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On Quantum Measurement Processes

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Abstract. We present a modification of von Neumann's theory of the measurement process, avoiding certain objections raised against the latter. We show that our considerations are non-vacuous by constructing explicitly a mathematical model satisfying the conditions put forward in this paper.

Introduction

The problem of the measuring process in quantum mechanics is to determine whether or not the so-called reduction of the wave-packet is consistent with a deterministic time-evolution. In his book [1] von Neumann gave a positive answer to this question. The mathematical correctness of his proof has not been challenged. The physical interpretation and relevance of von Neumann's result have however been the target of numerous criticisms centered around the meaning to be given to his assumptions and conclusions. It is therefore of paramount importance in the discussion of this problem to avoid inserting conditions which are demonstrably too restrictive on the mathematical formalization of the physical ideas one might have about the nature of the measuring process.

The aim of this note is to present a modification of von Neumann's scheme which avoids some of the criticisms justifiably directed at von Neumann's original treatment of the problem. We start from the following naive view of the process we will attempt to describe. A measuring process is an operation by which some information contained in the unknown state (ψ) of a physical system (Σ) of interest is transferred to the final state ($\eta\phi$) of a measuring apparatus (Υ) brought in interaction with (Σ). The mathematical physics literature abounds in conflicting formalizations and refinements of this idea. In order to present our contribution in as noncontroversial a light as possible we arranged the material of this note as follows:

In the first two sections we describe the expected effects of the measuring process on the system of interest (Σ) and on the apparatus (Υ). In the third section we state the conditions imposed on the admissible dynamical couplings between (Σ) and (Υ). In the fourth section we present a simple, exactly solvable model satisfying these conditions. We open the concluding section with a statement of our results. We then give a brief review of the literature, underlining the improvements that our model is thought to bring to some of the previous contributions to the understanding of the nature of the measuring process in quantum mechanics.

To avoid misunderstanding to occur from the start, we might mention here that we adhere, all along this note, to the Born [2] statistical interpretation of the state ψ of a physical system as representing a summary of the preparation of this system.

1. Effect of the Measuring Process on Σ

The physical system (Σ) of interest is described by its observables $\{A\}$ and its states $\{\psi\}$. For the sake of definiteness, we assume that the observables are the self-adjoint elements of a von Neumann algebra $\mathscr{A}$ acting on a Hilbert space $\mathscr{H}$, and that the states are positive (normal) linear forms on $\mathscr{A}$, normalized to 1. For instance, if Σ is the spin of an electron, $\mathscr{H} = \mathbb{C}^2$, the two-dimensional Hilbert space on the complex numbers; $\mathscr{A} = \mathscr{B}(\mathscr{H})$ the set of all two-by-two matrices with complex entries, equipped with its usual addition, multiplication and hermitian conjugation; the set $\mathscr{S}$ of all states on $\mathscr{A}$ is identified with the set of all density matrices ρ on $\mathscr{H}$, and the expectation value $\langle\psi;A\rangle$ of an observable A, when the system is in the state ψ, is given by $\langle\psi;A\rangle = \operatorname{Tr}\rho A$.

We now assume that we want to measure the observable M_0 in $\mathscr{A}$, or more generally all the observables generated as limits of polynomials in M_0. We denote by $\mathscr{M}$ the abelian von Neumann subalgebra of $\mathscr{A}$ generated by the observables to be measured. We assume furthermore that the spectrum of $\mathscr{M}$ is discrete and finite (although the latter is not essential), i.e. that there exists a (finite) family $\{E_i\}$ of pair-wise orthogonal projectors E_i in $\mathscr{M}$, adding up to I, and such that every self-adjoint element M in $\mathscr{M}$ can be written as $M = \Sigma_i m_i E_i$ with real m_i's.

