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Field Theory and Unstable Particles

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Abstract. The purpose of this paper is to give a field theoretical description of scattering processes involving long-lived unstable particles. A model with one kind of unstable particles, the B, and their stable decay products, the A, is discussed. The B are described by poles on the second sheet in the otherwise smooth Green's functions of a relativistic quantum field corresponding to the A. The decay is assumed to be due to a weak interaction, and the quantities of the theory are considered only in the two lowest non-vanishing orders in its coupling constant. Strong interactions of the B among themselves and the A among themselves are admitted and are rigorously taken into account. The consequences of unitarity are studied. It is shown that a pole with a factorizing residue on the second sheet of the 4-point function enforces the existence of such poles in the higher functions. The spatio-temporal development of scatterings involving B-particles, as seen in experiments, is analysed with the help of a formalism developed in an earlier work. A natural definition of incoming and outgoing scattering states and of the S-matrix is given.

1. Introduction

There are two types of unstable particles in high energy physics: *metastable particles* and *resonances*. The former are sufficiently long-lived to travel over macroscopic distances between production and decay. Resonances are never seen in a free state but manifest themselves only as humps in cross sections. In principle this distinction is not sharp, but in practice it turns out to be quite unambiguous, the metastable particles being the ones that decay through weak or electromagnetic interactions.

To both types the methods and concepts of scattering theory, originally developed for stable particles, are applied freely with little scruple about the lacking foundation in an exact underlying theory. In the case of resonances this is a highly questionable procedure, acceptable at best as a very crude approximation. But in the metastable case the method is clearly reasonable and, indeed, works very well. It is therefore desirable to find for it some underpinning in a rigorous theory. This is what we shall attempt to do in the present paper, the underlying theory being quantum field theory. In what follows, the word 'particle' will always denote stable or metastable ones, to the exclusion of mere resonances.

In relativistic field theory we have a rigorous definition of an S-matrix only for strictly stable particles. This is so because the notion of particle itself is only defined asymptotically, for $t \to \pm\infty$. (We exclude the trivial case of free fields.) But in this limit an unstable particle has long ceased, or not yet started, to exist. The definition of an S-matrix for unstable particles is therefore possible only in an approximate sense.

Consider the case of a strongly interacting particle B that can decay via a different, weak, interaction into two stable particles A. How do we describe scattering of B-particles ? An obvious idea is to simply forget about the weak interactions and treat the B as strictly stable. This is clearly impossible if we are interested in the decay process itself or in other processes that can only proceed weakly. But even if we are not interested in weak processes we are confronted with difficulties. In reality we cannot switch off the weak interaction, we can only neglect it, and this is a different thing mathematically. In particular we find that the state spaces are entirely different in the two cases. The Hilbert space of the exact theory is completely spanned by the asymptotic A-particle states. The approximation in which the weak interaction is neglected must be formulated in this space. On switching off the weak interaction, however, we obtain a much larger Hilbert space spanned by the asymptotic A-states *and* B-states together. The summation over a complete set of intermediate states occuring in the unitarity relations will thus look quite different in the two cases, and this apparent paradox has to be resolved.

We shall analyse such a model with the help of ideas introduced in an earlier paper [1], herafter quoted as **I**. It will be shown how, under certain natural assumptions, scattering processes involving the unstable B can be described satisfactorily. We shall define notions like 'incoming and outgoing particles', 'n-particle states' and 'S-matrix' in the two lowest non-vanishing orders in the weak coupling constant g. This is a sufficient approximation if g is small enough, which is the case in praxi for the weak and the electromagnetic interactions.

In **I** we distinguished the experimentalist's particle from that of the field theoretician. The latter is an object with a sharp mass, which definition disqualifies unstable particles from the start. The former can roughly be defined as an object which produces in a bubble chamber well defined straight tracks, or, more generally, which behaves under tracking with any sort of detectors like a classical particle. That such a tracking is, in practice, not possible for neutral particles is here of course irrelevant. We are only concerned with the principal aspects of the situation and shall generously assume the existence of counters which can detect any kind of particle directly (not via their decay products) without absorbing them. This experimental definition of a particle excludes resonances and includes the stable particles of field theory only if certain conditions are satisfied (see **I**). It is, however, the most appropriate definition for dealing with metastable particles.

As in **I** we consider the theory of a local field $A(x)$ with an energy-momentum spectrum corresponding to the existence of stable particles of mass $m > 0$. In **I** we have shown that such a theory describes stable particles in the above experimental sense, if its Green's functions are sufficiently smooth in p-space. We have also shown that unstable particles appear, if this smoothness is disturbed by poles on the second sheet near the real axis. In what follows we shall examine this model more closely.

In Sections 3 and 4 the consequences of asymptotic completeness (= 'unitarity') will be discussed. It will be shown that a pole with a factorizing residue on the second sheet of the 4-point function generates such poles also in the higher functions[1]). It will also be shown how the sudden increase of the number of intermediate states in the limit $g \to 0$ is produced in a natural way.

Section 5 is the central part of the paper. We define states of unstable B-particles and study the phenomenological development of scattering processes. This will lead to a natural definition of an S-matrix.

Section 6 is devoted to a brief discussion of a more conventional but less natural definition of the S-matrix with the help of some pseudo-asymptotic conditions.

Section 7 deals with the question whether a local field can be associated with the B-particles. The problem will be formulated, but no attempt at its solution will be made.

Note that we do not assume a priori the existence of a B-field but use the analytical structure of the Green's function for the introduction of unstable particles. In this our approach differs basically from the earlier work by Matthews and Salam [2].

2. The Model

We consider a model with two kinds of particles: the stable, pseudoscalar, uncharged A-particles with mass $m > 0$, and the unstable, scalar, uncharged B with mass M, $2\,m < M < 3\,m$. The B are strongly interacting among themselves. The A may also interact strongly among themselves but not with the B. Between A and B exists an interaction with a small coupling constant g, through which a B can decay into two A.

This model is unrealistic insofar as it does not allow strong production of the B. This deficiency could easily be eliminated: we could either admit a strong interaction between A and B with the strong decay $B \to 2\,A$ forbidden by a selection rule, or we could add another type of stable particles, the C, which interact strongly with the B but are too massive to allow the process $B \to 2\,C$. These expanded models can also be treated with our methods. No essential new features appear, wherefore we shall restrict ourselves to the simpler version.

The underlying field theory is the theory of a pseudoscalar, hermitian, local, asymptotically complete Wightman field $A(x)$ [3] with a spectrum corresponding to particles of mass m. Let $\mathcal{H}$ be the Hilbert space of this theory. It is generated from the vacuum Ω be repeated application of the asymptotic fields $A_{in}(x)$.

