Zeitschrift: Helvetica Physica Acta
Band: 47 (1974)
Heft: 1

Artikel: Remarks about Coulomb corrections to ^+p elastic scattering
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DOI: https://doi.org/10.5169/seals-114562

Helvetica Physica Acta
Vol. 47, 1974.

Birkhäuser Verlag Basel

Remarks about Coulomb Corrections to $\pi^+ p$ Elastic Scattering

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Abstract. Coulomb corrections to $\pi^+ p$ phase shifts below 300 MeV are calculated in a simple non-relativistic potential model. Comparison with more refined calculations justifies this simple treatment of the Coulomb barrier effect. The dependence of the Coulomb corrections on the use of different form factors in the Coulomb amplitudes is shown.

I. Introduction

The new exact measurements of $\pi^+ p$ scattering in the region 70 to 300 MeV [1] require the calculation of Coulomb and mass difference corrections to the nuclear phase shifts. For the coupled-channel case the first theoretical treatment of charge effects in a non-relativistic potential model was given by Oades and Rasche [2, 3]. Later, Auvil [4] formulated the same method in a slightly different way and this formulation was used by Bugg [5] to calculate Coulomb corrections which were included in a $\pi^+ p$ phase-shift analysis [6].

In this calculation Bugg has chosen a rather complicated nuclear potential whose first Born approximation reproduces the Born terms deduced from partial wave dispersion relations. However, in a potential model, the Coulomb corrections depend mainly upon the electromagnetic potential in the nuclear region about which little is known. Therefore, it seems sufficient to choose any nuclear potential which has a simple shape and an appropriate depth and range so that it reproduces the nuclear phase shifts. If such a potential model is inadequate in the low-energy region, then one should employ a full dispersion theoretical approach such as given by Hamilton, Tromborg and Øverbŏ [7].

In Section II we investigate formally how in the $\pi^+ p$ case the Coulomb corrections to nuclear phase shifts depend on the form factors included in the additive Coulomb amplitudes. The result is that the calculation of Bugg in connection with the phase shift analysis of Ref. [6] is only correct if one chooses the Coulomb potential

$$V_C^{(1)}(r) = [1 - \exp(-R)\,(1 + \tfrac{11}{16}R + \tfrac{3}{16}R^2 + \tfrac{1}{48}R^3)]\,2\eta k/r, \tag{1}$$

$\eta =$ Coulomb parameter, $R = m_\rho r$, $m_\rho \sim 5m_\pi$, which corresponds to the Coulomb amplitude

$$f_C^{(1)}(t) = f_C^{(p)}(t)\,(1 - t/m_\rho^2)^{-4}, \tag{2}$$

where $f_C^{(p)}$ is the amplitude for point charges. Bugg has used

$$V_C^{(2)}(r) = \operatorname{erf}(r/a)\,2\eta k/r \quad a^2 \sim 0.66 m_\pi^{-2} \tag{3}$$

which corresponds to the amplitude

$$f_C^{(2)}(t) = f_C^{(p)}(t) \exp(a^2 t/4). \tag{4}$$

In Section III π^+p Coulomb corrections are given which illustrate numerically the influence of different nuclear potentials and of different electromagnetic form factors. One of the results is that the numerical difference in the Coulomb corrections by using either potential (1) or (3) is unimportant.

II. Formalism for the Single Channel Case

The Coulomb amplitudes are separated from the total amplitudes in the usual way [8]

$$f(\theta) = f_C^{(f)}(\theta) + \sum_l P_l(\cos\theta)\,[(l+1) f_{l+} + l f_{l-}]$$

$$g(\theta) = g_C^{(f)}(\theta) + \sum_l P_l^1(\cos\theta)\,[f_{l+} - f_{l-}] \tag{5}$$

$$f_{lj} = [\exp(2i(\delta_{lj} + C_{lj}^{(f)})) - 1] \exp(2i\nu_l^{(f)})/(2ik). \tag{6}$$

The superscript f refers always to some electromagnetic form factors but is replaced by p in the case of point charges. δ_{lj} and $C_{lj}^{(f)}$ are the nuclear phase shifts and Coulomb corrections, respectively. For the Coulomb phases $\nu_l^{(f)}$ one makes the approximation that they correspond to the non-relativistic limit of the Coulomb amplitudes. They are therefore independent of j for given angular momentum l:

$$f_C^{(f)}(\theta) = (1/2ik) \sum_l P_l(\cos\theta)\,(2l+1)\,(\exp(2i\nu_l^{(f)}) - 1). \tag{7}$$

