here and in the following the assumptions $T_e = T_i = T$, $\gamma = \frac{5}{3}$ and the relations: (27), $a = 4.65$ cm,

$$\operatorname{Re} \lambda = \sqrt{\frac{c_s^2}{2\omega\nu_m^0}} = \sqrt{\frac{\gamma T}{\omega m_i \nu_m^0}}, \qquad f = \frac{\omega}{2\pi}\Big/ 10^5 \Big).$$

Thus, the pressure effects lead to values of $\sigma_\perp$ which are larger than those following from (27) by the factor $1/(1 - 0.068\,(N_0^{3/2} T^{1/2}/f)$. Presence of this factor allows to explain the fact that the values of $\sigma_\perp$ found in [3] appear to be systematically lower than those given by the theory [7] (see Fig. 1). The pressure effects, as described by the formula (11), explain as well the overall behaviour of the experimental curves $N(\omega)$ in [3] (Fig. 2), without using the fitted $\sigma_\perp$ values following from the formula $N = 1/|J_0(\kappa)|$, as was done in [3]. The pressure and viscous effects also explain the results in [4] where a systematic reduction of the ratios $N(P_0)$ measured as a function of the initial Argon pressure P_0 was found. As follows from the formula (11), with viscosity coefficient of the partially ionised Argon defined by the formula (24) in [1] and with sound velocity defined by $c_s = \sqrt{(\gamma T/m_i)(1 + x)}$ (we take here again $T_i = T_e = T$, $\gamma = \frac{5}{3}$), the theory accounting both for the pressure and viscous effects leads to much better agreement with experiment [4] than the theory disregarding these effects (Fig. 3).

These examples seem to show convincingly that the proper evaluation of the pressure and viscous effects, in the framework of the theory developed above, can be often necessary for the analysis of the plasma diagnostic experiments performed with the help of MAO in plasmas bounded by a solid wall.

6. Influence of the boundary effects on the impedance of the plasma column

The average energy W_a (per cm^2 and sec) absorbed in a plasma column (of radius a), heated by a HF-field of amplitude $H_Z(a) = H_{\text{ex}}$ can be represented by

