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Are Bell's Inequalities Concerning Hidden Variables Really Conclusive?

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(29. I. 75)

Abstract. We remark that the class of local hidden variable theories, introduced by J. S. Bell, lacks a certain desired stability property.

We then draw some conclusions from this type of unstability and on that basis discuss the present experimental situation.

1. Introduction

The present controversy on the interpretation of quantum mechanics (Q.M.) covers a basic divergence on the meaning of probabilities in microphysics. For the so-called 'Copenhagen School' of Bohr [1] and Von Neumann [2], quantum mechanical probabilities are 'complete', i.e. no knowledge is 'hidden' behind the probability distributions $|\psi|^2$ given by the wave function ψ. For their opponents (Einstein [3], de Broglie [4], Wiener, etc...) these probabilities should be interpreted in the sense described later by Von Mises [5], i.e. they do not cover all possible knowledge but reflect the physical limit of the distributions of the complex, uncorrelated stochastic behaviour of a sort of local (at present unmeasured) hidden variables. Thus, as proposed by Bohm and Vigier [6], the $|\psi|^2$ distributions do not form a complete description, but result from a deeper subquantal stochastic behaviour. The $|\psi|^2$ law can, in principle, be explained along the lines proposed by Einstein and Smoluchowski to interpret the laws of Brownian motion.

The new fact lies in the discovery of Bell [7] that one could turn the famous 'gedanken experiment' of Einstein, Podolski and Rosen [8] (E.P.R.) into realizable experiments which would yield *different results* for both types of interpretations. Let us recall its principle: one emits simultaneously two correlated particles a and b in the singlet state in a small domain. One then measures the spin of a (or b) once separated by a space-like interval. The measurement of the spin of $a(b)$ then predicts exactly the value of the spin of $b(a)$. This implies that if one produces two correlated photons γ_a and γ_b in the singlet state, one can, in principle, predict with certainty (probability 1) the helicity of

γ_b without any measurement on it, and there should exist a physical quantity (hidden parameter) attached to it. Bohr [9] has later shown this 'paradox' did not apply to his interpretation provided one accepted that no reality exists (in a microsystem) which is not really detected in a concrete measurement. The merit of Bell is that he has shown in the case of two fermions (the result has been later extended to bosons by Clauser et al. [10]) that this implies different predictions for the final correlated measurements of the angular correlated spin measurements, if the γ's have exchanged information only locally during their emission period. This result is summarized in Bell's inequalities, to which we shall return in Section 2.

It should be stressed that the renewed interest in hidden variables (HV) is not purely academic. Indeed, the hope of people believing in the necessity for introducing HV is that these variables, in special types of phenomena, will have a direct experimental manifestation. One should also mention that, as is completely clear from the works of the Geneva School (see e.g. Ref. [11a]), there exists a unified formalism and interpretation for classical and quantum mechanics. Moreover, HV can also be (and have been) introduced in classical theories (see e.g. Ref. [11b]). Therefore, there is no need to 'prove' *a priori* their existence or non-existence, except perhaps for philosophical reasons. Whether to utilize them or not should be related to what one can achieve with them as mathematical entities and to their possible experimental manifestation.

At last one should mention that the physical assumptions, under which the results of [7] were derived, have been recently criticized in an interesting paper [12] by L. de Broglie.

2. Bell's Inequalities and their Unstability

a) When we are dealing with physical theories in which only observed quantities occur, we are always interested in stability properties. For obvious reasons we like to have theories that can be 'deformed' to 'close' theories, giving predictions 'close' to those of the initial theory with respect to all notions and parameters appearing in the theory. The initial theory should then become a well-defined limiting case of the 'neighbouring' theories.

In many theories such a property is generally guaranteed by the stability with respect to a family of parameters or functions of certain types of differential equations. The non-existence of such a stability (as in certain perturbation expansions around some free solutions) always indicates that we should change our initial concepts and methods of calculation.

