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A Coupled Channel Approach to the Isomer Fission State[1])

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Abstract. It has been shown that the Isomer Fission States can originate from the simple channel coupling phenomenon. The set of coupled channel equations of the recently proposed theory of fission are treated in two approximations: (i) a simple two coupled channel case and (ii) a coupled system of N channels. In each case, under a certain physical assumption, it has been possible to find a transformation which uncouples the N channels. In the two channel case, the longer and the shorter half-lives can, respectively, be identified with the spontaneous and the isomer fission states. The N channel case reduces effectively to only 3 uncoupled equations. One of these corresponds to the scattering process for the parameters pertinent to the fission case. Thus, the N channel case reduces effectively to two independent decay channels. The half-lives associated with these two decay channels can again be identified with the decay of the spontaneous and isomer fission states. This treatment of the multichannel decay process is equally valid for the Bohr-Wheeler type of theory and can also be applied to uncouple the scattering problem, which fulfills the basic assumption of this model.

1. Introduction

Recent proposed theory of fission [1] is based on the sudden approximation and parallels a reaction theory (henceforth referred to as QMF). A simple version of that theory could (i) reproduce the observed spontaneous fission half-lives of ^{234}U, ^{240}Pu, ^{244}Cm, and ^{246}Cf, (ii) account for the mass distribution spectrum. In particular, the fission half-lives associated with the asymmetric decay modes are found to be considerably shorter than those associated with the symmetric decay modes. This means that, in agreement with observation, the yield curve is peaked towards asymmetric masses, and (iii) be made consistent with the kinetic energy spectra associated with the mass distribution. Further, it has been shown [2] that within the context of a single channel decay process, the observed kinetic energy spectra are not consistent with the Bohr-Wheeler-[3, 4] Strutinski [5] type (referred to further on as BWS) of barrier. The purpose of this note is to point out that a simple channel coupling in a decay process can give rise to the observed isomer fission states (denoted as IFS). Although we shall be discuss-

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ing and applying 'the coupled channel decay process' within the context of the QMF, it is equally applicable to the channel coupling problem associated with the BWS model and other reactions.

Of course, the isomer fission can easily be accommodated with the theory of Ref. [1], if the kinetic energy spectra associated with the mass distribution of the decay of the isomer states is on the average 30 to 40 MeV higher than those found in the case of spontaneous fission. As for example, in the decay of the isomer state of ^{240}Pu to ^{142}Ba and ^{98}Sr, if the kinetic energy is taken to be 208 MeV (which is about 36 MeV higher than that observed in the case of the spontaneous decay to this particular daughter pair), one obtains a half-life of 5×10^{-9} sec, using the same set of parameters given in Ref. [1]. This set of parameters can account for the spontaneous fission half-lives of ^{240}Pu to the same decay mode also. A somewhat more realistic barrier which replaces the square well type used in Ref. [1], can also account for the spontaneous half-lives of aforementioned nuclei, e.g., a potential of the following type

$$V(R) = \begin{cases} V_c & R < R_c \\ V_0 & R_c \leqslant R < R_b \\ V_{Coul} \exp\left[\dfrac{R - R_0}{d}\right] & R_b \leqslant R < R_0 \\ \dfrac{Z_1 Z_2 e^2}{R} & R_0 < R \end{cases} \tag{1.1}$$

where the standard value of 200 MeV is chosen for the core height

$$V_c,\ R_c = 8\,\text{fm},\ V_0 = V_{Coul} - 60\,\text{MeV},\ V_{Coul} = \frac{Z_1 Z_2 e^2}{R_0},\ R_b = R_0 + d \log\left(\frac{V_0}{V_{Coul}}\right),$$

Z_1 and Z_2 are the atomic numbers associated with the two daughter nuclei, $R_0 = [1.1(A_1^{1/3} + A_2^{1/3}) + 2.7]$ fm, and $d = 2.3$ fm, along with a preformation probability of 10^{-5} yields half-lives of 8×10^{16} years, 1×10^{11} years, 2×10^{7} years, 7×10^{3} years for ^{234}U, ^{240}Pu, ^{244}Cm, and ^{246}Cf, respectively, to the dominant decay modes of ^{142}Xe + ^{92}Sr, ^{142}Ba + ^{98}Sr, ^{144}Ba + ^{100}Zr, ^{144}Ce + ^{102}Zr, respectively using observed kinetic energy spectrum. If, in the decay of ^{240}Pu to ^{142}Ba + ^{98}Sr, the kinetic energy is increased to 205 MeV, using the same potential (1.1) we obtain a half-life of only 3×10^{-9} sec. This is consistent with the half-life of the IFS of ^{240}Pu.

