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Autor: Foroughi, F. / Pauletta, G.
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The coherence effect in many nucleons systems and some applications[1])

by **F. Foroughi** and **G. Pauletta**[2])

Institut de Physique de l'Université, Rue A.-L. Breguet 1,
CH-2000 Neuchâtel, Switzerland

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Abstract. A new demonstration of the coherence effect in the three nucleons system is presented and generalized for N nucleons. Application to proton–proton and alpha–alpha final state interaction is given with a comparison with the free two particles scattering.

1. Introduction

The coherence effect in the scattering of three nucleons, as described by the Faddeev equations, was found by Amado [1], and consists in a coherent addition of all terms in the expression of the break-up operator. As a consequence, a Möller operator appears as a common factor and contains all the information on the final state interaction (FSI) between a given pair of nucleons. However the demonstration given in [1] is restricted to the use of the Faddeev equations, and its extension to higher number of nucleons is not available. In the following section, we give a new demonstration [2] whose application to N nucleons ($N > 3$) can be easily generalized. In Section 3, we give two examples of application to FSI. In Section 4, we discuss the differences between free particle scattering and FSI. A general conclusion will be given in the last section.

2. Theory

2.1. The case of three nucleons

We consider only the final state of three nucleons

$$\text{anything} \longrightarrow 1 + 2 + 3$$

by anything we mean any incoming particles compatible with three outgoing nucleons (here $n + d$, for example).

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[1]) Work supported by the Swiss National Science Foundation
[2]) Present address: University of California, Los Angeles, California.

The corresponding Möller operator is given by:

$$\Omega(Z) = 1 + VG(Z) = 1 + V_iG(Z) + \bar{V}_iG(Z) \tag{2.1}$$

where

$$V = V_1 + V_2 + V_3, \qquad \bar{V}_i = V_j + V_k \quad i \neq j \neq k \tag{2.2}$$

V_i is the potential acting between the nucleon j and k (see Fig. 1 for illustration), $Z = E + i\varepsilon$ where E is the energy in the center of mass and $G(Z)$ is the total Green function.

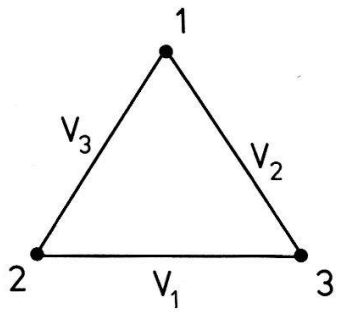

Figure 1
Illustration of the relation 2.2.

The Möller operator $\Omega(Z)$, when applied to the three nucleons outgoing plane wave, produces a scattering solution of the full Hamiltonian.

Using

$$G(Z) = G_i(Z) + G_i(Z)\bar{V}_iG(Z). \tag{2.3}$$

We find [2]

$$\begin{aligned}\Omega(Z) &= 1 + V_iG_i(Z) + V_iG_i(Z)\bar{V}_iG(Z) + \bar{V}_iG(Z) \\ &= (1 + V_iG_i(Z))(1 + \bar{V}_iG(Z)) = \omega_i(Z)\cdot\Omega_i(Z),\end{aligned} \tag{2.4}$$

where G_i is the sub-Green function for the pair i.

This result is known as the 'coherence effect' in 3 nucleons scattering, and is a property of the final state only. The Möller operator ω_i acts on the pair i while $\Omega_i(Z)$ acts on the three nucleons.

2.2 *The case of N free nucleons*

If we have N free nucleons in the final state:

$$\text{anything} \longrightarrow 1 + 2 + \cdots + N$$

the same result holds, because we still have

$$\bar{V}_i = V - V_i, \qquad V = \sum_{i=1}^{N} V_i.$$

Therefore the generalization to N free nucleons is trivial.

2.3. The case of three free composite particles

Consider now the more useful situation of 3 free composite particles in the final state

$$\text{anything} \longrightarrow 1 + 2 + 3$$

we still have

$$\Omega(Z) = 1 + V_i G(Z) + (V_j + V_k)G(Z) \qquad i \neq j \neq k \tag{2.5}$$

$$G(Z) = G_i(Z) + G_i(Z)\bar{V}_i G(Z), \tag{2.6}$$

but now V_i is the sum of all the potentials acting between the two particles belonging to the pair i, and $\bar{V}_i$ is given by

$$\bar{V}_i = V - V_i - V^j - V^k = V_j + V_k + V^i \quad i \neq j \neq k, \tag{2.7}$$

where V^i (see Fig. 2) is the sum of all the potentials that bind the nucleons belonging to the particle i. By repeating the same algebraic manipulations as before, we obtain:

$$\Omega(Z) = \omega_i(Z) \cdot \Omega_i(Z) - V^i G(Z). \tag{2.8}$$

In this case, the coherence effect is not complete because we have a 'background' containing the potential V^i which binds the nucleons in the third particle (here i).