Let $T(x_1, \ldots, x_n)$ be the amputated time ordered product of the fields $A(x_i)$. Formally:

$$T(x_1, \ldots, x_n) = (-i)^{n-1} \prod_1^n K_i \sum \theta(x_{i_1}, \ldots, x_{i_n})\ A(x_{i_1}) \ldots A(x_{i_n}) \tag{1}$$

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[1]) The reader who is willing to accept this not surprising result without proof may omit Section 4, which makes use of the not generally known generalized retarded functions.

where $K_i = -\Box_{x_i} - m^2$, $\theta(\ldots) = 1$ if the arguments are in chronological order, $\theta = 0$ otherwise, and the sum extends over all permutations $(i_1, \ldots, i_n)$ of the indices $(1, \ldots, n)$. The vacuum ecpectation value of T is called $\tau(x_1, \ldots, x_n)$, its truncated (= connected) part $\tau^T(x_1, \ldots, x_n)$. $\tilde{\tau}$ and $\tilde{\tau}^T$ are the Fourier transforms of τ and τ^T. $\tilde{\tau}$ is of the form

$$\tilde{\tau}(p_1, \ldots, p_n) = \delta^4(p_1 + \cdots + p_n)\, \hat{\tau}(p_1, \ldots, p_n)\,, \tag{2}$$

where $\hat{\tau}$ is only defined on the manifold $p_1 + \cdots + p_n = 0$. In what follows this restriction of the arguments of $\hat{\tau}$ will always be tacitly assumed. Equation (2) holds also for the truncated part.

We develop $\tilde{\tau}$, as well as all the other quantities of the theory, into a power series in g, which we assume to be asymptotic:

$$\tilde{\tau}(p_1, \ldots, p_n) = \sum_{\nu=0}^{N} g^\nu\, \tilde{\tau}_\nu(p_1, \ldots, p_n) + O(g^{N+1})\,. \tag{3}$$

This equation is to be understood in the sense of distributions, i.e. it holds after integration over sufficiently smooth test functions.

Relevant to the discussion of scattering is the behaviour of the Green's functions $\hat{\tau}$ on the mass shell and in its vicinity. We assume that in this region the $\hat{\tau}^T$ are smooth apart from the physically necessary singularities, including second sheet poles corresponding to the B and the threshold singularities generated by them.

More exactly: consider the variables $p_1, p_2, q_1, q_2, Q = q_1 + q_2$, in the region

$$p_1 \in V_-,\ p_i^2 \sim m^2,\ q_i \in V_+,\ q_i^2 \sim m^2,\ Q^2 \sim M^2\,, \tag{4}$$

with $V_\pm$ the forward and backward cones, and the sign $\sim$ denoting approximate equality. In this region we assume $\hat{\tau}^T(p_1, p_2, q_1, q_2)$ to be of the form

$$\hat{\tau}(p_1, \ldots, q_2) = a(p_1, \ldots, q_2) + \frac{b(p_1, \ldots, q_2)}{Q^2 - M^2 + i\, M\, \Gamma} \tag{5}$$

with a, b smooth, i.e. infinitely differentiable and without strong oscillations. This means that the derivative $\partial a/\partial p_i$ is at most of the same order of magnitude as the difference quotients $[a\,(p_i + \Delta p_i) - a(p_i)]/\Delta p_i$ with macroscopic Δp_i, and the same for b. This property shall hold independently for all coefficients of the perturbation expansion of a and b. a and b are, of course, only defined on $p_1 + \cdots + q_2 = 0$.

Here we have assumed that M is roughly in the middle of the elastic interval $(2\,m, 3\,m)$, so that interferences between the Q-pole and the $2\,A$ and $3\,A$ thresholds can be neglected.

For Q^2 far from M^2 we assume $\hat{\tau}$ to be as smooth as is compatible with the possible vicinity of physical singularities.

We assume Γ and b to start with terms of order g^2:

$$\Gamma = \Gamma_2\, g^2 + \cdots\,, \qquad b = b_2\, g^2 + \cdots\,. \tag{6}$$

A possible g-dependence of M will be ignored for simplicity.

The pole in (5) shall occur only in the s-wave, so that b depends only on the variables p_i^2, q_i^2, Q^2. For the moment we admit a possible dependence of Γ on p_i^2, q_i^2. We do not exhibit this dependence explicitly because it will turn out later that Γ must be constant.

The residue b shall, at the pole emplacement $Q^2 = M^2$, factorize in the two lowest orders:

$$b_2(p_1, \ldots, q_2) = 2\pi\, \hat{\tau}_1(p_1, p_2, Q)\, \hat{\tau}_1(q_1, q_2, -Q)\,,$$

$$b_3(p_1, \ldots, q_2) = 2\pi\, \{\hat{\tau}_1(p_1, p_2, Q)\, \hat{\tau}_2(q_1, q_2, -Q) + \hat{\tau}_2(p_1, p_2, Q)\, \hat{\tau}_1(q_1, q_2, -Q)\}\,. \tag{7}$$

The factors $\hat{\tau}_i(p_1, p_2, Q), \ldots$ are defined on $p_1 + p_2 + Q = 0$, $Q^2 = M^2$, and are smooth functions. $\hat{\tau}_i(p_1, p_2, Q)$ is a Green's function of two A and one B. It is *not* equal to the 3-point function $\hat{\tau}_i(p_1, p_2, q)$ for $q = Q$. We shall also use this notation in the future: small letters denote A-variables, capital letters B-variables, differences of small and capital letters (e.g.: $Q - k$) are A-variables. We hope that this will not lead to confusion.

The definition (7) does not fix the factors $\hat{\tau}_i$ unambiguously. The definition becomes unique up to a sign, if we add the CTP relation

$$\hat{\tau}(p_1, p_2, Q) = \hat{\tau}(-p_1, -p_2, -Q) \tag{8}$$

as a condition. This condition can be satisfied, since $\hat{\tau}(p_1, \ldots, q_2) = \hat{\tau}(-p_1, \ldots, -q_2)$ due to the CTP theorem [3]. We obtain

$$\hat{\tau}(p_1, p_2, Q) = (2\pi)^{-1/2}\, [b(p_1, p_2, -p_1, -p_2)]^{1/2} \tag{9}$$

for $(p_1 + p_2)^2 = M^2$. The Lorentz invariance of $\hat{\tau}(p_1, \ldots, q_2)$ gives invariance of $\hat{\tau}(p_1, p_2, Q)$, which depends then only on p_1^2, p_2^2. Furthermore, it is invariant under exchange of p_1 and p_2. The value of $\hat{\tau}$ on the mass shell will be called t:

$$t = \hat{\tau}(p_1, p_2, Q) \quad \text{for } p_1^2 = p_2^2 = m^2\,. \tag{10}$$

The higher functions shall contain the singularities that are necessary for satisfying unitarity and shall be smooth otherwise. The poles generated in the higher functions by the 4-point pole (5) will be discussed in Section 4.