During the measurement process the state ψ on $\mathscr{A}$ is expected to experience a transformation $\psi \to \eta\psi$, usually referred to as the 'reduction of the wave-packet'. In agreement with the traditional description of this transformation we require that $\eta\psi$ be a mixture $\Sigma_i \lambda_i \psi_i$ of states ψ_i on $\mathscr{A}$ satisfying the following properties: i) the relative weight λ_i of the component ψ_i is given by $\lambda_i = \langle\psi;E_i\rangle$; ii) the components ψ_i entering in $\eta\psi$ are dispersion-free on $\mathscr{M}$ and satisfy $\langle\psi_i;M\rangle = m_i$; in other words M assumes the value m_i with certainty on ψ_i. $\eta\psi$ is uniquely determined by these conditions if the spectrum of M_0 is simple (i.e. nondegenerate). In any case, we should remark that whereas $\langle\eta\psi;A\rangle$ is in general distinct from $\langle\psi;A\rangle$ on an arbitrary observable A in $\mathscr{A}$, $\eta\psi$ and ψ nevertheless coincide when restricted to $\mathscr{M}$.

If we consider again the example of the spin of the electron, we can take for M_0 the z-component σ^z of the spin, and then for $\mathscr{M}$ the algebra of all matrices diagonal with respect to $\{\Psi_i | i = +,-\}$ where $\sigma^z\Psi_\pm = \pm\Psi_\pm$. If now ψ is any (pure) state on $\mathscr{A}$ defined by a normalized vector $\Psi = \Sigma_i c_i \Psi_i$, the state $\eta\psi$ on $\mathscr{A}$ is uniquely defined by the above requirements; it is the state described by the density matrix

$$\rho = \Sigma_i |c_i|^2 E_i = \begin{pmatrix} |c_+|^2 & 0 \\ 0 & |c_-|^2 \end{pmatrix}$$

where $E_i (i = +,-)$ is the one-dimensional projector on the subspace $\{\lambda\Psi_i | \lambda \in \mathbb{C}\}$. For a general state ψ on $\mathscr{A}$ the measuring process changes the corresponding density matrix ρ into the density matrix $\eta\rho$ defined by $(\eta\rho)_{ij} = \rho_{ii}\delta_{ij}$.

Clearly the process $\psi \to \eta\psi$ is different from that given by the ordinary time-evolution $\psi \to \psi^t$ generated by any Hamiltonian H on $\mathscr{H}$. In particular, ψ pure implies ψ^t pure, whereas $\eta\psi$ is in general an (incoherent) mixture.

One way to produce the transformation $\psi \to \eta\psi$ would be to define $\eta\psi$ as the ergodic average of ψ^t for some Hamiltonian time-evolution:

$$\langle \eta\psi; A\rangle = \lim_{T\to\infty} \frac{1}{T} \int_0^T dt \langle \psi^t; A\rangle;$$

in the present case $H = -B\sigma^z$ would do. This procedure is, however, unsatisfactory on at least two accounts: a) it does not clearly relate with what the physicist seems to be doing when he measures a physical observable; b) it does not involve, except in an overly implicit way, the measuring apparatus and its interaction with Σ.

2. Effect of the Measuring Process on Υ

The measuring apparatus (Υ) is also described by its observables $\{B\}$ and its states $\{\phi\}$. We also assume that its observables are the self-adjoint elements of a von Neumann algebra $\mathscr{B}$ acting on a Hilbert space $\mathscr{G}$. The corresponding description is given for the states in complete analogy with the conventions taken in Section 1.

We now suppose that Υ is devised in such a manner as to provide a measurement of the observables in $\mathscr{M}$. Specifically we assume that the measuring apparatus has exactly as many mutually exclusive outcomes as M_0 has distinct eigenvalues. Mathematically this means that there exists in $\mathscr{B}$ a (finite) family $\{F_i\}$ of mutually orthogonal projectors F_i acting on $\mathscr{G}$ such that the index set of $\{F_i\}$ is the same as that of $\{E_i\}$ (i.e. in particular $E_i \neq E_j$ implies $F_i \neq F_j$). Let $\mathscr{N}$ be the algebra generated by all those observables on Υ which are of the form $N = \Sigma_i n_i F_i$. This algebra is evidently abelian.