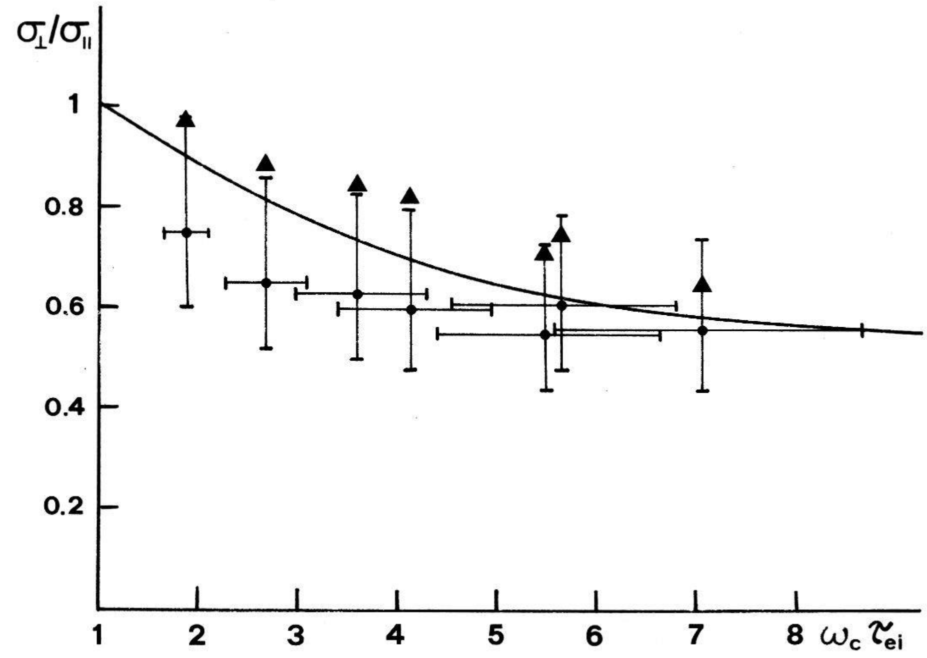

Figure 1

Values ▲ of the transverse conductivity (referred to $\sigma_\parallel = 1.95(n_2/m_e)$, $\sigma_\perp/\sigma_\parallel$, as calculated from formula (28) with ratios N_0 taken from the Table II in [3]. The temperatures, measured in [3] spectroscopically, were taken from the Table III in [3]. Here τ_{ei} is the electron-ion collision time and $\omega_c = eB_0/m_e c$. Full line corresponds to a theoretical formula for $\sigma_\perp$ derived in [7] (see also formula (6) in [3]). ● are the experimental points from [3]. Experimental errors indicated in Fig. 1 stem mainly from the uncertainty of the spectroscopically measured temperatures. (The same uncertainty affect, of course, also the points ▲ found with the formula (28)).

the formula

$$W_a = \frac{c}{8\pi} \operatorname{Re} E_\phi(a) H_{\text{ex}}^* = W \operatorname{Re} Z, \tag{29}$$

where $W = (c/8\pi)\,|H_{\text{ex}}|^2$ is the total energy (per cm^2 and sec) fed into the plasma and $Z = (E_\phi(a))/(H_{\text{ex}})$ is a complex impedance. Restricting ourselves to homogeneous plasmas satisfying the conditions $|\varepsilon_1| \ll 1$, $|\varepsilon_2| \ll \varepsilon_1$ for the low-frequency case ($\sigma_\perp$ is real) and using an expression for $E_\phi(a)$ derived in [1] just under these conditions

$$E_\phi(a) = iH_{\text{ex}} \frac{c_A}{c} \frac{J_1(\kappa)}{J_0(\kappa) - i\lambda J_1(\kappa)} \tag{30}$$

(κ and λ are defined in (11)),

we arrive at an explicit formula for Re Z

$$\operatorname{Re} Z = \frac{c_A}{c}\phi, \qquad \phi = \operatorname{Re} \frac{iJ_1(\kappa)}{J_0(\kappa) - i\lambda J_1(\kappa)}. \tag{31}$$

Nearby the first MAR (31) simplifies similarily to (12) and reads as

$$\operatorname{Re} Z = \frac{c_A}{c}\phi, \qquad \phi = \frac{1}{q_0} \frac{\gamma}{\left(\dfrac{\omega - \omega_0^1}{\omega_0}\right)^2 + \gamma^2} \tag{32}$$

(ω_0^1 and γ are defined in (12)).

