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Minimum Radius of Particles with Spin

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(15. XII. 71)

According to the generalized first law (momentum-energy and centre of energy-angular momentum), the density tensor of any insulated system Σ_{00} satisfies, in restricted relativity (r.r.)

$$\partial_\alpha \Theta^{\alpha\beta}(x) = 0 \quad (\Theta^{\alpha\beta} = \Theta^{(\alpha\beta)})(x). \quad (1)$$

The energy is

$$H = \int_{\infty} (d^3 V h)(\vec{x}\vec{t}) = \int_{\infty} (d^3 V \Theta^{44})(x) = \int_{\tau(y)=0} (d\check{\sigma}_\alpha \check{\Theta}^{\alpha 4})(y) \quad (2)$$

and the momentum

$$\check{\vec{P}} = \check{\vec{\Pi}} = \{\check{\Pi}_i\} \quad \check{\Pi}_i = \int_{\tau(y)=0} (d\check{\sigma}_\alpha \check{\Theta}_i^\alpha)(y) = \int_{\infty} (d^3 V \pi_i)(\vec{x}\vec{t}).$$

The angular momentum $\check{\vec{M}} = \{\check{M}^i = M_{kl}, ikl \curvearrowleft 123\}$ has the components

$$\check{M}_{ik} = \int_{\tau(y)=0} (d\check{\sigma}_\alpha (y_i \Theta_k^\alpha - y_k \Theta_i^\alpha))(y) = \int_{\infty} (d^3 V (x_i \pi_k - x_k \pi_i))(\vec{x}\vec{t}) \quad (3)$$

and the centre of energy is determined by

$$\vec{M} = \vec{M} = \left\{ M_i = M_{i4} = \int (d\check{\sigma}_\alpha (y_i \check{\Theta}^{\alpha 4} - \check{y}^4 \Theta_i^\alpha))(y) = \int_{\infty} (d^3 V x_i h)(\vec{x}\vec{t}) - \check{t} \check{\Pi}_i \right\} \quad (4)$$

the $\check{M}_{\alpha\beta} = \check{M}_{(\alpha\beta)}$ being constants.

$$\left. \begin{aligned} \check{z}(\check{t}) &= \frac{\vec{M}}{H} + \check{t} \frac{\check{\vec{\Pi}}}{H} = \frac{1}{H} \int_{\infty} (d^3 V \vec{x} h)(\vec{x}\vec{t}) \\ \check{z}^4(\check{t}) &= \check{t} \end{aligned} \right\} \quad (5)$$

$d\check{z}(\check{t})/d\check{t} = \check{\vec{\Pi}}/H \equiv \check{v}$ is the 3-velocity of the centre of energy (c.e.) $\check{z}(\check{t})$.

¹⁾ Paper presented at the Swiss Physical Society Meeting of Lausanne, 1 May, 1971, see: Helv. phys. Acta, 44, 593 (1971).

In this frame τ (= proper time) $\stackrel{*}{=} \check{t}$, and $\vec{w} \stackrel{*}{=} \check{\vec{v}}$; $\check{w}^4 \stackrel{*}{=} +1$. So we may write (5) in a covariant way

$$\left. \begin{aligned} z^i(\tau) &= M^i + \tau w^i, \\ \check{z}^4(\tau) &= \tau \check{w}^4. \end{aligned} \right\} \quad (6)$$

Now we may choose in this frame $\vec{M} = 0$. Then the *world-line* (w.l.) $x^\alpha = z^\alpha(\tau)$ coincides with the $\check{t} = \check{x}^4$ -axis in this particular frame. Let $\tilde{M} \neq 0$ be the intrinsic angular momentum (spin: $|\tilde{M}|^2 = \hbar^2 S(S+1)$) in this frame. Then, in any other frame ' $x = Lx$ ', obtained by homogeneous Lorentz transformations L^2 from x , the w.l. ' $x = 'z(\tau)$ ' is *parallel* (or *anti-parallel*) to the w.l. $x = z(\tau)$ (depending whether '*not*' or '*yes*' time inversion is included). ' \tilde{M} ' however, as soon as $\tilde{M} \neq 0$, no longer disappears in general. That means *these (anti-)parallels are shifted by an amount*, which can be easily obtained by L :

$$'M'^4_i = 'M'^i'4 (\equiv 'M'^i) = (L'_i L'_k - L'_k L'_i) \tilde{M}^l (ikl \cup 123), \quad (7)$$

where, taking (without time inversion $\text{sig}(\check{t}) = \text{sig}(\check{t})$)

$$\left. \begin{aligned} 'x'^1 &= x^1 \\ 'x'^2 &= x^2 \\ 'x'^3 &= (1 - v_0^2)^{-1/2} (x^3 - \check{v}_0 \check{t}) \\ 't &= 'x'^4 = (1 - v_0^2)^{-1/2} (-\check{v}_0 x^3 + \check{t}) \end{aligned} \right\} \quad (8)$$

which determines

$$L'_3 = L'_4 = (1 - v_0^2)^{-1/2} \quad \check{L}'_3 = -\check{v}_0 (1 - v_0^2)^{-1/2} = \check{L}'_4 \quad (9)$$

all other L'_β being zero or one. One thus obtains ($v_0^3 = v_0$; $v_0^2 = v_0^1 = 0$), taking account of ' $H = M(1 - v_0^2)^{-1/2}$ ', for the displacement of the c.e.:

$$\frac{'\vec{M}}{'H} = -\frac{[\vec{v}_0 \wedge \vec{M}]}{M} \quad (10)$$

The set of all c.e.'s fills a disk, normal to \vec{v}_0 and to \vec{M} , whose radius R is given by $|\vec{v}_0| = 1 - \epsilon$, $\epsilon \rightarrow +0$:

$$R = \frac{|\vec{M}|}{M} (1 - \epsilon) \Rightarrow \frac{\hbar}{M} \sqrt[+] {s(s+1)}, \quad s = 0, \frac{1}{2}, 1, \frac{3}{2}, \dots \quad (11)$$

Now $\hbar/M = h/2\pi M = \lambda_c$ ($c = 1$) the *Compton wave length* of a particle with mass M . The result is quite satisfactory, because the optimum location of a particle of mass

²⁾ We choose

$$g^{ii} = g_{ii} = -g^{44} = -g_{44} = 1; \quad g_{\alpha\beta} = g^{\alpha\beta} = 0 \text{ for } \alpha \neq \beta \quad (7a)$$

' $x = Lx$ ' is given by

$$'x'^\alpha = L'^\alpha x^\alpha \quad 'g'^{\alpha'\beta} = L'^\alpha_\alpha L'^\beta_\beta g^{\alpha\beta} \equiv g'^{\alpha'\beta} \quad (7b)$$

M is given by the invariant wave packet $D_{\kappa}^+(x)$ ($\kappa = \hbar^{-1}M = \lambda_c^{-1}$), which decreases exponentially for space-like events $x^2 = x_\alpha x^\alpha > 0$

$$D_{\kappa}^+(x) \propto |x|^{-3} \exp\left(-\frac{|x|}{\lambda_c}\right), \quad |x| > \lambda_c \quad (12)$$

while for time-like events, it behaves $t^2 = -x^2 = -x_\alpha x^\alpha > 0$

$$D_{\kappa}^+(x) \propto |t|^{-3} \exp\left(+i\frac{|\check{t}|}{\lambda_c}\right), \quad |\check{t}| > \lambda_c \quad (13)$$