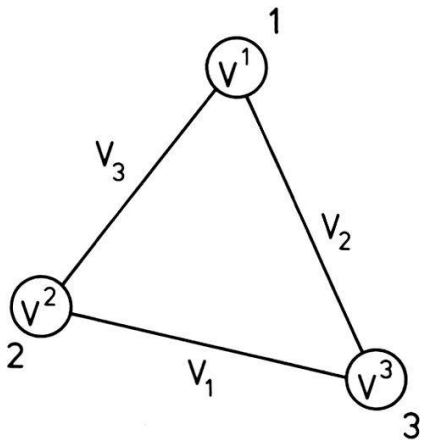

Figure 2
Illustration of the relation 2.7.

3. Application of the coherence effect to FSI scattering

3.1. p–p FSI

Considering the p–p pair (quoted 3 in the following) in the break-up reaction $p + d \rightarrow p + p + n$, we have (see 2.3)

$$\Omega(Z) = \omega_3(Z)\Omega_3(Z).$$

Consider only the operator $\omega_3(Z)$. Acting on a plane wave $\langle \mathbf{k}, \mathbf{q}|$, this operator gives

$$\langle \mathbf{k}, \mathbf{q}|\omega_3(Z) = \langle \mathbf{q}, \psi(Z, \mathbf{k})| \tag{3.1}$$

where $\psi(Z, \mathbf{k})$ is a diffusion state for two protons, and $\mathbf{q}$, $\mathbf{k}$ are the Faddeev–Lovelace [3] variables, associated with the pair 3.

If we consider only S wave scattering then, from $\psi(Z, \mathbf{k})$ we can factorize the inverse of the Jost function [4] $L_0^+(k)$:

$$\psi(Z, \mathbf{k}) = F_0(k)\phi_0(Z, \mathbf{k}) \tag{3.2}$$

with

$$F_0(k) = \frac{e^{i\delta_0}}{|L_0^+(k)|} \tag{3.3}$$

and we get for the differential cross section

$$\frac{\partial^3\sigma}{\partial E_p\, \partial\Omega_p\, \partial\Omega_n} = \text{cte}\, |F_0(k)|^2 \cdot M(\mathbf{k}, \mathbf{q}; Z|^2 \cdot \rho \tag{3.4}$$

where M contain the remainder terms ($\Omega_3(Z)$ matrix elements) and ρ is the phase space factor [5].

The presence of the Coulomb potential makes the application of integral equations or dispersion relations, to the calculation of $L_0^+(k)$ very difficult. The function $F_0(k)$ was therefore calculated by solving the Schrödinger equation for the p–p scattering, and using the fact that the radial solution for small r behave like

$$F_0(k)r. \tag{3.5}$$

Calculations were performed using 7 p–p potentials: Gaussian, Yukawa, exponential [6], hard-core of Christian and Gamel and Thaler [7] and Ried soft and hard-core [8]. The function F_0 was also calculated for a separable interaction using the formalism of Ali et al. [9]. All the results obtained indicate a very marked stability of form (see Fig. 3 where we compare our results with the data of Brückmann's experiment [10]). On the other hand, large differences were found in the absolute value.

This stability of form permits us to make valuable comparison with the experimental results. As is seen in Fig. 3, there is certainly an S wave dominance in the p–p FSI.

