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On space-like fields

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Abstract. We develop here a tentative of a field theory in which there do not exist asymptotic free fields. The modifications relative to usual time-like field theories are conceived in a minimal way. Such space-like theories might be suggestive for models in hadron physics based on quarks and gluons.

1. Introduction

Attempts to modify the basic postulates of relativistic quantum field theory have been extremely numerous but they have led essentially nowhere. One of the main reason for their failure, is that as soon as one of the basic requirements is relaxed, the theory becomes undefined in the sense that one gets too much arbitrariness (the best example being the case of locality).

In a letter [1] published by one of us, it has been proposed to modify a postulate until now to our knowledge unchallenged, namely the boundary or asymptotic conditions. In the letter quoted above, it has been shown that a new type of field theory could be defined via its Green functions. This field has some very exotic features, but it is completely defined and there is no more freedom than in the standard approach.

The purpose of the present paper is to construct the operator form of this new field theory, and to put it, if not in a mathematically perfect state, at least in a reasonably clean form.

As basic principle, we shall try to minimize the changes to be made from the standard approach, but it is clear that all types of modifications proposed for it could also be adapted to this new version.

We shall discuss here the field theory itself. Whether this theory is capable of explaining the structure of the hadronic matter as hinted in [1], this is a separate question upon which we shall not elaborate in this paper, but we want to remark that if it were true, it would amount to the prediction of the existence of a hitherto unknown new type of matter.

To be a little bit more explicit, it was proposed in [1] that if you write the propagator for a gluon field of mass m in x-space and change m everywhere into $-m$, you still get a good Green function for the same field equations. In the static approximation, it is shown that if the gluon field is coupled in the usual way to a quark current, we get an effective potential between a quark and an antiquark of the form $r^{-1}\exp(+mr)$. Such a type of potential could be a good candidate to explain the hadron spectrum, the essentially free behaviour under deep inelastic scattering and Zweig's rule. The type of matter corresponding to this gluon field has then the property of never existing in a free state and of forcing the quarks to appear only as bound states. The change of m into $-m$ was used in order to get other solutions of the Klein-Gordon equation, homogeneous or inhomogeneous, but it is not clear whether it has a direct physical interpretation in itself.

2. Rewriting the neutral, scalar quantum field

The neutral, scalar quantum field is usually written as (in 3 space dimensions)

$$\varphi(x) = \frac{1}{(2\pi)^{3/2}} \int \frac{d\mathbf{k}}{\sqrt{2k_0}} \{a^*(\mathbf{k})\, e^{-ikx} + a(\mathbf{k})\, e^{ikx}\} \tag{2.1}$$

with $k_0^2 = \mathbf{k}^2 + m^2$

We shall write it in the form

$$\varphi(x) = \int \{\bar{f}(x - y)a^*(\mathbf{y}) + f(x - y)a(\mathbf{y})\}\, d\mathbf{y} \tag{2.2}$$

with

$$f(x) = \frac{1}{(2\pi)^{3/2}} \int \frac{e^{ikx}}{\sqrt{2k_0}}\, d\mathbf{k} \quad \text{and} \quad a(\mathbf{y}) = \int e^{+i\mathbf{k}\mathbf{y}}\, a(\mathbf{k})\, d\mathbf{k}$$

the commutation relations for the creation and annihilation operators being the standard ones.

We want to obtain an explicit form for the function $f(x)$. Integration upon angles gives

$$\begin{aligned} f(x) &= -\frac{2\pi}{(2\pi)^{3/2}} \int_{-\infty}^{+\infty} d\rho\, \frac{\rho}{ir}\, e^{i\rho r - ik_0x_0} \cdot \frac{1}{\sqrt{2k_0}} \\ &= -\frac{1}{2\sqrt{\pi}\, r} \frac{\partial}{\partial r} \int_{-\infty}^{+\infty} d\rho\, e^{i\rho r - ik_0x_0} \cdot k_0^{-1/2}; \quad k_0^2 = \rho^2 + m^2 \\ &= \frac{1}{2\sqrt{\pi}\, r} \frac{\partial}{\partial r} \frac{\partial}{\partial ix_0} \int_{-\infty}^{+\infty} d\rho\, e^{i\rho r - ik_0x_0} \cdot k_0^{-3/2} \end{aligned} \tag{2.3}$$