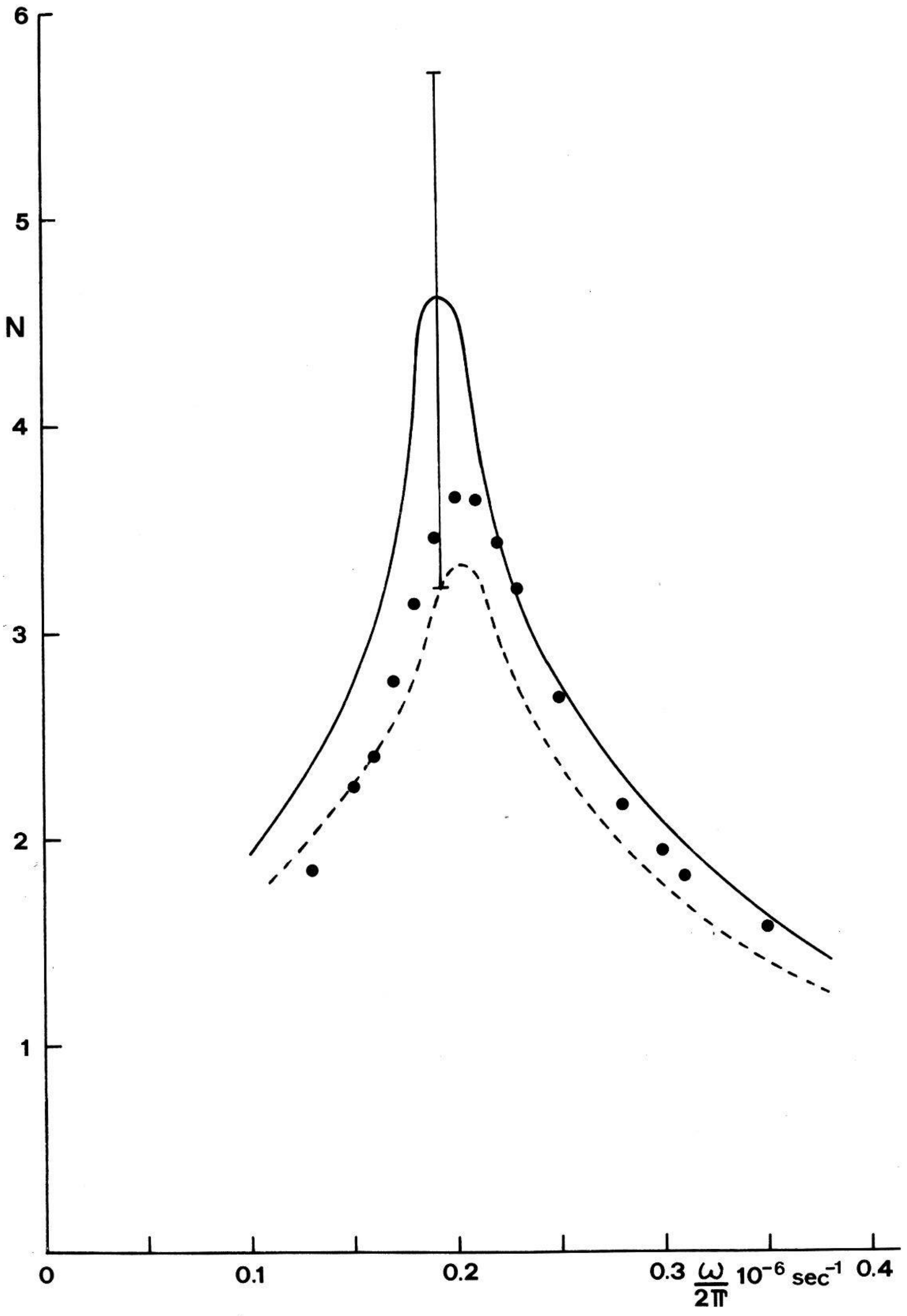

Figure 2
Distribution $N(\omega)$ for Argon plasma 2 in [3] found from the formula $N = 1/|J_0(\kappa)|$ used in [3] (full line) and from the formula (11) (dotted line). (Spectroscopical temperature $T = 1.54$ eV, $n = 6.2 \cdot 10^{15}$ cm^{-3}, $B_0 = 5.4$ kG, radius $a = 4.65$ cm). The calculations were performed with the value of the transverse conductivity $\sigma_\perp = 4.4 \cdot 10^{13}$ cm^{-1} found with these parameters from the theoretical formula (6) in [3]. ● are the experimental points from [3]. The vertical bar at $(\omega/2\pi) = 1.9 \cdot 10^5$ sec^{-1} indicates the error due to uncertainty of temperature.