For point charges we have

$$\nu_l^{(p)} = \arg\Gamma(l + 1 + i\eta). \tag{8}$$

The Coulomb corrections $C_{lj}^{(p)}$ corresponding to point charge amplitudes for the single channel case to first order in η are given in Ref. [9]:

$$C_{lj}^{(p)} = -k \int_0^\infty dr\, r^2\,[V_C(r)\, R_{lj}^2(r) - V_C^{(p)}(r)\, j_l^2(kr)], \tag{9}$$

where j_l are the spherical Bessel functions, V_C is the model dependent Coulomb potential, $V_C^{(p)}$ the potential for point charges and R_{lj} are the model dependent radial wave functions for partial wave lj in the absence of V_C.

For arbitrary electromagnetic form factors one has to change the second term of equation (9). This can be seen at once because the following identity has to hold:

$$\nu_l^{(f)} + \delta_{lj} + C_{lj}^{(f)} = \nu_l^{(p)} + \delta_{lj} + C_{lj}^{(p)} \tag{10}$$

or

$$C_{lj}^{(f)} = C_{lj}^{(p)} - (\nu_l^{(f)} - \nu_l^{(p)}). \tag{11}$$

To first order in the Coulomb parameter we have

$$\nu_l^{(f)} - \nu_l^{(p)} = -k \int_0^\infty dr\, r^2 (V_C^{(f)}(r) - V_C^{(p)}(r))\, j_l^2(kr). \tag{12}$$

The last two equations generalize equation (9) for the Coulomb corrections

$$C_{lj}^{(f)} = -k \int_0^\infty dr\, r^2 [V_C(r)\, R_{lj}^2(r) - V_C^{(f)}(r)\, j_l^2(kr)]. \tag{13}$$

Choosing V_C equal $V_C^{(f)}$ one gets the formula which Bugg has used in Ref. [5]. But this means that the Coulomb potential is given by equation (1), because it is fixed via the fact that the nuclear cross sections extracted from the new πp data [1] are given with respect to the electromagnetic form factors $(1 - t/m_\rho^2)^{-4}$.

As an example, the differences in the Coulomb phases or Coulomb corrections for the potentials $V_C^{(1)}$ and $V_C^{(2)}$ given by equations (1) and (3), as well as for

$$V_C^{(p)}(r) = 2\eta k/r \tag{14}$$

are calculated to first order in η from equation (12). We have

$$\begin{aligned} \nu_0^{(1)} - \nu_0^{(p)} &= [\ln(1+\gamma_1) + (\gamma_1/6)\,(1+\gamma_1)^{-3}\,(18 + 27\gamma_1 + 11\gamma_1^2)]\,\eta/2 \\ \nu_1^{(1)} - \nu_1^{(p)} &= [\ln(1+\gamma_1) - (\gamma_1/6)\,(1+\gamma_1)^{-3}(6 + 5\gamma_1 + \gamma_1^2)]\,\eta/2, \end{aligned} \tag{15}$$

where $\gamma_1 = 4k^2/m_\rho^2 \sim 0.16 k^2 m_\pi^{-2}$.

$$\begin{aligned} \nu_0^{(2)} - \nu_0^{(p)} &= [\mathrm{Ein}(\gamma_2)]\,\eta/2 \\ \nu_1^{(2)} - \nu_1^{(p)} &= [\mathrm{Ein}(\gamma_2) + (1 - \gamma_2 - \exp(-\gamma_2))\,2/\gamma_2]\,\eta/2 \end{aligned} \tag{16}$$

where $\gamma_2 = a^2 k^2 \sim 0.66 k^2 m_\pi^{-2}$

$$\mathrm{Ein}(x) = \int_x^\infty dt(\exp(-t)/t) + \ln(x) + 0.57722\ldots.$$

III. Results

Two completely different but simple nuclear potentials are chosen to show the weak influence on the Coulomb corrections. To avoid numerical integration in equation (13) the nuclear potentials are not given explicitly but, rather, implicitly by the expressions for the wave functions R_{lj}. The two approximations are

i) the 'polynomial' approximation

$$R_{lj}(r) = \begin{cases} (kr)^l [A_{lj}^{(p)} + B_{lj}^{(p)} kr] & r \leqslant r_N \\ j_l(kr\cos\delta_{lj} - n_l(kr)\sin\delta_{lj} & r \geqslant r_N \end{cases} \tag{17}$$

ii) the 'Bessel' approximation

$$R_{lj}(r) = \begin{cases} A_{lj}^{(b)} j_l(kr) + B_{lj}^{(b)}[(kr)^{-1} + (kr)^l n_l(kr)/(2l-1)!!] & r \leqslant r_N \\ j_l(kr)\cos\delta_{lj} - n_l(kr)\sin\delta_{lj} & r \geqslant r_N \end{cases} \tag{18}$$

The constants A and B are fixed by the condition that the wave functions and their derivatives are continuous at r_N, which is a nuclear range parameter. The functions (17) and (18) lead to potentials shown in Figure 1 for the p_{33} partial wave.