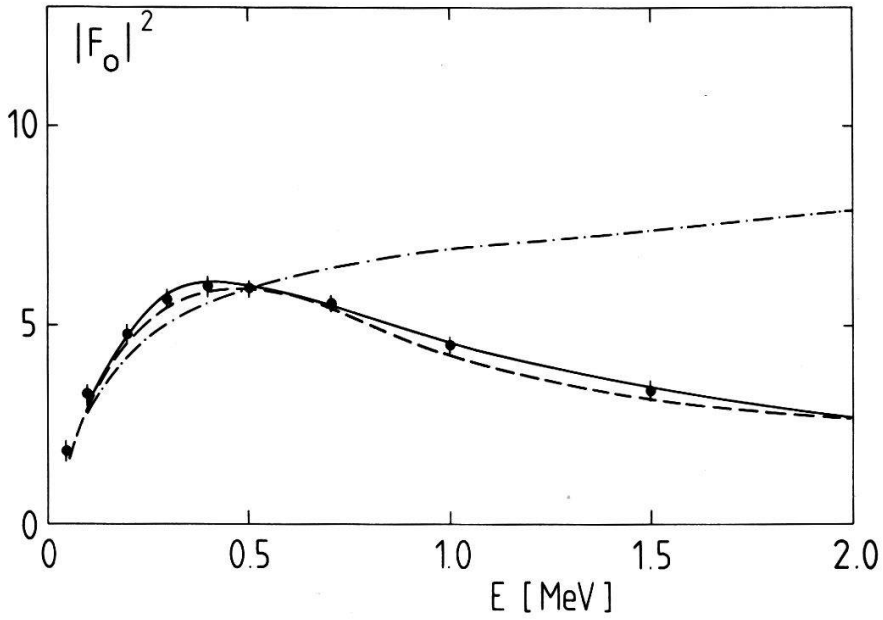

Figure 3
Comparison of the factorization for various potential. —— Gaussian potential, — — Ried soft-core potential, ● Brückmann's data, —·— C_0^2 (pure Coulomb).

The same comparison was done by Brückmann [10], but using an effective range expansion [11], and an asymptotic form of the wave function calculated for some matching radius.

We explain in Appendix A why his result was still reliable.

3.2. α–α *FSI*

We repeated the same analysis for two α particles. However in this case there is an essential difference due to the composite nature of the α particles. We can either use

(i) phenomenological potentials,
(ii) or a resonating group method (RGM).

For the first case, we have calculated the factorization for the potential of Ali–Bodmer [12] and Chien–Brown [13] and for RGM we used the formalism developed by Thompson et al. [14] (concerning the validity of the factorization see Appendix B).

Subsequent calculations showed the same overall stability for each of the two possibilities for low energy (see Fig. 4). It is not possible to reproduce the resonance of the ^{8}Be O^+ ground state by either model.

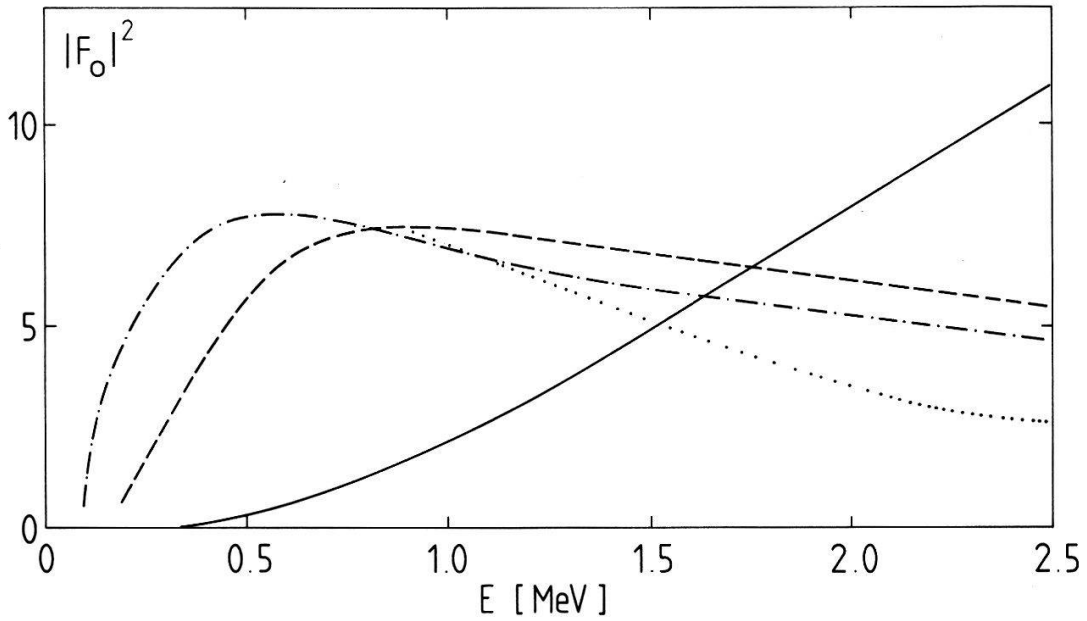

Figure 4
Comparison of the factorization in the α–α case. —·—· potential of Ali and Bodmer and Chien et al., — — potential of Thompson et al. (RGM), · · · potential of Brown et al. (RGM), —— C_0^2 (pure Coulomb).