In the framework of the so-called hidden variable (HV) theories, one has to be even more prudent: these hidden variables (the non-existence of which has been 'demonstrated' by many physicists since the work of Von Neumann) are not directly measureable quantities by definition. At the present time we can at best measure some indirect consequences of the existence (or non-existence) of a certain type of HV theories. To impose restrictions on the HV is already quite a strong and *ad hoc* assumption. If, in addition, HV theories are classified according to some criterion in such a way that this criterion is answered by yes or no, with no possibility of having intermediate situations (as far as predictions are concerned) between yes and no, then the criterion does not seem to be tenable. Such a criterion will also not be tenable if (as many of those interested in HV believe), at some later date, HV have a direct experimental manifestation.

This is, indeed, what happens with local HV. As we shall see later, we have to deal with a continuous hidden variable, λ, a continuous density, ρ, which in the local case is a function of λ only, and an observable (spin or helicity direction), A, taking only the values ± 1, which in the local case depends only on λ and on the orientation of the corresponding measuring apparatus. Now the measured quantities, and also the inequalities they satisfy in the local case (Bell's inequalities), can be made stable with respect to ρ. This means, as can be easily seen, that small non-localities in ρ (dependence on other parameters than λ for suitably regular ρ's) will cause only small changes in Bell's inequalities.

However, since the quantity A takes only the discrete values ± 1, there is no sense in which stability under non-local deformations of A can be guaranteed: one *cannot* have, in this connection, any 'almost local' theory 'close' to the local one. Now, as we explained before, A is not even a measured quantity. Locality is already quite a strong assumption, but since stability does not exist here, even if one will be able to measure quantities like $A(\lambda)$ in the future, how can one ensure no experimental contamination of locality which would make our 'local predictions' completely useless?

Moreover, if we choose our models at random, for any type of non-locality, almost every model will give predictions different from the local case, and we can choose models of $\rho(\lambda)$, $A(\lambda, a, b)$ (which depends *a priori* at random on b), etc..., such that we can have a maximal deviation from the local case.

The deviations from Bell's inequalities in the non-local case will be presented later. These new inequalities are rather trivial mathematically and cannot be used (unfortunately) to distinguish between HV theories and quantum mechanics as in the local case. The point is, whenever we do not have a local HV theory, we *cannot, a priori* (without going to a very specific *ad hoc* HV model), obtain better inequalities, due to the lack of stability discussed above.

It should be mentioned, as a matter of fact, that Bell's result indicates more of a conflict between *quantum mechanics and locality* (here imposed on unmeasurable quantities) than a conflict between HV theories and quantum mechanics.

We shall also try to draw conclusions from the lack of stability and compare them with the recent conflicting experimental results. But let us now pass to the non-local version of Bell's inequalities.

b) Following Bell's arguments [7], we consider two polarization measure apparatus A and B, characterized by the directions of their polarization axes, denoted respectively by a and b. The result of the measurement by A of a system, with hidden variable λ, is denoted by $A(a, \lambda, f((a,b),\alpha)) = \pm 1$, where f expresses the non-locality, i.e. the influence of B on the measure by A, α denoting all parameters (e.g. the distance) associated with the couple (A, B) other than their relative direction (a, b). A theory is said to have local measurement if $A(a, \lambda, f) = A(a, \lambda)$ for all b and α.

Let us note (supposing that the sets of λ's where $A(a, \lambda, f) = +1$ and -1 are measurable)

$$P_f(a,b) = \int \rho(\lambda)\, A(a, \lambda, f((a,b),\alpha))\, B(b, \lambda, f((a,b),\alpha))\, d\lambda, \tag{1}$$

where $\rho(\lambda)$ is the (local) density of the hidden variable λ, and by $P_0(a,b)$ the same expression in the local case. Bell's inequality can then be written:

$$|P_0(a, b) - P_0(a, b')| + |P_0(a', b) + P_0(a', b')| \leqslant 2 \tag{2}$$

for all directions a, a' of A and b, b' of B. We shall show that this inequality does not hold in the non-local case, namely that to the right-hand side of (2) we must then add a term which is almost always strictly positive.

