Scattering of protons from argon and hydrogen at 1 MeV

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Zusammenfassung. Frühere Präzisionsmessungen von elastischen Proton–Proton Wirkungsquerschnitten für 25°, 45° und 90° Schwerpunktswinkel wurden bei 0.992 MeV auf eventuelle systematische Fehler geprüft. Als Test für die angewandte Messtechnik wurden relative $^{40}A(p,p)$ Querschnitte bestimmt. Diese und die neu ausgewerteten relativen Querschnitte für p,p-Streuung sind in Uebereinstimmung mit theoretischen Werten. Die absoluten Querschnitte der p,p-Streuung zeigen aber nur Uebereinstimmung, wenn die experimentellen Werte um 0.3 bis 0.4 Prozent erhöht werden. Die Begründung für eine solche Normierung ist unbekannt.

The precision proton–proton differential cross-section measurements made at this laboratory by Mühry *et al.* [1], could be fitted only by reducing the strength of the vacuum polarization interaction by an arbitrary factor [2]. This discrepancy has led us to look for systematic errors which might not previously have been accounted for. Attention has been paid in particular to cross-section ratios, as they are determined experimentally more precisely than absolute cross-sections.

As a check of the measurement technique used in ref. [1] we have measured relative cross-sections for $^{40}A(p, p)$ elastic scattering at laboratory angles of 12°, 25° and 46°. The energy of 1.093 MeV was chosen to be free of effects of the reported resonances [3] in proton Argon scattering. Cross-section ratios were determined as a function of target gas pressure (0.1–4 Torr), and of beam current (1–40 nA), and extrapolated to zero pressure and current. Experimental results, together with predicted cross-section ratios, are shown in Table 1. A check of the reasonableness of the extrapolation is provided by the quantity $r_1 r_3/r_2$, which should be equal to unity. The deviation from 1.0, due to the independent extrapolation of each ratio, is in this case equal to 10^{-4}.

Table 1

Center of mass cross-section ratios for $A(p,p)A$ at 1.093 MeV and c.m.-angles $\theta_1 = 12.30_1^\circ$, $\theta_2 = 25.61_3^\circ$, $\theta_3 = 46.02_4^\circ$

	Experiment	Theory
$r_1 = \sigma(\theta_1)/\sigma(\theta_2)$	18.2880 ± 0.0091	18.2862
$r_2 = \sigma(\theta_1)/\sigma(\theta_3)$	176.44 ± 0.16	176.70
$r_3 = \sigma(\theta_2)/\sigma(\theta_3)$	9.6490 ± 0.0096	9.6634

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The calculations (see Appendix) include the vacuum polarization interaction as well as the shielding by atomic electrons, the Mott-Schwinger scattering and the nuclear interaction. Relativistic effects are taken approximately into account. It is seen that agreement of experiment with theory is well within two error bars.

The re-evaluated 0.9919 MeV proton–proton data corrected to full c.m. angles are presented as cross-section ratios in Table 2 (the deviation of $r_1 r_3 / r_2$ from one is 6×10^{-5}). As it was suspected that the measurements of [1] and [2] were made with detectors set at incorrect angles, a number of runs were repeated with the scattering table reassembled. Comparison of the two data sets shows that the angle error in the previous work was no more than 0.001° (i.e. within the quoted error bar).

Fit 1 of Table 2 was made by fixing the central combination Δ_c^c of p-wave phase-shifts at $-0.0059°$ [2], [4], and then adjusting the s-wave phase-shift to obtain the best χ^2 fit to the relative cross-sections. The best fit s-wave phase-shift $K_0 = (32.429 \pm 0.007)$ deg. is consistent with the Noyes–Lipinsky effective range expansion [5] with a pion-nucleon coupling constant $G^2 = 14.0$. Fit 2 corresponds to $\Delta_c^c = -0.010°$ [6] and $K_0 = (32,431 \pm 0.007)$ deg. The phase-shift error bars are found in each case by requiring an increase of χ^2 of 1. The recent summary of low energy p–p scattering by de Swart [7] favours values of Δ_c^c closer to zero than that used in fit 1. If one allows therefore a generous error of Δ_c^c of $\pm 0.006°$ so as to include $\Delta_c^c = 0$ [7] and $\Delta_c^c = -0.01$ [5], the s-wave phase-shift error bar of fit 1 increases to $\pm 0.0073°$. The vacuum polarization interaction is used everywhere at full strength.