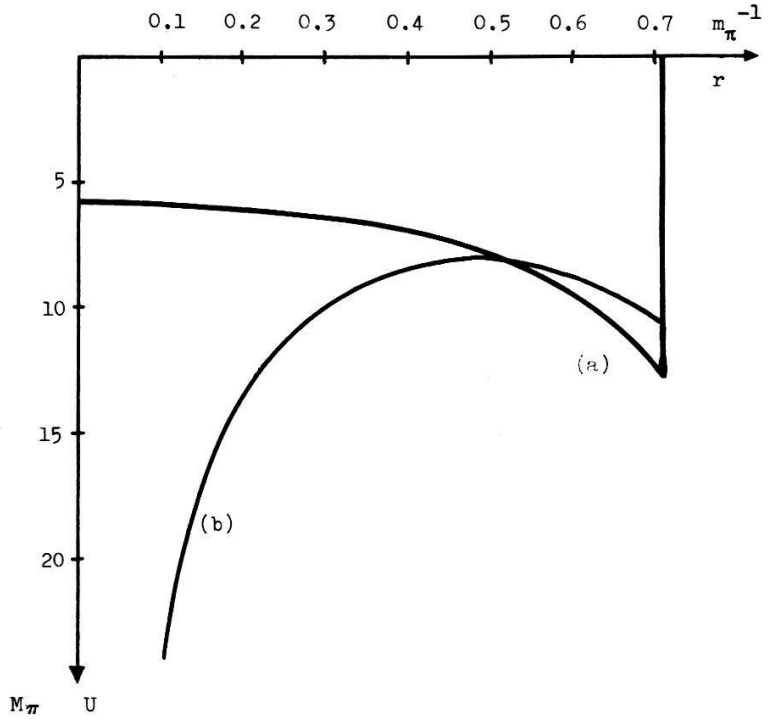

Figure 1
Nuclear potentials for partial wave p_{33} at resonance (1236) for (a) the 'Bessel' and (b) the 'polynomial' approximation.

The Coulomb potential is chosen as

$$V_C(r) = \begin{cases} 2\eta k/r_C[1.5 - 0.5r^2/r_C^2] & r \leqslant r_C \\ 2\eta k/r & r \geqslant r_C \end{cases} \tag{19}$$

where r_C is an electromagnetic range parameter.

The Coulomb corrections to the π^+p phases of Ref. [6] corresponding to electromagnetic form factors $(1 - t/m_p^2)^{-4}$ are calculated and given in Tables I to III. The first two tables contain only p_{33} corrections because these are the only important ones in the low energy range above the atomic region.

Table I gives the corrections for the two approximations (17) and (18). The difference between the corresponding corrections should be compared with the uncertainty arising from the choice of V_C. The next column gives the corrections for a different charge range parameter r_C and comparison shows that there is no need to use more accurate nuclear potentials.

Table I

p_{33} corrections in degrees for different nuclear and electromagnetic potentials (columns 1–3). The numbers in the last column are obtained by numerical integration of the Schrödinger equation. $r_N = 0.71 m_\pi^{-1}$.

Approximation	→	Bessel	Polynomial	Polynomial	Polynomial all orders in η
$r_C(m_\pi^{-1})$	→	0.71	0.71	1.08	0.71
T_{lab} (MeV)	$\delta_{p_{33}}^3$ (degrees)				
94.5	19.82	−0.65	−0.68	−0.61	−0.68
114.1	30.09	−0.93	−0.97	−0.85	−0.97
124.8	36.89	−1.08	−1.13	−0.98	−1.12
142.9	50.93	−1.26	−1.33	−1.11	−1.32
166.0	71.63	−1.16	−1.25	−0.99	−1.25
194.3	94.27	−0.65	−0.72	−0.50	−0.72
214.6	107.63	−0.28	−0.33	−0.17	−0.34
236.3	117.49	−0.04	−0.07	+0.04	−0.07
263.7	126.38	+0.13	+0.11	+0.17	+0.11
291.4	133.54	+0.21	+0.20	+0.23	+0.20

Table II

p_{33} corrections in degrees for different range parameters r_N and r_C, calculated in the polynomial approximation.