The main difference between (i) and (ii) comes from the nodal behavior of the wave function resulting from the RGM calculation [15]. This nodal behavior in the absence of bound state is due to the non locality of the 'potential' [16]. But in the particular case of the RGM, it is an artefact coming from the description of the wave function by Gaussian [15]. Thus *a priori* it is not sure that the RGM result is more reliable than the phenomenological one.

We performed α–α FSI experiment with the ^{6}Li(α, αd)α reaction [17] and as it is shown in Fig. 5, no potential diffusion was observed but only the ^{8}Be O^+ ground state.

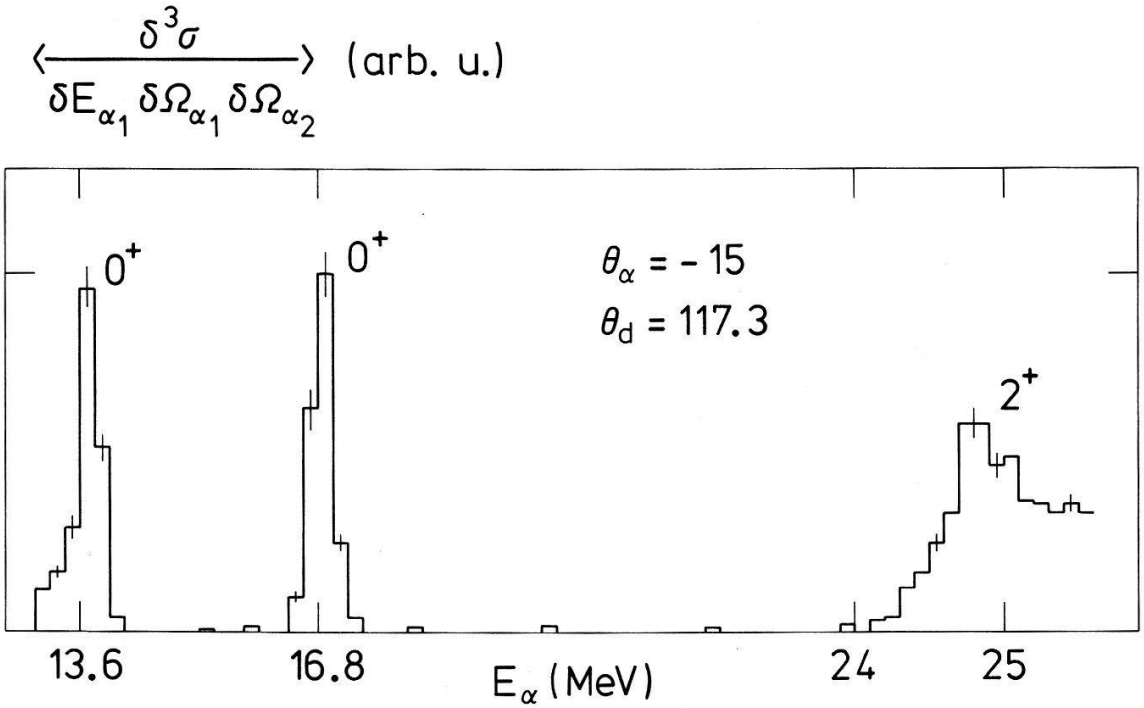

Figure 5
Observed α–α FSI in the ^{6}Li(α, α*d*)α reaction at 42 MeV.

4. Discussion

If the free *p–p* scattering, in the contrary of the break-up reaction, the factorization which is the same does not account for the form. Only the argument of the Jost function contributes and not its module. This can be easily shown if we recall the expression of the diffusion function (for uncoupled case) in term of the Jost solution $f_l^{\pm}(kr)$:

$$\psi_l(kr) = \frac{a}{r}\frac{k^l}{L_l^+(k)}\{L_l^+(k)f_l^-(kr) - L_l^-(kr)f_l^+(kr)\}$$
$$= \frac{ak^l}{r}\left\{f_l^-(kr) - \frac{L_l^-(k)}{L_l^+(k)}f_l^+(kr)\right\} \tag{4.1}$$

where the constant a depends on the choice of units and normalization. It is clear from the expression (4.1) that only the argument ($L_l^{\pm} = |L_l|e^{\mp i\theta_l}$) of the Jost function contribute to the amplitude f_l:

$$f_l(k) = \langle\psi_l(k)|V|lk\rangle \tag{4.2}$$

where $|lk\rangle$ is the l partial plane wave.

However, if we consider the function $\psi_l(kr)$ only in a neighborhood of the origin, since for r near zero $\psi_l(kr)$ behave as r^l and

$$L_l^{\pm}(kr) = \lim_{r=0} f_l^{\pm}(kr). \tag{4.3}$$

We get

$$\psi_l(kr) \simeq \frac{ak^l}{L_l^+(k)} r^l \tag{4.4}$$

which depends of the module of L^+.