We further require that during the measuring process the initial state ϕ of the measuring apparatus evolve to a state, the restriction $(\eta\phi)$ of which to $\mathscr{N}$ is a mixture $\Sigma_i \lambda_i \phi_i$, where $\lambda_i = \langle \psi; E_i\rangle$ (we recall that ψ is the state of the system of interest, see Section 1) and ϕ_i are dispersion-free states on $\mathscr{N}$ with $\langle \phi_i; N\rangle = n_i$. We thus will have $\langle \eta\phi; N\rangle = \Sigma_i \lambda_i n_i$ so that we can compute λ_i from the observed values of the observables N in $\mathscr{N}$, and hence compute $\langle \psi; M\rangle = \Sigma_i \lambda_i m_i$ for every M in $\mathscr{M}$. These are the quantities of interest and a physical system Υ satisfying the above assumptions qualifies therefore as a measuring apparatus for the set $\mathscr{M}$ of observables on Σ.

Again, in the case of the measurement of the spin of the electron, the measuring apparatus should admit exactly two outcomes, i.e.

$$\mathscr{N} = \{\Sigma_i z_i F_i | z_i \in \mathbb{C}, i = +, -\}$$

and

$$\langle \eta\phi; N\rangle = \lambda_+ n_+ + \lambda_- n_-.$$

3. Deterministic Processes

We now consider the composite system (Σ, Υ). Its algebra of observables is $\mathscr{C} = \mathscr{A} \otimes \mathscr{B}$. We assume that for $t \leqslant 0$ the two systems are uncoupled, so that the initial state of the composite system is $\chi = \psi \otimes \phi$. We then turn the interaction on.

We say that a *measuring process is deterministic* if it is possible to find a deterministic (unitary) time-evolution $\chi \to \chi^t$ on (Σ, Y), and a family $\mathcal{T}_0$ of initial states ϕ of Y, such that for all ϕ in $\mathcal{T}_0$ and all states ψ of Σ,

$$\begin{cases} \lim_{t\to\infty} \langle \chi^t ; A \otimes I \rangle = \langle \eta\psi ; A \rangle & \text{for all } A \text{ in } \mathscr{A} \\ \lim_{t\to\infty} \langle \chi^t ; I \otimes N \rangle = \langle \eta\phi ; N \rangle & \text{for all } N \text{ in } \mathscr{N} \end{cases}$$

where $\eta\psi$ and $\eta\phi$ have been defined in the preceding sections. The family $\mathcal{T}_0$ will be referred to as the *admissible* family of states of Y for the process considered.

Our first problem is to give a constructive proof that deterministic processes do exist. This will be done in the next section.

Our next problem will be to show that the above limits can be reached in a finite time for a large enough class $\mathcal{T}_0$ of initial states of the measuring apparatus Y. This problem will be solved in the concluding section of this note.

4. A Model

We want to give a model for the measurement of the z-component of the spin of the electron. We have thus (see Section 1) $\mathscr{A} = \mathscr{B}(\mathbb{C}^2)$ and $M_0 = \sigma^z$. In this 'gedanken experiment', inspired by the Stern–Gerlach experiment, the unknown information we are seeking is thus $\langle \psi ; \sigma^z \rangle$. We want it to be transferred to the final configuration state of the electron in such a manner that we can compute $\langle \psi ; \sigma^z \rangle$ from as crude a measurement of its position as is possible. We take thus for the algebra $\mathscr{B}$ generated by the observables on Y, the algebra $\mathscr{B}(\mathscr{G})$ of all bounded operators acting on $\mathscr{G} = \mathscr{L}^2(\mathbb{R}, dx)$. Let $F(a)$ be the projector on $\mathscr{G}$ defined by $\{(F(a)\Phi)(x) = 0$ for all $x < a$, and $(F(a)\Phi)(x) = \Phi(x)$ for all $x \geqslant a\}$, and take $F_+ = F(0)$, $F_- = I - F_+$.