The above example stresses the importance of obtaining more accurate information on the kinetic energy spectra for the decay modes of the isomer fission states. Unfortunately, our information on this point is still very sketchy. On the other hand, the initial measurements of Ferguson, Plasil, Alam, and Schmitt [6] imply that the *average* kinetic energy associated with the decay of the isomer fission state is *about the same* as that of the decay of the spontaneous fission state. It is therefore a plausible conjecture that the kinetic energy spectra or at least their maxima in the two cases are nearly equal. Based on this hypothesis, we propose to examine the mechanism of the decay of the (IFS) within the framework of the coupled channel theory of fission enunciated in Ref. [1]. We, first, present a simple case and then a more sophisticated version. A preliminary version of this theory is recorded in Ref. [7].

A key advantage of ascribing the IFS to the channel coupling phenomenon, is that the effective potentials in the spontaneous fission and in the IFS channels are quite different. This can be seen from equation (2.4) or equation (3.6). As a result a radiative transition from the IFS to the spontaneous fission state is very unlikely. This is in conformity with the present experimental knowledge [8, 9].

The key to this theory is the set of coupled equations [1] governing the decay process:[4])

$$\left[-\frac{\hbar^2}{2\mu}\nabla_{\boldsymbol{R}}^2 - E_{n\alpha} + U_{n\alpha\alpha}(\boldsymbol{R}) + K_{n\alpha\alpha}(\boldsymbol{R})\right] f_{n\alpha}(\boldsymbol{R})$$

$$= \sum_{\beta\neq\alpha}^{N} [U_{n\alpha\beta}(\boldsymbol{R}) f_{n\beta} + K_{n\alpha\beta}(\boldsymbol{R}) f_{n\beta}] \tag{1.2}$$

where $\boldsymbol{R}$ is the relative location of the centers of mass of the two daughter nuclei, μ is their reduced mass, $E_{n\alpha}$ is the channel kinetic energy, at large $|\boldsymbol{R}|$, associated with a given channel $(n\alpha)$, $K_{n\alpha\beta}$ is a non-local potential originating from the Pauli exclusion principle, and the potential $U_{n\alpha\beta}(\boldsymbol{R})$ is the scalar product

$$(\varphi_{n\alpha}, H_{int}\,\varphi_{n\beta}) \tag{1.3}$$

with $\varphi_{n\alpha}$ being the eigenfunctions of the sum of the two *intrinsic* daughter Hamiltonians, and H_{int} is the interaction Hamiltonian between the two daughter nuclei. We shall use N for the number of possible channels related to different modes of excitations of a given daughter pair and n refers to the decay of the parent nucleus to a specific pair.

We proceed to examine the effect of such a coupled system to the tunneling process in two important approximations.

2. The Simple Model of Two Channels

For simplicity, let us assume that there exists only *two* important coupled channels; i.e., $N = 2$ in equation (1.2) and we expect these two channels to be related to the spontaneous and isomer fission modes. Our hypothesis is that the channel kinetic energy, for large values of $|\boldsymbol{R}|$, in two cases are nearly equal. Representing for the sake of simplicity the sum of the potential $U_{n\alpha\beta} + K_{n\alpha\beta}$ by an effective local potential $V_{n\alpha\beta}$ and assuming this potential is to be spherically symmetric, we may apply partial wave analysis to (1.2). For the zero angular momentum case, equation (1.2) reduces to

$$\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dR^2} - E_n\mathbf{1} + \mathbf{V}_n(R)\right]\mathfrak{F}_n(R) = 0 \tag{2.1}$$

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[4]) A number of authors in trying to relate either the resonating group method or the ion-ion potential to the fission process have implicitly assumed such a set of equations. They are: H. FAISSNER and K. WILDERMUTH, Phys. Lett. *2*, 212 (1962); Nucl. Phys. *58*, 177 (1964); W. NORENBERG, Phys. Lett. *31B*, 621 (1970); Phys. and Chem. of Fission (1969); U. FACCHINI, E. GADIOLI-ERBA, and E. SAETTA-MENICHELLA, Phys. Lett. *28B*, 534 (1969); P. G. SONA and E. GADIOLI-ERBA, Phys. Rev. Lett. *22*, 406 (1969); U. FACCHINI and E. SAETTA-MENICHELLA, preprint (1970) unpublished.

where **1** is the 2×2 identity matrix, $E_n = E_{n1} = E_{n2}$ is the energy associated with the two channels, the 2×2 matrix $\mathbf{V}_n(R)$ and the column vector $\mathfrak{F}_n(R)$ are defined to be

$$\mathbf{V}_n(R) = \begin{pmatrix} V_{n11}(R) & -V_{n12}(R) \\ -V_{n21}(R) & V_{n22}(R) \end{pmatrix} \quad \text{and} \quad \mathfrak{F}_n(R) = \begin{pmatrix} F_{n1}(R) \\ F_{n2}(R) \end{pmatrix}. \tag{2.2}$$

Here $F_{n1}(R)$ and $F_{n2}(R)$ are $R \times f_{n1}$ and $R \times f_{n2}$, respectively.