The last integral is tabulated by Oberhettinger [2, p. 12, no 97] and we get

$$f(x) = \frac{1}{2\sqrt{\pi}} \frac{1}{r} \frac{\partial}{\partial r} \frac{\partial}{\partial ix_0} \{[K_{1/4}(u)]^{-2} K_{1/4}(v) K_{1/4}(t)\} \tag{2.4}$$

with the notations

$$u = \frac{i}{2} m x_0; \qquad v = \frac{m}{2}[(r^2 - x_0^2)^{1/2} - r]; \qquad t = \frac{m}{2}[(r^2 - x_0^2)^{1/2} + r]$$

and, finally

$$
\begin{aligned}
-f(x) = {} & \frac{1}{2\sqrt{\pi}}\cdot\frac{m}{K^2_{1/4}(u)}\left\{\left(\frac{mK^{-1}_{1/4}(u)K_{3/4}(u)}{v-t} - \frac{m^2}{(v+t)^2}\right)K_{1/4}(v)K_{1/4}(t)\right. \\
& + m^2\left(-\frac{u}{(v+t)^3} + \frac{u}{2(v-t)(v+t)^2} - \frac{tK^{-1}_{1/4}(u)K_{3/4}(u)}{(v-t)(v+t)}\right)K_{1/4}(v)K_{3/4}(t) \\
& - m^2\left(+\frac{u}{(v+t)^3} + \frac{u}{2(v-t)(v+t)^2} - \frac{vK^{-1}_{1/4}(u)K_{3/4}(u)}{(v-t)(v+t)}\right) \\
& \left.\times\, K_{3/4}(v)K_{1/4}(t)\right\} \\
\equiv {} & -f(x, m)\ ^{1)}
\end{aligned}
\tag{2.5}
$$

$f(x, m)$ decreases in time-like direction like x_0^{-3}, and in space-like direction like $r^{-7/4}\exp(-mr)$, as can be easily seen from the known asymptotic expansions of Hankel functions.

The standard development of the standardized scalar neutral field can then be developed as usual, only in a somehow more complicated form.

3. Space-like fields

The solutions of the wave equation used so far have the property of decreasing exponentially in space-like directions, whereas they decrease polynomially in time-like directions. For this reason, we shall refer to the standard approach to quantum field theory as the time-like theory.

If we assume that one of the basic requirements is to consider solutions of the Klein-Gordon equation, we have made tacitly in the time-like theory the choice of boundary conditions which exclude solutions which would grow at infinity. This is implicit in the fact that one assumes the existence of Fourier-transforms (in the usual sense) for the fields and is easily seen in the fact that one is making an expansion in plane waves of the form e^{ikx}, with the condition $k_0^2 = \mathbf{k}^2 + m^2$.

In choosing such an expansion, one is excluding the solutions which would grow at infinity in a space-like direction, for instance of the form $e^{\rho x}$, with the condition $\rho_0^2 = \boldsymbol{\rho}^2 - m^2$ (notice the formal analogy with the tachyon fields, but the tachyon fields considered so far are always of the time-like sort). The physical reasons for excluding such solutions (which correspond to what we shall call space-like theories) are essentially two:

— The cluster property in its usual formulation is not fulfilled and therefore one does not have asymptotically free particles. (This could be seen directly in the static approximation.)
— For solutions growing at infinity, the free Hamiltonian is always diverging.

We can therefore immediately conclude: 'If a space-like field theory exists, it cannot correspond to asymptotically free particles and the free Hamiltonian is not defined.'

The purpose of this paper is to show that such a space-like theory exists.

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1) Recall that $K_\nu(iz) = (i/2)\pi\, e^{-(i/2)\nu\pi} H_\nu^{(2)}(z)$.

As it turns out that only an interacting space-like field theory can be conceived, we shall not be able to prove with mathematical rigour its existence, but we hope, however, to make it sufficiently plausible.

We need a simple procedure to construct the space-like solutions from the time-like ones. Of course, they are all well-known to mathematicians. We shall only consider the symmetries of the Klein-Gordon equation in order to get practical recipes.

Obviously, the Klein-Gordon equation is invariant under the change m to $-m$ or under the inversion of all space and time coordinates. Under the same transformations therefore, a solution will be transformed in another solution. In particular, our solution (2.3) to (2.5) is still a solution if we change the signs of u, v and t in (2.4) for instance, we call this m to $-m$ symmetry, although m is only one of the factors. We have to verify that $f(x, -m)$ is defined at all and then study its asymptotic properties. As the Hankel functions entering its definition are multivalued, we have to choose on which part of the Riemann surface we want to stay. Typically,

$$K_\nu(z\, e^{ij\pi}) = e^{ij\pi\nu}\, K_\nu(z) - i\pi \frac{\sin(j\pi\nu)}{\sin(\pi\nu)} I_\nu(z) \tag{3.1}$$

for j integer. We shall systematically choose $j = +1$ in making the passage m to $-m$.

All asymptotic properties we shall use here follow from the expansions valid for large z

$$I_\nu(z) \approx \frac{e^z}{\sqrt{2\pi z}}\left\{1 - \frac{4\nu^2 - 1}{8z} + \cdots\right\} \tag{3.2}$$

$$K_\nu(z) \approx \frac{e^{-z}}{\sqrt{2z}}\sqrt{\pi}\left\{1 + \frac{4\nu^2 - 1}{8z} - \cdots\right\} \tag{3.3}$$

$f(x, -m)$ thus behaves in space-like directions as $r^{-7/4}\exp(mr)$ and it follows that a field based on an expansion in such functions cannot be tempered.