The Hilbert space of the composite system is thus $\mathbb{C}^2 \otimes \mathscr{G} = \mathscr{G} \oplus \mathscr{G}$. Consequently every C in $\mathscr{C} = \mathscr{A} \otimes \mathscr{B}$ is a two-by-two matrix with entries in $\mathscr{B}(\mathscr{G})$, i.e. $C = (C_{ij})$ with $C_{ij} \in \mathscr{B}(\mathscr{L}^2(\mathbb{R}, dx))$.

We take $V(t) = \exp\{-i(\sigma^z \otimes P)t\}$ (with P self-adjoint extension of $[-i(d/dx)]$ defined on $\mathscr{S}(\mathbb{R})$) and notice that $V(t) = (U_i(t)\delta_{ij})$ with $(U_\pm(t)\Phi)(x) = \Phi(x \mp t)$. The mapping $C \to C^t \equiv V(-t)CV(t)$ clearly gives a deterministic (unitary) time-evolution on the composite system (Σ, Y).

The model is thus completely defined and we now prove that it leads to a deterministic process for *all* initial states ϕ on Y.

Let $\Psi = \Sigma_i c_i \Psi_i$ with $\sigma^z \Psi_\pm = \pm \Psi_\pm$ be an arbitrary vector in $\mathbb{C}^2$, ψ be the state on $\mathscr{A}$ defined by $\langle \psi ; A \rangle = (\Psi, A\Psi)$, Φ be an arbitrary vector in $\mathscr{G}$ and ϕ be the corresponding state on $\mathscr{B}$. Let $\chi = \psi \otimes \phi$ be the initial state of the composite system. We thus have for any C in $\mathscr{C}$: $\langle \chi ; C \rangle = \Sigma_{ij} c_i^* c_j (\Phi, C_{ij} \Phi)$ and then:

$$\langle \chi^t ; A \otimes I \rangle = \Sigma_{ij} c_i^* c_j a_{ij} (\Phi, U_i(-t) U_j(t) \Phi)$$

$$\langle \chi^t ; I \otimes B \rangle = \Sigma_i |c_i|^2 (\Phi, U_i(-t) B U_i(t) \Phi).$$

Since P has absolutely continuous spectrum, and since $U_\pm(t) = U(\pm t)$, we have by Lebesgue–Riemann's lemma:

$$\lim_{t\to\infty} (\Phi, U_i(-t) U_j(t) \Phi) = \delta_{ij}$$

and hence for all A in $\mathscr{A}$:

$$\lim_{t\to\infty} \langle \chi^t; A \otimes I\rangle = \Sigma_i |c_i|^2 a_{ii} = \langle \eta\psi; A\rangle.$$

On the other hand, we notice that $U(-t)F(a)U(t) = F(a-t)$ so that

$$\lim_{t\to\infty} (\Phi, U_i(-t)F_j U_i(t)\Phi) = \delta_{ij};$$

we have hence for all N in $\mathscr{N}$

$$\lim_{t\to\infty} \langle \chi^t; I \otimes N\rangle = \Sigma_i |c_i|^2 n_i = \langle \eta\phi; N\rangle.$$

We remark that $\eta\phi$ does *not* depend on the vector state ϕ chosen as the initial state of the measuring apparatus. The above results are immediately extended by linearity to any arbitrary state ψ on $\mathscr{A}$ and to any (normal) state ϕ on $\mathscr{B}$.

Concluding Remarks

We constructed explicitly a model satisfying the conditions stated in Section 3 for a deterministic measuring process. We conclude therefore that there is no contradiction in principle between the reduction of the wave-packet and a unitary time-evolution.