Since we shall be computing the decay probabilities of these two channels to the same daughter pair having about the same excitation energy, the diagonal terms of the interaction potential, $\mathbf{V}_n(R)$, is expected to be almost equal. At the most, because of the slightly different excitation energy near zero separation, this is expected to differ only in the interior region (i.e., in the region $R < R_c$ in Fig. 1). However, as shown

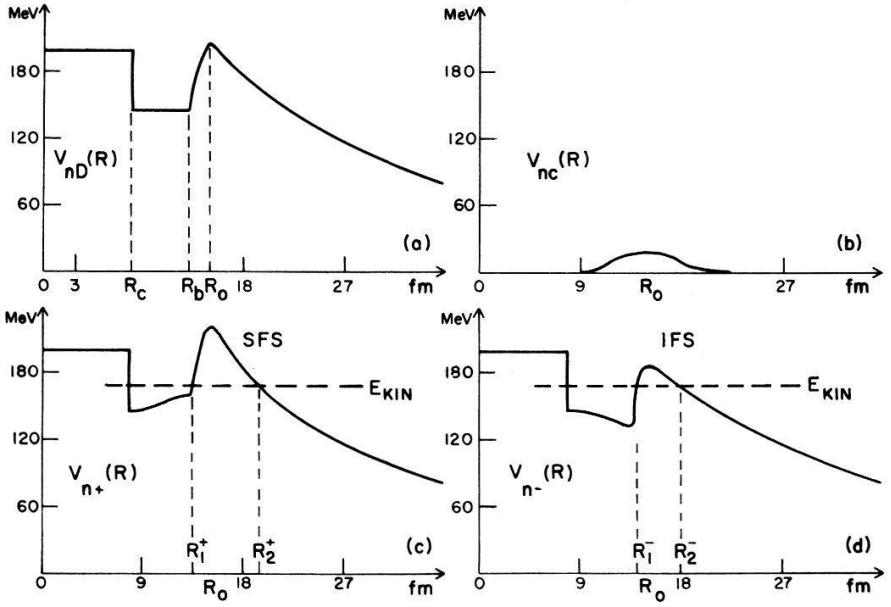

Figure 1

The diagonal and the coupling potentials for the *two channel model* are respectively shown in Fig. 1a and Fig. 1b. The *effective* potentials $V_{n\pm}(R)$ associated with the IFS and the spontaneous fission state (SFS) are represented in Fig. 1c and Fig. 1d. All quantities are plotted for the fission of $^{240}Pu \rightarrow {}^{142}Ba + {}^{98}Sr$.

theoretically in Ref. [2], this region's contribution to the half-life is negligible. This conclusion of Ref. [2]'s theoretical analysis is clearly borne out by the actual computations repeated in Ref. [1]. Therefore, it is reasonable to assume $V_{n11}(R) \simeq V_{n22}(R)$. Because of this assumption and the fact that $\mathbf{V}_n(R)$ is symmetric, i.e., $V_{n12}(R) = V_{n21}(R)$, we can find a constant 2×2 matrix $\mathbf{S}_2$ which can diagonalize the potential matrix $\mathbf{V}_n(R)$

$$\mathbf{S}_2 = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \tag{2.3}$$

Subjecting equation (2.1) to the above transformation, we obtain then a set of uncoupled equations

$$\left[-\frac{\hbar^2}{2\mu} \frac{d^2}{dR^2} - E_n + V_{n-}(R) \right] F_{n-}(R) = 0$$

$$\left[-\frac{\hbar^2}{2\mu} \frac{d^2}{dR^2} - E_n + V_{n+}(R) \right] F_{n+}(R) = 0 \tag{2.4}$$

where

$$V_{n\pm}(R) = V_{nD}(R) \pm V_{nc}(R),\ F_{n\mp}(R) = F_{n1}(R) \pm F_{n2}(R)$$

with

$$V_{nD}(R) = V_{n11}(R) \simeq V_{n22}(R) \text{ and } V_{nc}(R) = V_{n12}(R) = V_{n21}(R).$$

Given the actual form of the diagonal and the coupling interactions, the half-lives $T_{\frac{1}{2}}$ associated with the decay of each state '$F_{n-}(R)$' and '$F_{n+}(R)$' can now be computed in the JWKB approximation. As shown in Ref. [2] for the type of the ion-ion potential, this is

$$T^{\pm}_{\frac{1}{2}} = \left[\frac{2\mu}{E_n}\right]^{\frac{1}{2}} d\delta^{\pm}(k_n)/dk_n$$

$$\simeq 2\mu\left[\int_{R_c}^{R_1^{\pm}} \frac{dR}{[E_n - V_{n\pm}(R)]^{\frac{1}{2}}}\right] \cdot \exp\left[2\int_{R_1^{\pm}}^{R_2^{\pm}} \left[\frac{2\mu}{\hbar^2}(V_{n\pm}(R) - E_n)\right]^{\frac{1}{2}} dR\right] \tag{2.5}$$

where

$$k_n = \left[\frac{2\mu}{\hbar^2} E_n\right]^{\frac{1}{2}}$$

are the wave numbers associated with the two channels, $\delta^{\pm}(k_n)$ are the phase shifts associated with potentials $V_{n\pm}$, R_c is the core radius, and $R_1^{\pm}$ and $R_2^{\pm}$ are the two turning points where the energy E_n equals the barrier parts of potentials $V_{n\pm}(R)$ and are shown in Figure 1.