Table 2
Center of mass cross-section ratios for $H(p,p)H$ at 0.9919 MeV and angles $\theta_1 = 24°$, $\theta_2 = 50°$, $\theta_3 = 90°$

	Experiment	Fit 1	Fit 2
$r_1 = \sigma(\theta_1)/\sigma(\theta_2)$	13.9570 ± 0.0078	13.9691	13.9627
$r_2 = \sigma(\theta_1)/\sigma(\theta_3)$	15.775 ± 0.012	15.7671	15.7714
$r_3 = \sigma(\theta_2)/\sigma(\theta_3)$	1.13019 ± 0.00072	1.12872	1.12954

It should be noted that, while the relative cross-sections are well fitted by calculations with the above parameters (and, in fact, an exact fit is obtained for $\Delta_c^c \simeq -0.0065°$), the calculated absolute cross-sections indicate an upward normalization of the data of ref. [2] by 0.3% (fit 1) -0.4% (fit 2). As the error bar of the absolute normalization of this data was $\sim 0.1\%$, it was considered necessary to search for possible experimental normalization errors. Independent estimates of background, dead-time and multiple-scattering corrections all agreed with those of [1] and [2]. The effects of energy smearing due to thermal motion of the target molecules, of energy averaging in the finite thickness target and of dissociation of hydrogen molecules by the beam are all estimated to be negligible. The molecular effect is thought to be negligible. The partial pressure due to residual gas in the chamber (deuterium, air, pump oil), however, does point to the need to normalise the cross-sections of [2] upward by $(0.05 \pm 0.03)\%$, the error bar arising from the uncertainty of the composition of the residual gas. Bending under pressure of the nickel foil at the entrance to the Faraday cup would increase the efficiency of charge collection by approximately 0.01%, producing a total normalization factor of (1.0006 ± 0.0003). The possibility of there being other normalization errors is not excluded, but their origins are presently not understood.

Appendix

Theoretical treatment of elastic Proton–Argon scattering

We start from the following Hamiltonian for the scattering of a proton in the field of an Argon nucleus:

$$H = T + V_{\text{COUL}} + V_{\text{MS}} + V_{\text{NUC}} + V_{\text{POL}} + V_{\text{VACPOL}} + V_{\text{SCREEN}} + V_{\text{REL}}$$

Where T is the kinetic energy operator and V_{COUL} denotes the Coulomb potential of a charged sphere. V_{MS} is the Mott–Schwinger potential describing the scattering from the nuclear magnetic moment.

The nuclear interation is described by means of the usual Woods–Saxon potential V_{NUC}. The effect of the polarizability of the target in the electric field of the proton (including dipole and higher order corrections) can be well described by the potential V_{POL} which is given explicitly in ref. [8]. Vacuum polarization effects are included by V_{VACPOL}. It is easy to see [9] that in a case like ours only the first order Uehling potential has to be taken into account. Higher order contributions are completely negligible.

The influence of the shielding by the atomic electrons can be described by V_{SCREEN}. We have chosen the analytical expression of Gaspar [10]. Finally, relativistic orbit corrections can be taken into account by an additional potential V_{REL} and a shift of the bombarding energy [11].

Disregarding for the moment the potentials V_{SCREEN} and V_{REL}, the differential cross-section can be calculated by solving the Schroedinger equation. In this process the resulting badly convergent partial wave series is conveniently summed up with the aid of a Pade approximation [12]. Our calculations show that the effects due to the Mott–Schwinger interaction and the polarizability give a contribution of the order 10^{-5} to the cross-section, i.e. they can be neglected. Some care must be taken for the nuclear interaction, whereas the mean optical potential (which is designed to describe the elastic scattering at higher energies) again only gives contributions of the order 2×10^{-4} at $\Theta = 45°$ and of the order 1×10^{-4} at $\Theta = 12°$. The presence of isobaric analogue resonances would lead to large fluctuations ($> 1\%$) of the cross-section at the angles of the experiment. However they do not affect the cross-section for the bombarding energy chosen in this experiment.

It is well known that for nuclear reactions the atomic screening potential and the potential for relativistic orbit correction can not at present be included by exactly solving the Schroedinger equation because of their extreme long range behavior. To overcome this difficulty, we have performed a full classical recalculation of the scattering process, including only V_{COUL} and V_{VACPOL}. Since our bombarding energy is well below the Coulomb barrier, it is expected that a classical description is quite accurate. Indeed our calculations show that the agreement between the classical and the quantum-mechanical treatment is better than $10^{-3}\%$. Therefore it seems

reasonable to assume that the influence of the two long-range potentials can be accurately described by the classical method, too.

The results of these calculations are displayed in Table 3, where the non-negligible contributions from the different potentials are given separately. It is seen that the three effects have approximately the same importance in our case.

Table 3

Corrections $\Delta_i = (\sigma_i - \sigma_{RUTH})/\sigma_{RUTH}$ to the nonrelativistic Rutherford cross section σ_{RUTH} at the c.m.-angles used in the experiment, where σ_i is the cross section which includes only the effect denoted by the index i, i.e. vacuum polarisation, screening or relativistic kinematics, and

$$\Delta_{TOTAL} = \Delta_{VACPOL} + \Delta_{SCREEN} + \Delta_{REL}$$

c.m.-angle (degrees)	Δ_{VACPOL} (%)	Δ_{SCREEN} (%)	Δ_{REL} (%)	Δ_{TOTAL} (%)
12.301	0.10	-0.24	0.10	-0.04
25.613	0.25	-0.18	0.08	0.15
46.024	0.38	-0.17	0.05	0.26

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