We should remark that the class $\mathscr{T}_0$ of admissible initial states of the apparatus Υ extends to *all* (normal) states on $\mathscr{B}$. We emphasize that this means exactly that *whatever* the initial state ϕ of the apparatus Υ, the evolution of the combined system (Σ, Υ) is such that, in the limit where t tends to infinity, the expectation of *all* observables A in $\mathscr{A}$, and *all* observables N in $\mathscr{N}$, will tend to the prescribed values which depend *only* on A (or N) and on the initial state ψ of Σ, but *not* on the initial state ϕ of Υ.

We want furthermore to emphasize that the speed of convergence of the above process can be controlled by a mild control of the initial state ϕ of Υ. In particular, for any a, b in $\mathbb{R}$ with $-\infty < a < b < +\infty$, define the projector $F(a,b) = F(a) - F(b)$ acting on $\mathscr{G}$; for the definition of $F(a)$ see Section 4. For fixed a and b, to impose that a (normal) state ϕ of Υ satisfies $\langle\phi; F(a,b)\rangle = 1$ means that ϕ is (at worst) a mixture of vector states the wave functions of which have support in (a,b), or in more physical terms that the electron is located in the finite interval (a,b). One verifies easily that for any fixed a and b, there exists a finite time T (depending on a and b) such that for all χ of the form $\psi \otimes \phi$ with ψ arbitrary on $\mathscr{A}$, all ϕ satisfying $\langle\phi; F(a,b)\rangle = 1$, and all $t \geqslant T$ we have

$$\langle \chi^t; A \otimes N\rangle = \Sigma_i \lambda_i \langle \psi_i; A\rangle n_i$$

so that χ^t, restricted to $\mathscr{A} \otimes \mathscr{N}$ becomes a mixture of the form $\Sigma_i \lambda_i \psi_i \otimes \phi_i$; we have thus in particular:

i) $\langle \chi^t; A \otimes I\rangle = \langle \eta\psi; A\rangle$
$= \Sigma_i \lambda_i \langle \psi_i; A\rangle$ for all A in $\mathscr{A}$

ii) $\langle \chi^t; I \otimes N\rangle = \langle \eta\phi; N\rangle$
$= \Sigma_i \lambda_i n_i$ for all N in $\mathscr{N}$

iii) $\langle \chi^t; E_i \otimes F_j\rangle = \lambda_i \delta_{ij}$

and in particular:

iv) $\langle\chi^t; P\rangle = \Sigma_i \lambda_i p_i$ for all $P = \Sigma_i p_i E_i \otimes F_i$

(we recall that $\lambda_i \equiv \langle\psi; E_i\rangle$ depend only on the initial state of Σ).

These relations, and in particular i) and ii), mean that by an appropriate, but minor, restriction of the class of admissible initial states, namely $\langle\phi; F(a,b)\rangle = 1$ for some finite a and b, ensures that the measuring process is completed in a *finite* time T. This lower bound on the time necessary to complete the measurement depends evidently on a and b, i.e. on the care with which one is willing to prepare the apparatus. It is nevertheless remarkable that T is finite as long as a and b are finite.

We now want to compare these results with those found in some of the literature [3].