We should note that the equation (2.5) is the analog of equation (27) of Ref. [2] and the difference is due to the fact that we did not assume that potentials $V_{n\pm}(R)$ are constants in the region where $R_c \leqslant R \leqslant R_1^{\pm}$. Of course if we make the above assumption, equation (2.5) then becomes identical with equation (27) of Ref. [2].

Within the context of the QMF, the life-time is then

$(T^{\pm}_{\frac{1}{2}})$ (preformation probability in each channel)$^{-1}$.

The preformation probability in both channels will be taken to be 10^{-5} as it has been estimated in Ref. [1].

Using the guide of the ion-ion potential, we may assume the diagonal part of the interaction in (2.1), $V_{n11} = V_{n22} = V_{nD}$, is again given by the potential (1.1) except that d and R_0 are assumed to be arbitrary parameters, to be found empirically.

We also use an empirically found coupling potential, $V_{n21} = V_{n12} = V_{nc}$, of the form

$$V_{nc}(R) = \gamma e^{-|R - R_0/c|^3}. \tag{2.6}$$

We fix *all four* parameters, c, γ, d, and R_0, to reproduce the observed isomer and spontaneous fission half-lives of ^{240}Pu to (^{142}Ba + ^{98}Sr). Next knowing R_0 for this case we choose a reasonable set of r_0 and t so that $R_0 = r_0(A_1^{1/3} + A_2^{1/3}) + t$. Then we leave the empirically found set, c, γ, d, r_0, and t *unchanged* in all other cases, including those of the IFS. However, note that since R_0, for any case, is now defined in terms of t and r_0 as $R_0 = r_0(A_1^{1/3} + A_2^{1/3}) + t$, the half-lives will be A_1 and A_2 dependent. Using these unchanged sets of parameters we calculate half-lives of the decay of ^{234}U, ^{236}U, ^{240}Pu,

Table 1

Comparison of the computed half-lives in the two-channel model with the observation. The values of the parameters related to equations (2·4) and (2.6) are $r_0 = 1.3$ fm, $t = 2.2$ fm, $\gamma = 15.8$ MeV, $c = 4.2$ fm, and $d = 5.2$ fm. Columns three to ten are, respectively, the observed kinetic energies K.E. (exp), kinetic energy used, position (R_b) where the outer barrier of the diagonal part of the potential begins, observed IFS half-lives in sec.

Parent	Daughters	K.E. (exp)	E_{kin}	R_b	IFS (exp) sec	IFS (th) sec	SFS (exp) y	SFS (th) y
			161			4×10^{-5}		4×10^{17}
^{234}U	$^{142}Xe + {}^{92}Sr$	163 ± 2 (a)	163	13.0	$<2 \times 10^{-9}$ (b)	8×10^{-7}	1.6×10^{16} (c)	2×10^{15}
			165			2×10^{-8}		1×10^{13}
			163			4×10^{-7}		1×10^{15}
^{236}U	$^{142}Xe + {}^{94}Sr$	165 ± 2 (d)	165	13.0	1.1×10^{-7} (e)	1×10^{-8}	2×10^{16} (f)	8×10^{12}
			167			4×10^{-10}		6×10^{10}
			170			6×10^{-8}		6×10^{13}
^{240}Pu	$^{142}Ba + {}^{98}Sr$	172 ± 2 (g)	172	13.2	4×10^{-9} (h)	2×10^{-9}	1.34×10^{11} (i)	4×10^{11}
			174			1×10^{-10}		3×10^{9}
			183			2×10^{-10}		2×10^{9}
^{244}Cm	$^{144}Ba + {}^{100}Zr$	185.5 ± 5 (j)	185	13.3	5×10^{-7} (k)	1×10^{-11}	1.34×10^{7} (l)	2×10^{7}
			187			1×10^{-12}		2×10^{5}
			193.7			1×10^{-12}		1×10^{5}
^{246}Cf	$^{144}Ce + {}^{102}Zr$	195.6 ± 2 (m)	195.6	13.4	(n)	1×10^{-13}	2×10^{3} (o)	1×10^{3}
			197.6			2×10^{-14}		2×10^{1}