3. Elastic Unitarity

Let $\delta_+(k) = \theta(k_0)\, \delta\, (k^2 - m^2)$ be the δ-function of the positive mass shell. Define

$$\delta_Q k = \delta_+(k)\, \delta_+(Q - k)\, d^4k\,. \tag{11}$$

The unitarity equation for the 4-point function reads in the region (4)

$$\hat{\tau}^T(p_1, \ldots, q_2) - \hat{\tau}^{T*}(p_1, \ldots, q_2) = -\frac{i}{2}\,(2\pi)^4 \int \delta_Q k\, \hat{\tau}^{T*}(q_1, q_2, -k, -Q + k)\, \hat{\tau}^T(p_1, p_2, k, Q - k)\,. \tag{12}$$

We develop this equation in powers of g and consider the terms of second order, this being the lowest order in which the B-poles appear. The pole itself can be expanded:

$$\frac{1}{Q^2 - M^2 + i\,M\,\Gamma} = \frac{1}{Q^2 - M^2 + i\,\varepsilon}\left\{1 - \frac{i\,\Gamma_2\,g^2}{Q^2 - M^2 + i\,\varepsilon}\right\} + O(g^3)\,, \tag{13}$$

which equation holds again in the sense of distributions.

The pole terms on the left-hand side of (12) are in second order

$$2\pi\,\frac{\hat\tau_1(p_1, p_2, Q)\,\hat\tau_1(q_1, q_2, -Q)}{Q^2 - M^2 + i\,\varepsilon} - 2\pi\,\frac{\hat\tau_1^*(p_1, p_2, Q)\,\hat\tau_1^*(q_1, q_2, -Q)}{Q^2 - M^2 - i\,\varepsilon}\,. \tag{14}$$

On the right-hand side we obtain poles from the terms with one factor of second order and one of zeroth order:

$$-\frac{i}{2}\,(2\,\pi)^5\left\{\frac{t_1^*\,\hat\tau_1^*(q_1, q_2, -Q)}{Q^2 - M^2 - i\,\varepsilon}\,J_0(p_1, p_2, Q)\right.$$
$$\left. + \frac{t_1\,\hat\tau_1(p_1, p_2, Q)}{Q^2 - M^2 + i\,\varepsilon}\,J_0^*(q_1, q_2, -Q)\right\} \tag{15}$$

with

$$J_0(p_1, p_2, Q) = \int \delta_Q k\,\hat\tau_0^T(p_1, p_2, k, Q - k)$$
$$J_0(q_1, q_2, -Q) = \int \delta_Q k\,\hat\tau_0^T(q_1, q_2, -k, -Q + k)\,. \tag{16}$$

But we have also poles of second order from the terms in which *both* factors are of second order. This is so because on inserting the exact poles in both factors we obtain the product $(Q^2 - M^2 - i\,M\,\Gamma)^{-1}\,(Q^2 - M^2 + i\,M\,\Gamma)^{-1}$, and this diverges for $g \to 0$ like g^{-2}. We can develop:

$$\frac{1}{Q^2 - M^2 - i\,M\,\Gamma}\,\frac{1}{Q^2 - M^2 + i\,M\,\Gamma} = \frac{1}{2\,i\,M\,\Gamma}$$
$$\times\left\{\frac{1}{Q^2 - M^2 - i\,M\,\Gamma} - \frac{1}{Q^2 - M^2 + i\,M\,\Gamma}\right\} = \frac{1}{2\,i\,M\,\Gamma_2}\left\{1 - g\,\frac{\Gamma_3}{\Gamma_2} + O(g^2)\right\}$$
$$\times\left\{\frac{1}{Q^2 - M^2 - i\,\varepsilon} - \frac{1}{Q^2 - M^2 + i\,\varepsilon} + (g^2)\right\}, \tag{17}$$

and obtain in (12) the additional pole term

$$-\frac{(2\,\pi)^6}{4\,M\,\Gamma_2}\,I\,|t_1|^2\,\hat\tau_1(p_1, p_2, Q)\,\hat\tau_1^*(q_1, q_2, -Q)\times$$
$$\left\{\frac{1}{Q^2 - M^2 - i\,\varepsilon} - \frac{1}{Q^2 - M^2 + i\,\varepsilon}\right\} \tag{18}$$

with

$$I = \int \delta_Q k\,, \qquad Q^2 = M^2\,, \tag{19}$$

a phase space factor.

The residues of the pole $(Q^2 - M^2 + i\,\varepsilon)^{-1}$ must be the same on both sides of (12). This yields, after division by $2\,\pi\,\hat\tau_1(p_1, p_2, Q)$:

$$\hat{\tau}_1(q_1, q_2, -Q) = -\frac{i}{2}(2\pi)^4 t_1 J_0^*(q_1, q_2, -Q)$$
$$+ (2\pi)^5 \frac{I\,|t_1|^2}{4\,M\,\Gamma_2}\,\hat{\tau}_1^*(q_1, q_2, -Q)\,, \tag{20}$$

for $Q^2 = M^2$. For $q_i^2 = m^2$ this becomes, with the definition

$$J_0 = J_0(q_1, q_2, -Q)\,\Big|_{\,q_1^2 = q_2^2 = m^2,\; Q^2 = M^2}\,, \tag{21}$$

after division by t_1:

$$1 = -\frac{i}{2}(2\pi)^4 J_0^* + (2\pi)^5 \frac{I\,t_1^{*\,2}}{4\,M\,\Gamma_2}\,. \tag{22}$$

(We exclude the case $t_1 = 0$ in which the B are stable in first order.)

Define

$$\varrho_0 = 1 - \frac{i}{2}(2\pi)^4 J_0\,. \tag{23}$$

ϱ_0 is the s-wave part of the S-matrix for A–A-scattering in order g^0:

$$\varrho_0 = \exp(2\,i\,\delta_{0,0}) \tag{24}$$

with δ_0 the s-wave phase shift, $\delta_{0,0}$ its zero order contribution.

(22) becomes

$$\varrho_0 = (2\pi)^5 \frac{I\,t_1^2}{4\,M\,\Gamma_2} \tag{25}$$

Γ_2 must be real positive. Hence

$$t_1 = |t_1|\,e^{i\,\delta_{0,0}}\,, \tag{26}$$

the phase of t_1 is the zero-order s-wave phase shift. This is a special case of Watson's final state theorem [4].