$r_N(m_\pi^{-1})$ →	0.71	0.71	0.90	1.00	Results from
$r_C(m_\pi^{-1})$ →	1.22	0.71	0.71	1.08	Ref. [6]
T_{lab} (MeV)					
94.5	−0.58	−0.68	−0.56	−0.48	−0.58
114.1	−0.80	−0.97	−0.77	−0.63	−0.78
124.8	−0.91	−1.13	−0.86	−0.69	−0.91
142.9	−1.02	−1.33	−0.95	−0.70	−1.08
166.0	−0.89	−1.25	−0.78	−0.48	−1.06
214.6	−0.12	−0.33	−0.01	+0.17	−0.40
236.3	+0.07	−0.07	+0.17	+0.28	−0.16

Furthermore, Table I contains the corrections for the 'polynomial' approximation when one does not employ equation (13) which gives only the first-order corrections in η but numerically integrates the Schrödinger equation for potentials corresponding

Table III

Phases (upper numbers) and Coulomb corrections (lower numbers) in degrees. The Coulomb phase differences represent the change of corrections applying different electromagnetic form factors. $r_N = 0.71 m_{\pi}^{-1}$, $r_C = 1.08 m_{\pi}^{-1}$.

T_{lab} (MeV)	$\nu_0^{(1)}$ $-\nu_0^{(p)}$	$S31$	$P31$	$\nu_1^{(1)}$ $-\nu_1^{(p)}$	$P33$	$D33$	$\nu_2^{(1)}$ $-\nu_2^{(p)}$	$D35$
94.5		−8.48	−1.77		19.82	0.04		−0.11
	0.16	0.07	0.03	0.01	−0.61	−0.002	0.001	0.002
114.1		−8.62	−1.97		30.09	0.07		−0.16
	0.18	0.05	0.03	0.02	−0.85	−0.003	0.001	0.002
124.8		−9.80	−3.45		36.89	0.09		−0.19
	0.19	0.06	0.05	0.02	−0.98	−0.003	0.002	0.003
142.9		−11.28	−4.04		50.93	0.11		−0.24
	0.21	0.06	0.05	0.02	−1.11	−0.004	0.002	0.003
166.0		−13.73	−3.14		71.63	0.14		−0.31
	0.24	0.07	0.03	0.03	−0.99	−0.006	0.003	0.003
194.3		−15.34	−5.12		94.27	0.20		−0.45
	0.26	0.06	0.05	0.04	−0.50	−0.008	0.005	0.004
214.6		−17.04	−6.05		107.63	0.23		−0.54
	0.28	0.06	0.06	0.04	−0.17	−0.009	0.006	0.004
236.3		−17.96	−5.63		117.49	0.25		−0.64
	0.30	0.05	0.05	0.05	+0.04	−0.011	0.008	0.004
263.7		−18.88	−7.32		126.38	0.25		−0.77
	0.32	0.04	0.06	0.06	0.17	−0.012	0.010	0.004
291.4		−20.08	−7.44		133.54	0.25		−0.91
	0.34	0.04	0.05	0.07	0.23	−0.014	0.013	0.004

to the radial functions of equation (17) and the Coulomb term of equation (19) and which give exact total phases. One can conclude that first-order results are good enough even in the resonance region.

Table II contains the p_{33} corrections for different range parameters r_N and r_C in the 'polynomial' approximation. It shows the uncertainty of the results if one does not use a phase shift analysis or a dispersion theoretical approach to the nuclear potential as in [5]. The last column shows Bugg's corrections [6] which agree with the values of this calculation for a suitable set of parameters.

All Coulomb corrections up to $l = 2$ are listed in Table III together with the Coulomb phase differences (15). The corrections with respect to point charge amplitudes are obtained from the given corrections by adding these Coulomb phase differences.

The Coulomb corrections of Bugg are calculated without inclusion of mass difference effects. Because it is not clear how these effects influence the phase shift analysis of the low energy $\pi^- p$ data, results for Coulomb corrections with inclusion of mass difference effects will be published in a forthcoming paper.

Acknowledgments

I am grateful to Professors G. Rasche, G. C. Oades and W. S. Woolcock for helpful comments.

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