Von Neumann [1] considers the case where $\mathscr{A} = \mathscr{B}(\mathscr{H})$, $\mathscr{B} = \mathscr{B}(\mathscr{G})$ and M_0 has simple, discrete spectrum. There exists then an orthonormal basis $\{\Psi_i\}$ in $\mathscr{H}$ such that $M = \Sigma_i m_i E_i$ where E_i is the one-dimensional projector on Ψ_i. Let us denote by ψ_i the state on $\mathscr{A}$ defined by $\langle\psi_i; A\rangle = (\Psi_i, A\Psi_i)$ for all A in $\mathscr{A}$. Our condition on the effect of the measuring process on Σ reduces to von Neumann's form: $\eta\psi = \Sigma_i \lambda_i \psi_i$; in particular, if Ψ is an arbitrary vector in $\mathscr{H}$, $\lambda_i = |c_i|^2$ with $c_i = (\Psi_i, \Psi)$. Let, further, $\{\Phi_i\}$ be an orthonormal basis in $\mathscr{G}$, the elements of which are in *one-to-one correspondence* with the elements of $\{\Psi_i\}$; let F_i be the corresponding *one-dimensional* projectors in $\mathscr{G}$. von Neumann now imposes his famous 'consistency conditions' that for every Ψ in $\mathscr{H}$ and for a *fixed* Φ in $\mathscr{G}$ and a *fixed* time τ (identified as the duration of the measuring process):

$$\begin{cases} X^\tau \equiv V(\tau)\Psi \otimes \Phi \\ (X^\tau, E_i \otimes F_j X^\tau) = \lambda_i \delta_{ij}. \end{cases}$$

These conditions imply that $X^\tau = \Sigma_i c_i \Psi_i \otimes \Phi_i$ with $|c_i|^2 = \lambda_i = |(\Psi, \Psi_i|^2$. Upon defining χ^τ on $\mathscr{C} = \mathscr{A} \otimes \mathscr{B}$ by $\langle\chi^\tau; C\rangle = (X^\tau, CX^\tau)$ we get immediately that this vector state on $\mathscr{C}$ satisfies formally the properties i) to iv) stated above for our model, and that actually iii) can be strengthened, in von Neumann's case, to:

iii)′ $\langle\chi^\tau; I \otimes B\rangle = \Sigma_i \lambda_i (\Phi_i, B\Phi_i)$ for all B in $\mathscr{B}$

(and not only, as in iii), for all N in $\mathscr{N}$). The formal similarity of these properties is emphasized by our notation; one should however realize, in particular, that in von Neumann's treatment the projectors F_i are one-dimensional projectors. We do not require this very stringent condition in our approach; as we shall now argue, this extension allows us to avoid several of the difficulties encountered with von Neumann's original description of the measuring process in quantum mechanics.

Firstly, von Neumann's description requires that the duration τ of the measuring process be *sharply* calibrated. If the time t in which Σ and Υ are kept in interaction is not exactly equal to τ, the measurement is ruined, and so is, in particular, the reduction of the wave-packet. (We could point out in this respect that the one-to-one correspondence between $\{\Psi_i\}$ and $\{\Phi_i\}$ assumed by von Neumann implies, in case the spectrum of M consists of finitely many simply eigenvalues, that $\mathscr{H} \otimes \mathscr{G}$ is finite-dimensional, so that if Σ and Υ are left in contact $\langle\chi^t; A\rangle$ will be at best an almost periodic function of time.) This is clearly unacceptable. Our model avoids this difficulty: when ϕ is appropriately chosen, there exists a finite time T in which the relevant expectation values reach their prescribed values and *after* which *they do not change*, even if Σ and Y are kept in interaction with one another. Hence the duration of the measuring process in our model need not be sharply calibrated; it must only exceed a minimum time T which depends only on how well we are willing to prepare the initial state(s) of the measuring apparatus.

Secondly, in von Neumann's approach, given $V(\tau)$, the initial state of the measuring apparatus should exactly be the vector state ϕ corresponding to Φ where the latter is determined by the consistency conditions $\Psi \otimes \Phi = V(-\tau)X^\tau$ and $X^\tau = \Sigma_i c_i \Psi_i \otimes \Phi_i$. Our model avoids this difficulty too. Indeed given T we can find (a, b) such that all initial states ϕ which satisfy $\langle \phi; F(a, b)\rangle = 1$ are admissible (see discussion in the beginning of this section).

To our knowledge, none of the alternatives to von Neumann's approach so far presented in the literature have considered and solved these two difficulties.