From (25) we obtain finally

$$4\,M\,\Gamma_2 = (2\pi)^5\, I\,|t_1|^2\,, \tag{27}$$

another well-known relation.

As a side remark we can now sketch the proof that Γ_2 is a constant. If Γ_2 depends on p_i^2, q_i^2, then $2\,\Gamma_2$ gets replaced in (20) by $\Gamma_2(p_1^2, p_2^2, m^2, m^2) + \Gamma_2(m^2, m^2, q_1^2, q_2^2)$. According to (20) this expression cannot depend on p_i^2 and for reasons of symmetry also not on q_i^2. It is therefore equal to its mass shell value $2\,\Gamma_2$. If we consider now the terms of order g^4 in (12) and use (13), we find double poles $(Q^2 - M^2 + i\,\varepsilon)^{-2}$. Equalizing the residues of these double poles on both sides yields an equation, which is obtained from (20) by multiplying on the left with $\Gamma_2(p_1^2, \ldots, q_2^2)$, on the right with Γ_2. This is compatible with (20) only if $\Gamma_2(p_1^2, \ldots, q_2^2) = \Gamma_2$, q.e.d.

Equation (20) becomes with (27):

$$\hat{\tau}_1(q_1, q_2, -Q) - \hat{\tau}_1^*(q_1, q_2, -Q) = -\frac{i}{2}(2\pi)^4 \int \delta_Q k\, \hat{\tau}_0^*(q_1, q_2, -k, -Q+k)$$
$$\times\, \hat{\tau}_1(k, Q-k, -Q)\,. \tag{28}$$

(Remember that $Q - k$ is an A-variable.) This equation has the form of an unitarity equation, if we interprete $\tilde{\tau}(q_1, q_2, -Q) = \delta^4(q_1 + q_2 - Q)\, \hat{\tau}(q_1, q_2, -Q)$ as amputated Green's function of two A-fields and one B-'field', the latter being defined only on $Q^2 = M^2$ (see Section 7).

The second order contributions to (12), including the extraordinary term (18), are in extenso

$$\hat{\tau}_2^T(p_1, \ldots, q_2) - \hat{\tau}_2^{T*}(p_1, \ldots, q_2) =$$
$$-\frac{i}{2}(2\pi)^4 \sum_{\nu=0}^{2} \int \delta_Q k\, \hat{\tau}_\nu^{T*}(q_1, q_2, -k, -Q+k)\, \hat{\tau}_{2-\nu}^T(p_1, p_2, k, Q-k)$$
$$-i\,(2\pi)^2\,\delta(Q^2 - M^2)\, \hat{\tau}_1^*(q_1, q_2, -Q)\, \hat{\tau}_1(p_1, p_2, Q)\,. \tag{29}$$

The last term on the right has the form that would be obtained by inserting a stable 1-B-state as an intermediate state. In other words: if we write down the unitarity equations for $\hat{\tau}^T$ in a larger Hilbert space $\mathcal{H}'$ spanned by A-states *and* B-states considered as independent and develop the right-hand side according to the simple, as we know faulty, rule $(A\,B)_\mu = \Sigma_{\nu=0}^{\mu} A_\nu B_{\mu-\nu}$, then we obtain in second order the equation (29). This is the solution of the paradox mentioned in the introduction.

4. The Higher Functions

For the discussion of the higher Green's functions it is convenient to work with the generalized retarded functions (g.r.f.), since they satisfy simpler unitarity relations than the time-ordered functions. We use the definitions and notations of reference [5], hereafter quoted as **II**. We are especially interested in the ordinary retarded products $R(x_1, \ldots, x_n)$ and the products $R(x_1, \ldots, x_n \uparrow y_1, \ldots, y_m)$, $R(x_1, \ldots, x_n \downarrow y_1, \ldots, y_m)$ associated to the following cells $C_\updownarrow$ (the symbol $\updownarrow$ means $\uparrow$ or $\downarrow$).

Let X be the set $\{x_1, \ldots, x_n\}$, Y the set $\{y_1, \ldots, y_m\}$. Then

$$\sigma_X = \begin{cases} - \text{ in } C_\uparrow \\ + \text{ in } C_\downarrow \end{cases}, \qquad \sigma_Y = \begin{cases} + \text{ in } C_\uparrow \\ - \text{ in } C_\downarrow \end{cases}. \tag{30}$$

The other σ_I are the same in both $C_\updownarrow$, namely

$$\sigma_I = \begin{Bmatrix} + \\ - \end{Bmatrix} \text{ if } I \subset Y \text{ and } y_1 \begin{Bmatrix} \in \\ \notin \end{Bmatrix} I$$
$$\sigma_I = \begin{Bmatrix} + \\ - \end{Bmatrix} \text{ if } I \subset X \text{ and } x_1 \begin{Bmatrix} \in \\ \notin \end{Bmatrix} I \tag{31}$$
$$\sigma_{I_1 \cup I_2} = \sigma_{I_1} \text{ if } I_1 \subset X\,, \quad I_2 \subset Y\,, \quad I_1 \neq \phi, X\,.$$

We shall work only with totally amputated products, i.e. with Klein-Gordon operators in all variables applied to them (compare (1)). The vacuum expectation values of $R(..)$, $R(.. \updownarrow ..)$ are called $r(..)$, $r(.. \updownarrow ..)$. Their Fourier transforms are of the form

$$\tilde{r}(p_1, \ldots, p_n) = \delta^4(p_1 + \cdots + p_n)\, \hat{r}(p_1, \ldots, p_n)\,,$$
$$\tilde{r}(p_1, \ldots, p_n \updownarrow q_1, \ldots, q_m) = \delta^4(\Sigma p_i + \Sigma q_i)\, \hat{r}(p_1, \ldots \updownarrow \ldots, q_m)\,. \tag{32}$$

The $\hat{r}$ are defined on the manifold $\Sigma p_i = 0$, or $\Sigma p_i + \Sigma q_i = 0$, respectively.