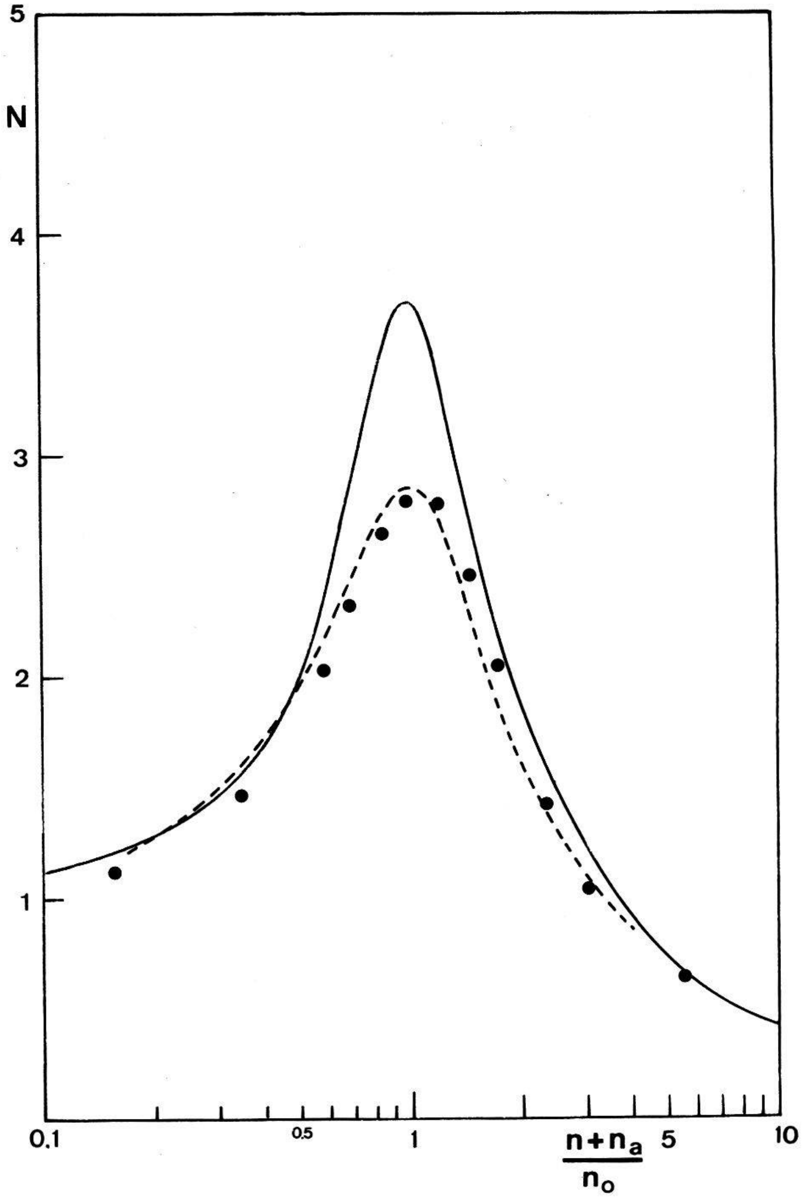

Figure 3
Ratio $N = (H_Z(0)/H_{ex})|$ as a function of the normalised particle density $(n+n_a)/n_0$ (density n_0 is defined by the equation $(\omega a/c_A) = 2.4$, $c_A = (B_0/\sqrt{2\pi\rho})$ for $a = 4.65$ cm, $T = 1.6$ eV, degree of ionisation of the Argon plasma $x = 0.64$, $(\omega/2\pi) = 0.3$ MHz, $B_0 = 5.65$ kG (see Fig. 12b in [4]). Full line was calculated with the formula $N = 1/|J_0(\kappa)|$ used in [4], dotted line was calculated from (11) with viscosity coefficient $\eta = 2 \cdot 10^{-3}$ Poise. ● are the experimental points from [4].

As follows from (32), the maximal value of W_a corresponding to $\omega = \omega_0^1$ is reduced under the influence of the viscous and pressure effects by the factor

$$\frac{q_0}{2}\frac{\nu_m}{c_A a}\bigg/\left(\frac{q_0}{2}\frac{\nu_m}{c_A a}+\frac{\mathrm{Re}\lambda}{q_0}\right).$$

This is a consequence of the reduction of the magnetic and electric fields in the bulk of the plasma by the same factor (see the formulas (25), (27) in [1]).

An example of calculation of W_a with (32) for a fully ionised homogeneous Hydrogen plasma under conditions similar to those used in [8] (where the calculations were performed with pressure effects neglected) is presented in Fig. 4. As shows Fig. 4, the pressure effects lead to a considerable reduction of the