Therefore, one possible explanation of the success of the factorization in the description of the form of the FSI is that the interaction hold in a small volume. As a consequence in the expression of the matrix element, for the break-up reaction, one replaces the function $\psi_0(kr)$ by $F_0(k)$ and integrate over a small volume. This is nothing but the old Watson–Migdal [18] idea.

One question arises, is the small volume connected with the fact that the relative energy is small? If yes, then low energy FSI is a special case. For higher relative energy FSI like ^{8}Be first excited state, it is possible that the factorization does not account for the form. Unfortunately, the different possible theoretical approaches do not give unambiguous results (see right part of Fig. 4).

In fact, as it is reported in the literature [19], some discrepancies exist which are more or less explained by off shell effects or interferences.

It remains to explain the S wave dominance. Our observed results for the α–α case seem to indicate that the many body dynamics favors strong interaction (only S wave) over electromagnetic one.

It should be noted that, since L_l^+ is dimensionless, the presence of the factor k^l prevent comparison at the level of the factorization.

5. Conclusion

The preceding calculations and comparison with the experimental data show that

(i) for p–p case the factorization of F_0 from the amplitude well describes the form of the FSI in the $d(p, pn)p$ break-up reaction and we have an S wave dominance;

(ii) for α–α case: the O^+ ground state of ^{8}Be dominates in the low relative energy domain, and no potential scattering is observed.

It will be possible to check the various hypothesis and explanation we presented, as soon as it becomes possible, taking into account the Coulomb potential to calculate the deuteron break-up reaction by a proton. The most promising method is that of Merkuriev et al. [20], which describes the three-body reaction in configuration space, in which the inclusion of Coulomb potential [21] is 'simple'. It will then be quite easy in the calculation of the matrix element, to restrict the integration to a small volume.

Our final conclusion is that if it is relatively simple to describe the form of the FSI, the description really depends on the complete dynamics rather than on a sub-process alone.

Appendix A

If we calculate $1/L_0^+(k)$ using a separable potential in the formalism of Rahman

et al. [9], we obtain

$$\frac{e^{i\sigma_0}}{2i} C_0(k) \frac{Z_0 - [h(\alpha)/R + kC_0^2(k)H(k)]}{Z_0 - h(\alpha)/R - ikC_0^2(k)}$$

where Z_0 is given by

$$Z_0(k) = C_0^2(k)k \operatorname{cotg} \delta_0 + \frac{h(\alpha)}{R}$$

$$R = \frac{\hbar^2}{Me}, \qquad C_0^2(k) = \frac{2\pi\alpha}{\exp\{2\pi\alpha\} - 1}, \qquad \alpha = \frac{\mu Z_1 Z_2 e^2}{\hbar^2 k}$$

and

$$H(k) = \frac{\int_0^\infty G_0(kr)q_0(r)\,dr}{\int_0^\infty F_0(kr)q_0(r)\,dr}$$

$h(\alpha)$ is a function evaluated by Jackson and Blatt [22], $q_0(r)$ is the nuclear form factor, and G_0, F_0 the irregular and regular Coulomb functions.

Brückmann's [10] expression is

$$\frac{e^{i\sigma_0}}{2i} C_0(k) \frac{Z_0 + [1/r_m + \beta \ln(\beta r_m) + 2\gamma - 1]}{Z_0 - h(\alpha)/R - ikC_0^2(k)}$$

where $\beta = 1/R$ and γ is the Euler constant, and r_m a matching radius.

We see that the difference arises from the terms in brackets which for very low relative energy do not differ in their behavior.

Appendix B

In the resonating group calculation, the radial antisymmetrized wave function is given by

$$u_0^A(r) = u_0(r) + \int_0^\infty N_0(r, r')u_0(r')\,dr'$$

where $N_0(r, r')$ is the kernel resulting from the antisymmetrization, given that the kernel of the integro-differential equation for $U_0(r)$ is hermitian, the generalized theorems [23] remain valid and we still have

$$u_0(r) = \frac{k}{L_0^+(k)} \phi_0(r)$$

where $\phi_0(r)$ is the regular solution ($\phi_0(r) \sim r$ near origin) and thus

$$u_0^A(r) = \frac{k}{L_0^+(k)} \left\{ \phi_0(r) + \int_0^\infty N_0(r, r')\phi_0(r')\,dr' \right\}.$$

This shows that the principles of factorization remain valid.

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