Indeed, if we follow the same lines of calculation in the non-local case, we obtain the following inequality (which may be an equality, as in Bell's case):

$$|P_f(a,b) - P_f(a,b')| + |P_f(a',b) + P_f(a',b')| \leqslant 2 + \Delta_f(a,a';b,b') \tag{3}$$

where, writing to simplify notations $A(a,\lambda,f((a,b),\alpha) = A(ab)$, and so on,

$$\begin{aligned}\Delta_f(a,a';b,b') &= \Big| \int d\lambda \rho(\lambda) \, [A(ab)\, B(ba)\, A(a'\, b')\, B(b'\, a') \\ &\qquad - A(ab')\, B(b'\, a)\, A(a'\, b)\, B(ba')] \Big| \\ &= \Big| \int d\lambda \, \rho(\lambda)\, N(a,a';b,b';\lambda) \Big|.\end{aligned} \tag{4}$$

Obviously, the correction Δ_f, due to non-locality, is zero in the local case. It is also zero if $a = a'$ or $b = b'$, in which case, however, inequalities (2) and (3) are trivially satisfied, and therefore cannot be used to prove or disprove anything. In the non-local case, Δ_f is, *a priori*, bounded by 2, and the correction can be significant from the very moment that there exists some non-locality in the hidden variables.

Let us write

$$A(ab') = A(ab) + J_A, \quad B(ba') = B(ba) + J_B$$

and, similarly,

$$A(a'\, b') = A(a'\, b) + J'_A, \quad B(b'\, a') = B(b'\, a) + J'_B,$$

where J_A, J'_A, J_B, J'_B are functions of λ also and express the jumps due to non-locality between the corresponding measurements. They have values 0 or ± 2. We can then decompose $N = N_A + N_B + N_{AB}$, where

$$\left.\begin{aligned} N_A &= B(ba)\, B(b'\, a)\, (A(ab) J'_A - A(a'\, b) J_A) \\ N_B &= A(ab)\, A(a'\, b)\, (B(ba) J'_B - B(b'\, a) J_B) \\ N_{AB} &= A(ab)\, B(ba) J'_A J'_B - A(a'\, b)\, B(b'\, a) J_A J_B. \end{aligned}\right\} \tag{5}$$

N_A (respectively N_B) expresses the pure contribution from non-locality for A (respectively B) to the integrand, while N_{AB} is the mixed contribution. It is clear from relations (4) and (5) that Δ_f can vanish only in very special cases. For example, even if we have $A(ab)J'_A = A(a'\, b) J_A$ we shall still have $N = A(ab)\; A(a'\, b')(B(ba)J'_B - B(b'\, a)J_B)$ which, in general, will not give zero when integrated with weight ρ.

Let us now express Δ_f in experimental situations such as those described in Refs. [13], [14] or [15]. We assume that, like $P_0(a,b)$, f depends only on the angle $\theta = (a,b)$. One considers the configurations for which the quantum analogue to the left-hand side of (2) predicts a maximal violation (equality to $2\sqrt{2}$) to Bell's inequality, namely $\cos(a,b) = \cos(a',b') = \cos(a',b) = \cos(\pi/8)$ or $\cos(3\pi/8)$, and $\cos(a,b') = \cos(3\pi/8)$ or $\cos(9\pi/8)$ (respectively). Expression (2) can then be written as:

$$|P_0(\pi/8) - P_0(3\pi/8)| \leqslant 1. \tag{6}$$

In the non-local case, with the obvious notations where the angles are expressed in units of $\pi/8$, the integrand N will be given, in the first configuration, by

$$N = A(a', 1)\,B(b, 1)\,[A(a, 1)\,B(b', 1) - A(a, 3)\,B(b', 3)].$$

Hence $\Delta_f \leqslant 2|P_f(1)|$, and similarly $\Delta_f \leqslant 2|P_f(3)|$ in the second configuration. We have equality if the deviation from locality has maximal effect on the measurements for almost all λ when comparing the cases with angles $\pi/8$ and $3\pi/8$. Thus (6) becomes

$$|P_f(1) - P_f(3)| \leqslant 2. \tag{7}$$

The correction to Bell's inequality that we obtain is of course compatible with all experiments.