(IFS (exp) sec), calculated IFS half-lives in sec (IFS (th) sec), observed spontaneous fission state half-lives in years (SFS (exp) y) and the calculated spontaneous fission half-lives in years (SFS (th) y). The preformation probability of 10^{-5} has been used in all cases. Energies are in MeV and lengths are in 10^{-13} cm. References to experimental numbers are:

(a, d, g) W. E. Steins, Phys. Rev. *108*, 94 (1957).
(b) S. M. Polkanov and G. Sletten, Nucl. Phys. *A151*, 656 (1970).
(c) A. Ghiorso, et al., Phys. Rev. *87*, 163 (1952).
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(i) D. E. Watt, et al., Phys. Rev. *126*, 264 (1962).
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(k) V. Metag, et al., Phys. and Chem of Fission (IAEA) 1969.
(l) D. Metta, et al., J. Inorg. Nucl. Chem. (1964).
(m) A. M. Friedman, et al., Phys. Rev. *131*, 1203 (1963).
(n) IFS not observed, and from our theoretical calculations, it follows that the life-time measurement of this decay is beyond the present experimental capabilities.
(o) E. K. Hulet, Phys. Rev. *89*, 878 (1953).

^{244}Cm, and ^{246}Cf to ^{142}Xe + ^{92}Sr, ^{142}Xe + ^{94}Sr, ^{142}Ba + ^{98}Sr, ^{144}Ba + ^{100}Zr, and ^{144}Ce + ^{102}Zr, respectively, in both channels. These are tabulated in Table 1 and compared with the observed data. Clearly, all the experimental data can be reproduced. Thus, isomer fission state originates from the simple channel coupling which is inherent to the SMF.

3. An N Channels Model

Although our two channel model can, successfully, account for the experimentally observed half-lives of the spontaneously fission and isomer fission states, it is legitimate to examine closely the validity of this two channel approach. In the case of fission, because of the availability of the large kinetic energy, many channels are energetically open. In that case, one may well argue that the incorporation of more channels would yield many different half-lives and clearly this expectation is not in agreement with the experiments. It is our purpose now to show that under physically valid approximations this two channels model is, in actuality, a manifestation of many channels effects. In other words, we proceed to show that an N channels coupled system can reduce to three channels under a realistic physical situation accompanying fission. Out of these three cases one is a pure scattering channel (or rather instantaneous decay channel with a half-life of $\sim 10^{-22}$ sec) because the kinetic energy of that channel is greater than the potential barrier and hence *only two* cases represent barrier penetration problem.

Physically, it is reasonable to assume that kinetic energies of all open channels are nearly the same, since the intrinsic excitation energy of the individual daughter pair is small in comparison to the channel kinetic energy. Moreover, the differently excited daughter pair is expected to give rise to the nearly same diagonal part of the ion-ion interaction. This is clearly true, if $U_{n\alpha\alpha}$ is computed using the prescription of Ref. [1]. Third assumption of this model is that all off-diagonal coupling terms which are directly coupled to the channel in consideration, have different strength but the same functional form. Lastly, we neglect those coupling terms which are not directly coupled to the channel in consideration; i.e., channel No. 1.

Thus

$$E_n = E_{n1} \simeq E_{n\alpha}, \qquad \alpha = 2, \ldots N \tag{3.1a}$$

$$V_{nD}(R) = V_{n11}(R) \simeq V_{n\alpha\alpha}(R), \qquad \alpha = 2, \ldots N \tag{3.1b}$$

and

$$V_{n\beta 1}(R) = V_{n1\beta} = \gamma_{n\beta} V_{nc}, \qquad \beta = 2, \ldots N \tag{3.1c}$$

and

$$V_{n\alpha\beta}(R) \simeq 0 \qquad \text{for all } \beta \neq \alpha \text{ and } \alpha, \beta = 2, \ldots N. \tag{3.1d}$$

The $\gamma_{n\beta}$ in (3.1c) are different coupling constants. (3.1a) and (3.1b) state that the channel kinetic energy and the diagonal interaction are nearly equal, respectively, in all channels. (3.1c) and (3.1d) state that only important coupling terms are those which are directly coupled to the channel 1. There are N such coupling terms. Making the usual partial wave analysis and considering only $l = 0$ wave, we have the following N coupled equations for this model

$$\left[-\frac{\hbar^2}{2\mu} \frac{d^2}{dR^2} - (E_n - V_{nD}(R))\, \mathbf{1} - V_{nc}(R) \mathbf{C}_N \right] \mathfrak{F}_n(R) = 0 \tag{3.2}$$

where **1** is the $N \times N$ dimensional unit vector, $E_n = E_{n\alpha}$, $V_{nD} = V_{n\alpha\alpha}$, and the $N \times N$ matrix $\mathbf{C}_N$ and the column vector $\mathfrak{F}_n$ are defined to be

$$\mathbf{C}_N = \begin{pmatrix} 0 & \gamma_{n2} & \gamma_{n3} & \cdot & \cdot & \cdot & \gamma_{nN} \\ \gamma_{n2} & 0 & 0 & & & & 0 \\ \gamma_{n3} & 0 & 0 & & & & 0 \\ \cdot & & & \cdot & & & \\ \cdot & & & & \cdot & & \\ \cdot & & & & & \cdot & \\ \gamma_{nN} & 0 & 0 & \cdot & \cdot & \cdot & 0 \end{pmatrix}, \qquad \mathfrak{F}_n(R) = \begin{pmatrix} F_{n1}(R) \\ F_{n2}(R) \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ F_{nN}(R) \end{pmatrix}$$