The g.r.f. are real in x-space. Hence, for g_μ any g.r.f.:

$$\tilde{g}_\mu(p_1, \dots, p_n) = \tilde{g}_\mu^*(-p_1, \dots, -p_n) . \tag{33}$$

For q_1, q_2 on the positive mass shell or sufficiently close to it, we have

$$\tilde{r}(p_1, \dots, p_n \uparrow q_1, q_2) = \tilde{r}(p_1, \dots, p_n, q_1, q_2) , \tag{34}$$

the difference $-i\,(\Omega, [\tilde{R}(p_1, \dots, p_n, q_2), \tilde{R}(q_1)]\,\Omega)$ of the two sides vanishing for $q_1^2 < 4\,m^2$. Furthermore, in the region (4):

$$\tilde{r}(p_1, p_2 \uparrow q_1, q_2) = \tilde{r}(p_1, p_2, q_1, q_2) = \tilde{\tau}^{T*}(p_1, p_2, q_1, q_2) ,$$
$$\tilde{r}(p_1, p_2 \downarrow q_1, q_2) = \tilde{\tau}^{T}(p_1, p_2, q_1, q_2) . \tag{35}$$

Hence $\hat{r}(p_1, p_2 \uparrow q_1, q_2)$ contains the same B-pole as $\hat{\tau}^{T*}$, etc.

We define

$$\hat{r}_1(p_1, p_2 \uparrow Q) = \hat{\tau}_1^*(p_1, p_2, Q) ,$$
$$\hat{r}_1(p_1, p_2 \downarrow Q) = \hat{\tau}_1(p_1, p_2, Q) . \tag{36}$$

The $R(.. \updownarrow ..)$ satisfy the identity

$$R(x_1, \dots, x_n \uparrow y_1, \dots, y_m) - R(x_1, \dots, x_n \downarrow y_1, \dots, y_m) =$$
$$-i\,[R(x_1, \dots, x_n), R(y_1, \dots, y_m)] , \tag{37}$$

from which we obtain the unitarity equations

$$\tilde{r}(p_1, \dots, p_n \uparrow q_1, \dots, q_m) - \tilde{r}(p_1, \dots \downarrow \dots, q_m) = i \sum_{l=1}^{\infty} \frac{(2\,\pi)^{2l}}{l!}$$
$$\times \int dk_1 \dots dk_l \left\{ \prod_1^l \delta_+(k_i) - \prod_1^l \delta_+(-k_i) \right\} \tilde{r}(p_1, \dots, p_n, k_1, \dots, k_l)$$
$$\times\, \tilde{r}(q_1, \dots, q_m, -k_1, \dots, -k_l) . \tag{38}$$

For the proof see **II**, equation (80) (note a sign error in this reference).

For $n = m = 2$, in the region (4), (38) is the same as (12) because of (35).

Consider the case $m = 2$, n arbitrary, for q_1, q_2 close to the positive mass shell, $Q^2 = (q_1 + q_2)^2 \sim M^2$. Equation (38) becomes

$$\hat{r}(p_1, \dots, p_n \uparrow q_1, q_2) - \hat{r}(p_1, \dots, p_n \downarrow q_1, q_2)$$
$$= \frac{i}{2}\,(2\,\pi)^4 \int \delta_Q k\, \hat{r}(p_1, \dots, p_n, k, Q - k)\, \hat{r}(q_1, q_2, -k, -Q + k) . \tag{39}$$

We expect $\hat{r}(.. \updownarrow q_1, q_2)$ and $\hat{r}(p_1, \dots, q_2)$ to contain poles of the form

$$\frac{b(p_1, \dots, p_n \updownarrow q_1, q_2)}{Q^2 - M^2 \mp i\,M\,\Gamma} , \quad \frac{b(p_1, \dots, p_n, q_1, q_2)}{Q^2 - M^2 - i\,M\,\Gamma} \tag{40}$$

respectively. The upper sign in the first denominator goes with $\uparrow$, the lower with $\downarrow$. We develop (39) in powers of g and take again the terms of second order, assuming the b_2 to be as smooth as is compatible with unitarity. This implies in particular that b_2 does not itself contain any B-poles. That the Γ_2 used here is independent of n can be shown by a similar argument as the one given after (27) to prove the constancy of

Γ_2 for $n = 2$. Comparison of the residues of $(Q^2 - M^2 + i\,\varepsilon)^{-1}$ on both sides yields

$$\begin{aligned}&- b_2(p_1, \ldots, p_n \downarrow q_1, q_2) = \frac{i}{2} (2\,\pi)^5\, t_1\, \hat{r}_1(q_1, q_2 \downarrow -Q)\\ &\times J_0(p_1, \ldots, p_n \uparrow Q) - \frac{1}{I\, t_1^*} \hat{r}_1(q_1, q_2 \downarrow -Q)\\ &\times \int \delta_Q k\, b_2(p_1, \ldots, p_n, k, Q-k)\end{aligned} \tag{41}$$

with

$$J_0(p_1, \ldots, p_n \uparrow Q) = \int \delta_Q k\, \hat{r}_0(p_1, \ldots, p_n, k, Q-k)\,. \tag{42}$$

The last term in (41) is analogous to (18). From (41) we see that $b_2(.. \downarrow ..)$ factorizes:

$$b_2(p_1, \ldots, p_n \downarrow q_1, q_2) = 2\,\pi\, \hat{r}_1(p_1, \ldots, p_n \downarrow Q)\, \hat{r}_1(q_1, q_2 \downarrow -Q) \tag{43}$$

with

$$\begin{aligned}&\hat{r}_1(p_1, \ldots, p_n \downarrow Q) = -\frac{i}{2} (2\,\pi)^4\, t_1\, J_0(p_1, \ldots, p_n \uparrow Q)\\ &+ \frac{1}{2\,\pi\, I\, t_1^*} \int \delta_Q k\, b_2(p_1, \ldots, p_n, k, Q-k)\,.\end{aligned} \tag{44}$$

From the $-i\,\varepsilon$ pole we obtain in the same way

$$\begin{aligned}&b_2(p_1, \ldots, p_n \uparrow q_1, q_2) = \frac{i}{2} (2\,\pi)^4 \int \delta_Q k\, b_2(p_1, \ldots, p_n, k, Q-k)\\ &\times \hat{r}_0(q_1, q_2, -k, -Q+k)\\ &+ \frac{1}{I\, t_1^*} \hat{r}_1(q_1, q_2 \downarrow -Q) \int \delta_Q k\, b_2(p_1, \ldots, p_n, k, Q-k)\,.\end{aligned} \tag{45}$$

We put $q_1 = k'$, $q_2 = Q - k'$, multiply both sides with $\hat{r}_0(q_1, q_2, -k', -Q+k')$ and integrate over $\delta_Q k'$. Using (35), (41) and (12) in 0th order, we obtain

$$\begin{aligned}&\int \delta_Q k\, b_2(p_1, \ldots, p_n, k, Q-k)\, \hat{r}_0(q_1, q_2, -k, -Q+k)\\ &= \frac{t_1}{I\, t_1^*} J_0^*(q_1, q_2, -Q) \int \delta_Q k\, b_2(p_1, \ldots, p_n, k, Q-k)\,.\end{aligned}$$

Substitution of this expression in (45) yields, with the help of (28) and (36):

$$b_2(p_1, \ldots, p_n \uparrow q_1, q_2) = 2\,\pi\, \hat{r}_1(p_1, \ldots, p_n \uparrow Q)\, \hat{r}_1(q_1, q_2 \uparrow -Q) \tag{46}$$

with

$$\hat{r}_1(p_1, \ldots, p_n \uparrow Q) = \frac{1}{2\,\pi\, I\, t_1^*} \int \delta_Q k\, b_2(p_1, \ldots, p_n, k, Q-k)\,. \tag{47}$$

Thus this residue factorizes too.