3. Experimental Results

The interesting point of this discussion on the unstability of Bell's inequalities is that it might provide a theoretical basis to discuss conflicting results in recent experiments performed to test local hidden variables. These experiments – closely related to the E.P.R. paradox [8] – will now be briefly discussed.

a) Two experiments, using correlated γ pairs emitted in cascade process, have yielded conflicting results. Freedman and Clauser [13] used a cascade

$$4p^2\,{}^1S_0 \to \gamma_1 - 4p4s\,{}^1P_1 \to \gamma_2 - 4s^2\,{}^1S_0$$

in atomic calcium to produce a photon pair (γ_1, γ_2), and measured the coincidence rates $R(\theta)$ after passing through two polarizers at angle θ, and R_0 when both polarizers are removed. Whilst Bell's inequality gives in this case

$$\delta = |R(3\pi/8) - R(\pi/8)|\,R_0^{-1} - \tfrac{1}{4} \leqslant 0$$

they found $\delta = 0.050 \pm 0.008$, in agreement with the quantum prediction $\delta_{QM} = 0.051$, and with the 'non-local' inequality which gives $\delta \leqslant \frac{1}{4}$.

On the other hand, Holt [14] and Holt and Pipkin [15], using a $9\,{}^1P_1 \to 7\,{}^3S_1 \to 6\,{}^3P_0$ cascade in ^{198}Hg, found $\delta = -0.034 \pm 0.013$, in agreement with Bell's inequality but four mean deviations away from $\delta_{QM} = 0.016$. Though these authors remain cautious about their results, systematic errors sufficient to account for the discrepancy were not found.

It is interesting to note that the former experiment might provide an example of the unstability of Bell's inequalities. One can also argue that, in the former experiment, the density of calcium atoms provides the possibility that γ_2 would excite a second calcium atom which re-emits it, thus lengthening, as observed by Barrat [16], the lifetime of the $4\,{}^1P_1$ intermediate state. The latter experiment does not seem to correspond to such a situation [17].

It should be mentioned that the radiation-trapping discussed above, which could have existed only in the first experiment, would have pushed the quantum prediction towards the local HV predictions. Thus, if the experimental result of Ref. [13] is confirmed, it would indicate that if radiation trapping exists at all in such an experiment, it is cancelled by an additional (non-local) effect which accompanies it, and thus makes the naïve quantum-mechanical prediction work in this experiment. The observed lengthening of the intermediate state lifetime might be a manifestation of such an effect, which would increase the correlation and make the HV non-local.

b) Other experiments have used positron–electron pair annihilation to provide the γ pair in the singlet state. They also yield conflicting results. A first experiment by Langhoff [18], using a ^{22}Na source of positrons annihilating on ^{64}Cu, is in agreement with the quantum prediction, while a second experiment by Faraci et al. [19], using a different source of photons (the ^{22}Na positrons annihilating on a plexiglass envelope), falls exactly on Bell's limit.

Here also a radiation-trapping effect of the type mentioned above might occur: one, or both, of the γ's in the first experiment could be absorbed and re-emitted by electrons in the metal.

c) Experiments are being performed with non-relativistic fermions: preliminary results by Lamehi-Rachti and Mittig [20], using proton–proton scattering, seem to favour the quantum-mechanical prediction.

It is significant that we repeat the Holt type of measurement with non-relativistic neutrons, or with a completely controllable source of photons similar to the particular cascade recently suggested by Alfred Kastler: the excitation of the $7\,^3S_1$ level of ^{198}Hg by the 4047 Å laser line, producing a pair, γ_1 (4358 Å) and γ_2 (2537 Å).

It should be mentioned that experiments with non-relativistic fermions, however more difficult, are in a way more decisive. Believing in non-relativistic Q.M., we think that there is a good chance they will favour the quantum predictions. Indeed, for non-relativistic heavy fermions, Schrödinger Q.M. is more directly applicable and the notion of spin is clear. For photons, we have to consider helicities and, as a matter of fact, should apply quantum electrodynamics. Relativistic invariance is also then involved, and confirmation of Bell's prediction might also lead to problems with it.

4. Conclusion

There are therefore two main possibilities. Either experimental results completely favour Bell's local hidden variables – in which case, for such experiments, Q.M. (and possibly Poincaré invariance) are in trouble (we shall thus have, in general, non-local HV theories which under some particular situations become local and violate Q.M. predictions); or, an eventuality in which we believe, experimental results will coincide with the quantum-mechanical predictions. In such a case, we have shown that many theories of non-local hidden variables, of the type discussed in Section 2, can still exist and cannot be easily rejected.

Evidently, physical models showing contamination of locality can be produced. However, more experimentation is needed before we can suggest a realistic model of this type.

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