Nevertheless, we define as scalar neutral space-like field

$$\varphi(x) = \int \{\bar{f}(x - y, -m)a^*(\mathbf{y}) + f(x - y, -m)a(\mathbf{y})\}\, d\mathbf{y} \tag{3.4}$$

in perfect analogy with (2.1), the commutation relations remaining

$$[a(\mathbf{y}), a^*(\mathbf{y}')] = i\delta(\mathbf{y} - \mathbf{y}')$$

There are 3 problems to solve:

— Define the space of test functions on which φ is a good operator on a Fock space.
— Compute the commutation relations between $\varphi(x)$ and $\varphi(x')$.
— Compute the propagator.

In fact, once the first of these problems is suitably solved, the other two follow easily.

4. Test function space

The problem is not simple as appears immediately. Suppose that if $|0\rangle$ is the Fock vacuum, we want to have $\|\varphi(g)|0\rangle\|$ to be finite, this amounts to have

$$\int \bar{f}(x - y)f(y - z)g(x)g(z)\,dx\,dy\,dz < \infty \tag{4.1}$$

We can consider the function $f(x, -m)$ at a sharp time for a moment, in order to simplify the discussion; we get then

$$f(x, -m)\,|_{x_0=0} = -\frac{(-2m)^{5/4}}{i2^{3/2}}\frac{1}{\Gamma(1/4)}\frac{1}{r^{5/4}}K_{5/4}(-mr) \tag{4.2}$$

$$K_{5/4}(-mr) = e^{i5\pi/4}K_{5/4}(mr) - i\pi I_{5/4}(mr) \tag{4.3}$$

g is therefore a function which is such that its convolution with f belongs to L_2. In order to determine precisely the suitable class, let us write as usual (3.4) in the form $\varphi = \varphi_+ + \varphi_-$.

We have formally (omitting the index $-m$):

$$\varphi_-(x) = \int f(x - y)a(y)\,dy = \int f(-y)a(x + y)\,dy \tag{4.4}$$

This means that we would like to have, for any test-function g, the following relation – where we write formally the action of a generalized function on a test function as an integral, this having a rigorous meaning if we choose properly the test-function space:

$$\begin{aligned}\int \varphi_-(x)g(x)\,dx &= \int\int f(-y)a(x + y)g(x)\,dx\,dy = \\ &= \int\int f(-y)a(x)g(x - y)\,dx\,dy = \\ &\int f(-y)(a{*}g)(y)\,dy\end{aligned} \tag{4.5}$$

It is therefore sufficient to choose g in a space such that the regularized $a{*}g$ of the distribution a is a C^∞-function with decrease at infinity faster than any exponential. The subspace S_α [3] of $\mathscr{S}$ (with $\alpha > 0$) will do ($a{*}g$ is a C^∞-function when $g \in \mathscr{S}$ since $a \in \mathscr{S}'$ as in the usual theory).

Since we prefer to have a space stable under Fourier-transform, we shall take the space of type S of Gelfand-Shilov [3] having this property, namely the subspace $S_{1/2}^{1/2}$ of $S_{1/2}$ of C^∞-functions $g(x)$, restrictions to $\mathbb{R}^3$ of entire functions in $\mathbb{C}^3$ satisfying

$$|g(x + iy)| \leqslant c_1 \exp(-c_2|x|^2 + c_3|y|^2) \tag{4.6}$$

for some positive constants c_1, c_2, c_3.

For $g \in S_{1/2}^{1/2} a{*}g$ will be a C^∞-function with exponential decrease of order 2, which enables us to define $\varphi_-(g) = \int f(-y)(a{*}g)(y)\,dy$ and similarly for φ_+.

The application $g \to \varphi(g)$ thus defined will have all the desired properties. Notice that, as in the case of the free time-like field, no time smearing is necessary.

5. Commutator and propagator

In Section 4, we showed that our field could be defined on a test function space, namely $S_{1/2}^{1/2}$. It is evident that all of the Green's functions of the time-like approach are also defined on the same space. As a trivial consequence, all the convolutions of the form

$$\int \bar{f}(x - y, -m) f(y, -m)\, dy$$

when considered as defined on $S_{1/2}^{1/2}$, are exactly of the form

$$\int \bar{f}(x - y, m) f(y, m)\, dy$$

to which the operation m to $-m$ is applied. But as all Green's functions can be written with the help of such convolutions of the function f multiplied eventually by a suitable θ-function, which can be performed without difficulties because of the smoothness of our test functions space, we can conclude: 'All Green's function of our space-like theory can be obtained from the corresponding Green's function of the time-like theory by just changing m into $-m$.'

We have therefore verified the assertions made in [1]. The main tasks remaining are:

— Examine the structure of this space-like theory, specially the spectrum condition and what is to be understood for the causal behaviour.
— What can be said about the solutions of the Bethe-Salpeter equation with exchange of our type of gluons.
— Compute the bound states of quarks bound by space-like gluons and see whether the obtained levels can be made to fit the existing data.

We hope to come back to these questions in forthcoming publications.

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