Comparison of (44) with (47) gives

$$\hat{r}_1(p_1, \dots, p_n \uparrow Q) - \hat{r}_1(p_1, \dots, p_n \downarrow Q) = \frac{i}{2} (2\pi)^4 t_1 J_0(p_1, \dots, p_n \uparrow Q)$$

$$= \frac{i}{2} (2\pi)^4 \int \delta_Q k \, \hat{r}_0(p_1, \dots, p_n, k, Q - k) \, \hat{r}_1(Q, -k, -Q + k) , \tag{48}$$

where we have defined

$$\hat{r}_1(Q, -q_1, -q_2) = \hat{\tau}_1(-q_1, -q_2, Q) . \tag{49}$$

(48) is a generalization of (28), and can in the same way be considered as the first order part of an unitary equation with n A-fields and one B-field, the latter being defined on its mass shell only. It is an unitarity equation either in $\mathfrak{H}$ or in $\mathfrak{H}'$, since in the latter case intermediate states containing B do not contribute.

(48) shows that the $\hat{r}_1(p_1, \dots, p_n \updownarrow Q)$ cannot both vanish if t_1 and J_0 are different from zero: a B-pole must appear in at least one of the functions $\hat{r}_2(p_1, \dots, p_n \updownarrow q_1, q_2)$.

We proceed now to the case of arbitrary n and m. We expect that $\hat{r}(p_1, \dots, p_n \updownarrow q_1, \dots, q_m)$ contain B-poles in the variable $Q = \Sigma_1^m q_i$. To fix ideas, we assume $Q \in V_+$. $\hat{r}_2(.. \updownarrow ..)$ will contain a pole term

$$\frac{b_2(p_1, \dots, p_n \updownarrow q_1, \dots, q_m)}{Q^2 - M^2 \mp i\,\varepsilon} \tag{50}$$

in complete analogy to (40). In the by now familiar way we insert (50) into (38) and obtain, using the results of the case $m = 2$:

$$b_2(p_1, \dots, p_n \updownarrow q_1, \dots, q_m) = 2\pi \, \hat{r}_1(p_1, \dots, p_n \updownarrow Q) \, \hat{r}_1(q_1, \dots, q_m \updownarrow -Q) \tag{51}$$

with $\hat{r}_1$ defined by (44) and (47). The factorization (51) holds for $Q^2 = M^2$. The direction of the arrow is the same in all three intervening functions.

The general unitarity equations for any g.r.f. have been given in **II**, equation (80). (The overall sign on the right-hand side ought to be a + in this reference.) With their help we can derive the 2nd order B-poles in any g.r.f. from those in $\hat{r}_2(.. \updownarrow ..)$. The result is the following.

Let $\hat{g}_\mu(p_1, \dots, p_n, q_1, \dots, q_m)$ be a g.r.f. corresponding to the cell C_μ, with a δ-factor split off as in (32). Let $Q = \Sigma_1^m q_i$. Then the second order term $\hat{g}_{\mu 2}$ contains a pole

$$\frac{b_{\mu 2}(p_1, \dots, q_m)}{Q^2 - M^2 - i\,\sigma_Q\,\varepsilon} . \tag{52}$$

σ_Q is the sign attached to the set $\{q_1, \dots, q_m\}$ in C_μ.

The residue $b_{\mu 2}$ factorizes in $Q^2 = M^2$:

$$b_{\mu 2}(p_1, \dots, q_m) = 2\pi \, \hat{g}_{\alpha 1}(p_1, \dots, p_n, Q) \, \hat{g}_{\beta 1}(q_1, \dots, q_m, -Q) . \tag{53}$$

$\hat{g}_{\alpha 1}$ ($\hat{g}_{\beta 1}$) can be interpreted as the terms of first order in g.r.f. of n (m) A-fields and one B-field, whose cells C_α (C_β) are as follows: the sign of the one-variable set $\{\pm Q\}$ is the σ_Q of (52) and the σ_I for proper subsets I of $\{p_i\}$, $\{q_i\}$ are as in C_μ. The $\tilde{g}_{\alpha 1}$ satisfy the appropriate unitarity equations. Let C_α, C_β be two adjacent cells with the boundary $\{p_1, \dots, p_s\}$ (for simplicity) and the partial cells C_ρ in $\{p_1, \dots, p_s\}$ and C_ν in $\{p_{s+1}, \dots, p_n, Q\}$.

Then

$$\tilde{g}_{\alpha 1}(p_1, \dots, p_n, Q) - \tilde{g}_{\beta 1}(p_1, \dots, p_n, Q)$$
$$= i \sum_{l=1}^{\infty} \frac{(2\pi)^{2l}}{l!} \int \prod_1^l dk_i \, \{\Pi \delta_+(k_i) - \Pi \delta_+(-k_i)\}$$
$$\times \tilde{g}^+_{\varrho 0}(p_1, \dots, p_s; k_1, \dots, k_l) \, \tilde{g}^+_{\nu 1}(p_{s+1}, \dots, p_n, Q; -k_1, \dots, -k_l) \,. \tag{54}$$

For the notation see **II**. Note that we consider totally amputated g.r.f. so that the suffix *amp* of **II** can be dropped.

Again we can consider (54) as unitarity equation in $\mathfrak{H}'$, since intermediate states with B-particles will, in first order, contribute nothing to the right-hand side.

From the connection of the T-product with the retarded products (see **II**) we learn finally that the functions $\hat{\tau}$ have also the expected pole structure: $\hat{\tau}^T_2(p_1, \dots, p_n$ $q_1, \dots, q_m)$ contains the pole

$$2\pi \frac{\hat{\tau}^T_1(p_1, \dots, p_n, Q) \, \hat{\tau}^T_1(q_1, \dots, q_m, -Q)}{Q^2 - M^2 + i\varepsilon}, \tag{55}$$

where the residue is given at the pole emplacement $Q^2 = M^2$. The factors $\hat{\tau}^T_1(p_1, \dots,$ $p_n, Q)$ can be interpreted as truncated Green's functions of n A-fields and one B-field. i.e. they satisfy the appropriate unitarity equations. As usual, $\hat{\tau}^T_1(\dots, Q)$ is only defined on $\Sigma p_i + Q = 0$, $Q^2 = M^2$.