Thirdly, the most often [3] heard criticism of von Neumann's approach to the measurement problem in quantum mechanics is that von Neumann's consistency conditions only transfer the problem of measuring ψ on Σ to that of measuring χ^τ on (Σ, Y) and thus involves a chain of measurements ending somewhere in the observer's consciousness. Jauch [4] disposed of this criticism by noticing that we do not need to measure χ^τ on the whole system (Σ, Y) but only on the subalgebra $\mathscr{P}$ of observables of the form $\Sigma_i p_i E_i \otimes F_i$. This algebra is abelian and thus classical, so that any subsequent measurement of χ^τ with respect to these observables does not perturb the state χ^τ (in contradistinction to the quantum change $\psi \to \eta\psi$ occurring in Σ during the first measurement). As we noticed in the beginning of this section, our model also satisfies the condition $\langle \chi^t; P\rangle = \Sigma_i \lambda_i p_i$ for all $t \geqslant T$, so that Jauch's argument, originally presented in the more restricted framework of von Neumann's model, can indeed be extended to the situation considered here. This argument applies also to the state $\eta\phi$ on the abelian algebra $\mathscr{N}$, obtained as the restriction of χ^t to $I \otimes \mathscr{N}$. We remark here again that whereas $\mathscr{A}$ itself (and not just $\mathscr{M}$) is of interest, $\mathscr{N}$ (and not the whole algebra $\mathscr{B}$) really pertains in an essential manner to Y considered as an apparatus devised to measure $\langle \psi; M_0\rangle$.

In connection with the question just discussed, it is often asserted that the measuring apparatus should be 'classical and thus macroscopic', or that 'the measuring process should bring a macroscopic change in the state of the apparatus'. Jauch [4] also discussed this question, distinguishing the microscopic quantum process ('event') and its subsequent amplification ('datum'). In our model, the interaction induces the following change in the state of the apparatus. From an initial situation where ϕ is localized in a finite region (a, b), the apparatus is driven to a final situation where, depending on whether the spin is up or down (i.e. $\langle \psi; \sigma^z\rangle = +1$ or -1), the particle moves towards $+\infty$ or $-\infty$. The algebra $\mathscr{N}$ of the relevant observables on Y tests exactly this alternative. The reader will verify that we do not even need to define F_+ and F_- as sharply as we did. For instance, we could have taken $F_+ = F(\beta)$ and $F_- = I - F(\alpha)$ with α and β specified only within an interval of finite, but arbitrary, length ϵ. In this sense, the measurement process indeed induces a 'macroscopic' change in the state of the apparatus. Hepp [5] recently investigated the consequences of a more rigid definition of this requirement. He gave a precise mathematical meaning to the 'macroscopicity' condition using the (abelian) algebra of the 'observables at infinity' [6] on a local quantum system. Under a reasonable assumption on the continuity of $\chi \to \chi^t$ Hepp showed however that the resulting condition of asymptotic disjointness implies that a measurement which brings a macroscopic change in the apparatus can *not* be completed in a finite time. Aside from the fact that we did not request as much 'macroscopicity' from the measuring apparatus as Hepp does, our theory differs from Hepp's in at least one other aspect. In our Section 3, we did not require the convergence (as $t \to \infty$) of χ^t as a state on $\mathscr{C} = \mathscr{A} \otimes \mathscr{B}$, but only of its restriction to the 'relevant observables', namely those belonging to the subspace $(\mathscr{A} \otimes I) \oplus (I \otimes \mathscr{N})$ of $\mathscr{C}$. Incidentally, the subalgebra $\mathscr{A} \otimes \mathscr{N}$ considered by Prosperi et al. [7] seems to be larger than required for the purpose of understanding the measuring process; moreover their introduction of ergodic considerations in the measuring process rests on still unsettled premises which the simplicity of our model bypasses.

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