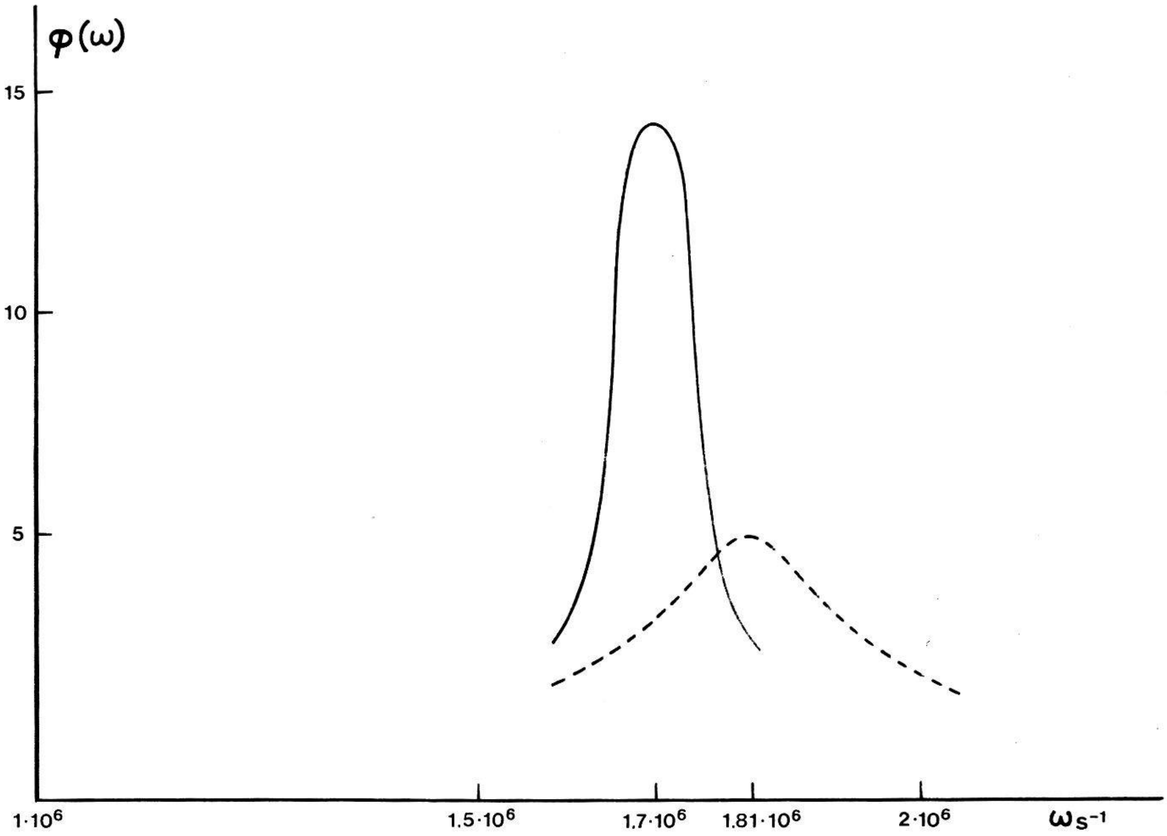

Figure 4
Function $\phi(\omega) = (c/c_A)\,\mathrm{Re}\,Z$ for a fully ionised atomic Hydrogen with $T = 2\,\mathrm{eV}$, $n = 10^{15}\,\mathrm{cm}^{-3}$, $B_0 = 10^3 G$, $a = 10$ cm. Full line was calculated from the formula (32) with pressure and viscous effects neglected ($\gamma = \gamma_0 = (q_0 m/2c_A a)$, dotted line was calculated from (32) with the pressure effects accounted for. (The viscous effects are negligibly small in this case).

maximal energy absorbed in the plasma. Besides, the frequency distribution $W_a(\omega)$ at the first MAR becomes noticeably broader.

In conclusion, we mention that the proper evaluation of the boundary effects arising as consequence of the presence of a solid wall surrounding a plasma could also be of interest in connection with the problem of additional heating of fusion reactors of the tokamac type by magneto-acoustic waves. According to existing estimations (see e.g. [9]), a layer of relatively cold gas (with temperatures of a few thousands degrees) must exist nearby the solid wall in such reactors, which, however, must be highly ionised. Besides, the magnetic field used for the plasma confinement remains strong at the wall, according to [9]. This means that the magneto-acoustic waves, if excited in such plasmas, will manifest themselves also nearby the solid wall and, hence, the boundary effects discussed above may be of interest also in this case. Of course, only the consequent calculations, allowing for the realistic distributions of temperatures, densities and confining magnetic fields, can confirm this conjecture.

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