As yet we have only discussed the B-poles in a single, arbitrarily chosen, partial sum of the variables p_i. We wish to know whether poles in different partial sums combine according to the rules that are valid for stable particles [6].

We have assumed that the residues $b_{\mu 2}$ of (52) are smooth outside the A-singularities. Products of B-poles can then not occur in second order. (B-poles in all possible partial sums occur of course additively.) In order to get products of two poles in different partial sums we must proceed to the terms of order g^3. It is easily shown that multiplicative poles in partially overlapping sets of variables cannot occur. A pole in a certain partial sum can equally well be written as a pole in the complementary sum. Hence we can always write a product of two poles as poles in non-overlapping partial sums. Consider the function $\hat{g}_{\mu 3}(p_1, \dots, p_n, p'_1, \dots, p'_m, p''_1, \dots, p''_l)$ and suppose that it contains the pole product

$$\frac{c_{\mu 3}(p_1, \dots, p''_l)}{(P'^2 - M^2 - i\,\sigma_{P'},\varepsilon)\,(P''^2 - M^2 - i\,\sigma_{P''},\varepsilon)}, \tag{56}$$

$P' = \Sigma p'_i$, $P'' = \Sigma p''_i$. $c_{\mu 3}$ is supposed to be smooth apart from the necessary A-singularities. $\sigma_{P'}$, $\sigma_{P''}$ are defined as in (52).

We insert this ansatz into the 3rd order unitarity equations and compare the residues of corresponding pole products on both sides, taking proper account of terms of the type (18). By using the same type of arguments as earlier, we find that $c_{\mu 3}$ factorizes as expected:

$$c_{\mu 3}(p_1, \dots, p''_l) = (2\pi)^2 \, \hat{g}_{\alpha 1}(p_1, \dots, p_n, P', P'') \, \hat{g}_{\beta 1}(p'_1, \dots, p'_{m'}, -P')$$
$$\times \hat{g}_{\gamma 1}(p''_1, \dots, p''_l, -P'') \tag{57}$$

for $P'^2 = P''^2 = M^2$. $\hat{g}_{\beta 1}$ and $\hat{g}_{\gamma 1}$ are the functions already occuring in (53). $\hat{g}_{\alpha 1}$ is a g.r.f. of n A-variables and two B-variables, with its cell C_α given by $\sigma_{P'}$, $\sigma_{P''}$ and the σ_I of C_μ for subsets I of $\{p_i\}$, including the full set. The $\hat{g}_{\alpha 1}(p_1, \dots, p_n, P', P'')$ satisfy the appropriate 1st order unitarity equations in $\mathfrak{H}'$.

In the same way we can successively build up products of any number of poles by going to sufficiently high orders in g. The procedure must be slightly modified when we reach the case in which all the variables p_i are contained in one of the partial B-pole sums. Let, e.g., $\{P_1\}, \dots, \{P_4\}$ be pairwise disjoint subsets of $\{P\} = \{p_1, \dots, p_n\}$, such that $\cup_i \{P_i\} = \{P\}$. Let $P_i = \Sigma_{\{P_i\}} p_j$. The g.r.f. $\hat{g}_\mu(p_1, \dots, p_n)$ will contain in a suitable order the pole product $\Pi (P_i^2 - M^2 \pm i\,\varepsilon)^{-1}$ with the residue (for $P_i^2 = M^2$)

$$c_\mu(p_1, \dots, p_n) = (2\,\pi)^3\, \hat{g}_\alpha(\{P_1\}, -P_1) \dots \hat{g}_\delta(\{P_4\}, -P_4)\, \hat{g}_\varepsilon(P_1, \dots, P_4). \tag{58}$$

The $\hat{g}_\alpha, \dots, \hat{g}_\delta$ are the functions introduced above, with lowest nonvanishing contributions in first order. Since we want the B to be strongly intracting we must admit a non-vanishing contribution of order g^0 in $\hat{g}_\varepsilon$. In this case the residue of the step before, i.e. of a three-pole product, cannot be maximally smooth but must be allowed to contain another B-pole. Our procedure can be modified in this sense without trouble. The $\hat{g}_{\varepsilon 0}(P_1, \dots, P_n)$ defined in this way satisfy the exact zero-order unitarity equations in $\mathfrak{H}'$, where this time intermediate states containing A-particles do not contribute but the B-states do.

Analogous results can again be proved for the time ordered functions $\hat{\tau}_1^T(p_1, \dots, p_n, P_1, \dots, P_m)$, $\hat{\tau}_0^T(P_1, \dots, P_m)$.

It can also be shown that not only the one-B-poles turn out as expected, but also the n-B-thresholds. We will not discuss this point explicitly since these higher singularities are not important in what follows.

Without proof we remark that the results of this and the preceding section are still valid in the next higher order in g. The same poles as above appear in this order, with residues which factorize as in (7). The factors $\hat{\tau}_2(p_1, \dots, p_n, P_1, \dots, P_m)$, $\hat{\tau}_1(p_1, \dots, P_m)$ satisfy the correct unitarity equations in $\mathfrak{H}'$, in the same way as (29). As analogon to (27) we obtain

$$2\,M\,\Gamma_3 = (2\,\pi)^5\, I\, |t_1\, t_2| \tag{59}$$

and as analogon to (26)

$$\arg t_2 = \delta_{0,0} + \operatorname{arc\,tg}\left[\frac{2\,\Gamma_2}{\Gamma_3}\,\delta_{0,1}\right]. \tag{60}$$

$\delta_{0,1}$ is the 1st-order term in the s-wave phase shift δ_0.

In still higher orders in g the higher terms of (13) have to be taken into account, so that poles of high degree appear. This means that the finite distance of the B-poles from the real axis becomes important. Factorization of the residues can then no longer be expected in $Q^2 = M^2$, but possibly in $Q^2 = M^2 \pm i\,M\,\Gamma$. This complicates the situation considerably. We have not considered this problem. The existence of poles in higher functions and their location will presumably come out as expected, but on the form of the residues it is more dangerous to make prognostications[2]).