It is shown in appendix A that a secular equation of $N \times N$ matrix $\mathbf{C}_N$ can only have *two non-zero* eigenvalues, regardless of its dimension, and they are

$$\lambda = \left[\sum_{j=2}^{N} \gamma_{nj}^2\right]^{\frac{1}{2}}$$

and

$$\lambda' = -\left[\sum_{j=2}^{N} \gamma_{nj}^2\right]^{\frac{1}{2}}. \tag{3.4}$$

Because of this fact, there clearly exists an $N \times N$ constant matrix $\mathbf{S}_N$ such that

$$\mathbf{S}_N \mathbf{C}_N \mathbf{S}_N^{-1} = \begin{pmatrix} \lambda & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\ \cdot & -\lambda & & & & & \\ \cdot & & 0 & & & & \\ \cdot & & & \cdot & & & \\ \cdot & & & & \cdot & & \\ \cdot & & & & & \cdot & \\ 0 & & & & & & 0 \end{pmatrix} \tag{3.5}$$

Next applying this constant matrix, $\mathbf{S}_N$, to the equations of (3.2), and making use of the equations of (3.5), we find that (3.2) reduces to only three different differential equations, *regardless of the number of channels involved.*

$$\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dR^2} - E_n + V_{nD}(R) + \lambda V_{nc}(R)\right] g_+(R) = 0$$

$$\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dR^2} - E_n + V_{nD}(R)\right] g_0(R) = 0$$

$$\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dR^2} - E_n + V_{nD}(R) - \lambda V_{nc}(R)\right] g_-(R) = 0 \tag{3.6}$$

where $g_\pm(R)$ and $g_0(R)$ are some linear combinations of elements of $F_n(R)$.

We may digress to note that *the above result may have a wide application in many fields of nuclear and other types of physics*. Given the form of the potentials $V_{nD}(R)$, $V_{nc}(R)$ and the coupling strength λ, the three equations can be solved for the tunneling problem and one may obtain the required half-lives for the case of the N coupled channels. Once more we choose $V_{nD}(R)$ to be given by (1.1), except that d and R_0 are assumed to be arbitrary and are to be found empirically. We choose $V_{nc}(R)$ to be defined by (2.6), with $\gamma = 1$ and c being a parameter.

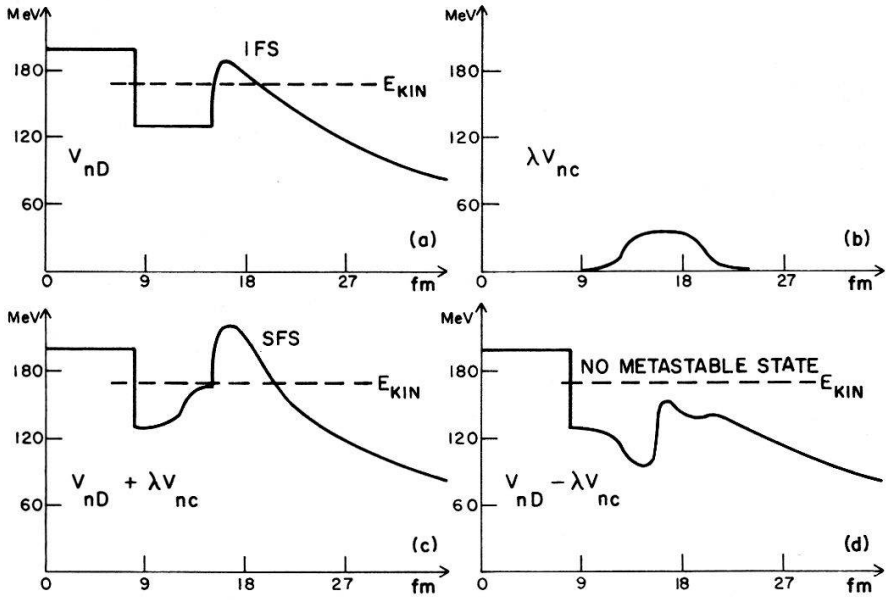

Figure 2

The diagonal and the effective coupling potentials for the N *channels model* are respectively shown in Fig. 2a and Fig. 2b. The effective potentials associated with the decay of $^{240}Pu \rightarrow {}^{142}Ba + {}^{98}Sr$ are shown in Fig. 2a, Fig. 2c and Fig. 2d.

In order to find the parameters, d, R_0, c and λ, we associate the state '$g_+(R)$' with spontaneous fission and state '$g_0(R)$' with isomer fission and next we fix the parameters from the observed isomer and spontaneous fission half-lives of ^{240}Pu using the estimated preformation probability of 10^{-5}. Similar to the case in section 2, we associate a reasonable r_0 and t to this R_0. Using the same set of parameters, d, r_0, t, c and λ in all cases we find the half-lives associated with the decay of ^{234}U, ^{236}U, ^{240}Pu, ^{244}Cm, and ^{246}Cf. It is interesting to note that this set of parameters is such that only two of the three equations in (3.6) involve barrier penetration, because the observed kinetic energies of 163, 165, 172, 185, 195 MeV for the decay of ^{234}U, ^{236}U, ^{240}Pu, ^{244}Cm, ^{246}Cf, respectively, to $^{142}Xe + {}^{92}Sr$, $^{142}Xe + {}^{94}Sr$, $^{142}Ba + {}^{98}Sr$, $^{144}Ba + {}^{100}Zr$, $^{144}Ce + {}^{102}Zr$ are in each case above the effective barrier for the channel, '$g_-(R)$.' This is shown in Figure 2d. Keeping this set of parameters fixed in all cases, we note that the computed half-lives, as they are tabulated in Table 2, are in good agreement with the observed half-lives of both spontaneous decay and the decay of the IFS.

Table 2

Comparison of the computed half-lives in the N channels model with the observed ones. The values of the parameters in equation (3.6) are $r_0 = 1.3$ fm, $t = 3.5$ fm, $\lambda = 34$ MeV, $c = 4.8$ fm, and $d = 3.2$ fm. The columns and references are the same as in Table 1.

Parent	Daughters	K.E. (exp)	E_{kin}	R_b	IFS (exp) sec	IFS (th) sec	SFS (exp) y	SFS (th) y
			161			4×10^{-5}		2×10^{17}
^{234}U	$^{142}Xe + {}^{92}Sr$	163 ± 2 (a)	163	14.8	$<2 \times 10^{-9}$ (b)	9×10^{-7}	1.6×10^{16} (c)	2×10^{15}
			165			3×10^{-8}		2×10^{13}
			163			5×10^{-7}		2×10^{15}
^{236}U	$^{142}Xe + {}^{94}Sr$	165 ± 2 (d)	165	14.9	1.1×10^{-7} (e)	2×10^{-8}	2×10^{16} (f)	2×10^{13}
			167			6×10^{-10}		2×10^{11}
			170			4×10^{-8}		5×10^{13}
^{240}Pu	$^{142}Ba + {}^{98}Sr$	172 ± 2 (g)	172	15.1	4×10^{-9} (h)	2×10^{-9}	1.34×10^{11} (i)	6×10^{11}
			174			8×10^{-11}		7×10^{9}
			183			5×10^{-11}		1×10^{9}
^{244}Cm	$^{144}Ba + {}^{100}Zr$	185.5 ± 5 (j)	185	15.1	5×10^{-7} (k)	4×10^{-12}	1.34×10^{7} (l)	2×10^{7}
			187			3×10^{-13}		3×10^{5}
			193.6			2×10^{-13}		7×10^{4}
^{246}Cf	$^{144}Ce + {}^{102}Zr$	195.6 ± 2 (m)	195.6	15.3	(n)	3×10^{-14}	2×10^{3} (o)	1×10^{3}
			197.6			5×10^{-15}		2×10^{1}

4. Discussion and Conclusion

This paper clearly indicates that the IFS can originate from the channel coupling alone. Although in this treatment, we have primarily discussed the channel coupling within the context of the QMF, *the entire treatment is equally valid for the coupled channel case in the multi-dimensional deformation space*, which is the customary description in the liquid drop version of fission. However, as it is pointed out in Ref. [2], the channel kinetic energy and not the customary Q-value must be used in computing the decay rate.

A few comments on the set of parameters r_0, t, λ are warranted. As it is noted in Ref. [1], the strength of the diagonal part of the interaction is consistent with the ion-ion interaction, as it is estimated from the nuclear matter energy-density curve. The factor $(1.3) = r_0$ in R_0 is quite within the tolerance limit of our knowledge of the nuclear radius. However, $(1.1) = r_0$ is probably more accepted value today. This slight increase of r_0 is probably indicative of the fact that the daughter nuclei are rather ellipsoidal which is expected. An ellipsoidal deformation of $\beta \simeq .2$ effectively increases the radius of the daughter pair to this value of r_0. The other interpretation, if we worry about this at all, is that the density on the surface of the daughter pair is more smeared out than in the normal unexcited nuclei. The strength of the channel coupling term 'λ' is not actually that large, if one recognizes that this represents an accumulative effect of the coupling of a dominant channel to *many other channels*. Therefore, individual coupling strength need not be strong.

The most important thing is to recognize that the computation is not at all sensitive to the detail behavior of the potential surface inside. This point is discussed in detail in Ref. [2]. The computated half-lives are not sensitive to the detailed shape of the coupling potential. As for example, if the coupling potential is taken to be of the volume type by extending it inside, the half-lives of both the isomer and the spontaneous fission states are not affected. The reason is clearly analysed in Ref. [2].

We have considered here one of the energetically most favored decay modes for each parent nucleus. Both experimentally and the theory of Ref [1] indicate that within a given range of the kinetic energy spectrum, a number of different decay modes occur. Some of these decay modes have spontaneous fission half-lives close to one another. Their IFS will also lie close to one another. In other words, there can be more than one IFS associated with the decay of a particular nucleus.

We also note that the IFS corresponding to the most dominant decay mode gets exceedingly short as the mass number of the parent gets large. However, two points in this regard must be investigated before one draws any specific conclusion: (a) the half-lives of the IFS associated with less dominant decay modes. As the half-lives of the IFS associated with the dominant ones get too short to be measured, those associated with the less dominant modes may become of the order of a nano second. (b) The behavior of the coupling parameters. We have restricted them severely by assuming them to be the same in all cases. In general, however, they depend on the channel kinetic energies, the intrinsic excitation energies, and the mass splitting.

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Appendix A

In order to show that the non-zero eigenvalues of $\mathbf{C}_N$ are only the ones given by (3.4) let us consider the following $N \times N$ matrix

$$\mathbf{K}_{nN} = \mathbf{C}_N - \lambda \mathbf{1}. \tag{A.1}$$

We would like to claim that

$$\text{DET}\,(\mathbf{K}_{nN}) = (-1)^N \left[\lambda^N - \lambda^{N-2} \sum_{j=2}^{N} \gamma_{nj}^2 \right] \tag{A.2}$$

it is easy to check that for the case of $N = 2$ or $N = 3$ equation (A.2) is true. To complete the proof for any N, we will use the induction method, that is, assume (A.2) is true for $N = N_0$, then to complete the proof we only need to show that (A.2) is also true for $N = N_0 + 1$. In other words assume that for $N = N_0$

$$\text{DET}\,(\mathbf{K}_{nN_0}) = (-1)^{N_0} \left[\lambda^{N_0} - \lambda^{N_0-2} \sum_{j=2}^{N_0} \gamma_{nj}^2 \right] \tag{A.3}$$

is true and now let us consider

$$\text{DET}\,(\mathbf{K}_{n(N_0+1)}).$$

Noting the definition of $\mathbf{K}_{nN}$ we find that

$$\text{DET}\,(\mathbf{K}_{n(N_0+1)}) = -\lambda\,\text{DET}\,(\mathbf{K}_{nN_0}) + (-1)^{2+N_0}\,\gamma_{n(N_0+1)}\,\text{DET}(\mathbf{L}) \tag{A.4}$$

where

$$\text{DET}\,(\mathbf{L}) = \begin{vmatrix} \gamma_{n2} & \gamma_{n3} & \cdot & \cdot & \cdot & \gamma_{nN_0} & \gamma_{n(N_0+1)} \\ -\lambda & 0 & & & & & 0 \\ 0 & -\lambda & & & & & 0 \\ \cdot & & \cdot & & & & \cdot \\ \cdot & & & \cdot & & & \cdot \\ \cdot & & & & \cdot & & \cdot \\ 0 & 0 & \cdot & \cdot & \cdot & -\lambda & 0 \end{vmatrix} = \lambda^{N_0-1}\,\gamma_{n(N_0+1)}$$

Substituting this fact and equation (A.3) in (A.4) we get

$$\text{DET}\,(\mathbf{K}_{n(N_0+1)}) = (-1)^{N_0+1} \left[\lambda^{N_0+1} - \lambda^{N_0-1} \sum_{j=2}^{N_0} \gamma_{nj}^2 \right] - (-1)^{N_0+1}\,\lambda^{N_0-1}\,\gamma_{n(N_0+1)}^2$$

$$= (-1)^{N_0+1} \left[\lambda^{N_0+1} - \lambda^{(N_0+1)-2} \sum_{j=2}^{N_0+1} \gamma_{nj}^2 \right]$$

Thus equation (A.2) must be true for any N. Now we find the eigenvalues of $\mathbf{C}_N$ by requiring that $\text{DET}\,(\mathbf{K}_{nN}) = 0$. From (A.2), it follows that $\mathbf{C}_N$ has $(N-2)$ trivial eigenvalues and two non-zero eigenvalues given by equations (3.4).

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