Let us briefly summarize the results of sections 3 and 4. We started from the Green's functions $\tilde{\tau}(p_1, \ldots, p_n)$ of the field A and assumed the presence of the pole (5), (7) in the 4-point function. Apart from that we assumed the $\tilde{\tau}$ to be maximally smooth. We then introduced Green's functions $\tilde{\tau}(p_1, \ldots, P_n, P_1, \ldots, P_m)$, $P_i^2 = M^2$, of n A-variables and m B-variables. They are defined up to third order in g for $m = 0$, to second order for $n \neq 0$, $m \neq 0$, to first order for $n = 0$, and satisfy, up to these orders, the unitarity equations

$$\tilde{\tau}_\sigma(p_1, \ldots, P_m) - \tilde{\tau}_\sigma^*(p_1, \ldots, P_m) = -i \sum_{l,h=0}^{\infty} \sum_{L,R} \sum_{\varrho=0}^{\sigma}$$
$$\times \frac{(2\pi)^{2(l+h)}}{l!\,h!} \int \prod_1^l \delta k_i \prod_1^h \delta k_j \, \tilde{\tau}_\varrho^*(\{p, P\}_L, -k_1, \ldots, -k_l, -K_1, \ldots, -K_h)$$
$$\times \tilde{\tau}_{\sigma-\rho}(\{p, P\}_R, k_1, \ldots, k_l, K_1, \ldots, K_h) \,. \tag{61}$$

Here

$$\delta k = \theta(k_0)\, \delta\,(k^2 - m^2)\, d^4k\,, \quad \delta K = \theta(K_0)\, \delta\,(K^2 - M^2)\, d^4K \tag{62}$$

and the L, R summation extends over all partitions of the set $(p_1, \ldots, P_m)$ into two complementary, non-empty, subsets $\{p, P\}_L$ and $\{p, R\}_R$. The subscripts σ etc., denote the order in g. The $\tilde{\tau}_\sigma$ are Lorentz invariant and symmetric under permutations of the p_i and the P_i separately.

This result is hardly surprising. It states exactly what has always been assumed to be true. What is new here is this: we have not only shown that the pole structure described above is consistent with unitarity, but that it is actually enforced by it, once we assume the presence of a pole with a factorizing residue in the 4-point function.

5. Scattering Processes

We want to give a theoretical description of scattering processes involving B-particles as seen by the experimentalist, i.e. as processes monitored by detectors. For simplicity we shall always talk of 'counters', but our considerations are applicable to other types of detectors like bubble chambers, etc. Even a target can be considered as a counter in our sense, since it does localize particles in a given region.

We use the formalism developed in **I**, of whose basic notions we shall now briefly remind the reader. A *state* is an ensemble of identical systems (particles or groups of particles), prepared in a certain prescribed way. It is represented by a vector ϕ in $\mathfrak{H}$. In a counting process these systems are allowed to interact with a counter. The systems

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[2]) Since the completion of this manuscript we have obtained the following result, which holds outside of perturbation theory: if the 4-point function contains a pole (5) with a factorizing residue, then all the higher functions contain the corresponding poles, also with factorizing residues. Assumed is that all the functions can be analytically continued sufficiently far across the relevant Q^2-cut.

which have triggered the counter form the new state ϕ'. This operation can be represented by a bounded operator C in the field algebra generated by A, which is essentially localized in a bounded space-time region G with diameter d_1. (See **I** for the definition of essential localization.) We choose G such that it contains the origin of our system of co-ordinates. Translation of the counter by the 4-vector x gives the counter $C(x)$, which is essentially localized in a neighbourhood of x. The Fourier transform

$$\tilde{C}(\zeta) = (2\pi)^{-2} \int d^4x \, e^{i\zeta x} \, C(x) \tag{63}$$

has an essential support of diameter $d_2 \ll m$. ζ is the momentum transfer to the counter from the observed particle.

We shall use the letters $\zeta_1, \zeta_2, \ldots$ for counter arguments in momentum space, while p_i, $P_i - p_i$, (P_i) retain their meaning of A- (B-) variables. $\tilde{\tau}(\zeta_1, \ldots, \zeta_\alpha, p_1, \ldots, p_\beta, P_1, \ldots, P_\gamma)$ and $\tilde{r}(\zeta_1, \ldots, P_\gamma)$ are the time-ordered and retarded functions of α counter-'fields' $\tilde{C}(\zeta)$, β A-fields and γ B-'fields', where the latter are restricted to the mass shell $P_i^2 = M^2$. $\tilde{\tau}$, $\tilde{r}$ are amputated with respect to the p_i and P_i but not the ζ_i. $\hat{\tau}$, $\hat{r}$ are defined as in (32). They are defined on the manifold $\Sigma \zeta_i + \Sigma p_i + \Sigma P_i = 0$. In what follows this restriction will always be understood.

The counter fields $\tilde{C}(\zeta)$ can be easily introduced into the considerations of the preceding sections: $\hat{\tau}^T(\zeta_1, \ldots)$ and $\hat{r}(\zeta_1, \ldots)$ have in the two lowest non-vanishing orders the expected singularity structure. We assume again that $\hat{\tau}^T$ and $\hat{r}$ are smooth outside the physical singularities, i.e. they are C^∞ and do not oscillate.

Let $|\boldsymbol{p}_1, \ldots, \boldsymbol{p}_n\rangle_{ex}$ be a n-particle *in* or *out* state, with the normalization

$$ {}_{ex}\langle \boldsymbol{q}_1, \ldots, \boldsymbol{q}_n \mid \boldsymbol{p}_1, \ldots, \boldsymbol{p}_n\rangle_{ex} = \frac{1}{n!} \sum_{(j_1, \ldots, j_n)} \prod_{i=1}^{n} \left[2\,\omega(\boldsymbol{q}_i)\, \delta^3(\boldsymbol{q}_i - \boldsymbol{p}_{j_i})\right],$$
$$\omega(q) = (\boldsymbol{q}^2 + m^2)^{1/2}. \tag{64}$$

Let

$$\begin{aligned}&\tilde{\tau}_{MS}(\zeta_1, \ldots, \zeta_\alpha, \boldsymbol{p}_1, \ldots, \boldsymbol{p}_n, -\boldsymbol{q}_1, \ldots, -\boldsymbol{q}_m) = \\ &\tilde{\tau}(\zeta_1, \ldots, p_1, \ldots, -q_1, \ldots)\Big|_{p_{i0} = \omega(\boldsymbol{p}_i),\, q_{i0} = \omega(\boldsymbol{q}_i)}\end{aligned} \tag{65}$$

be the mass-shell restriction of $\tilde{\tau}$ and define $\tilde{r}_{MS}(\zeta_1, \ldots, -\boldsymbol{q}_m)$ analogously. Then, according to the LSZ reduction formulae (see **II**):

$$\begin{aligned}&(n!\,m!)^{-1/2}\,(2\pi)^{n+m}\,\hat{\tau}_{MS}(\zeta_1, \ldots, -\boldsymbol{q}_m) \\ &\qquad = {}_{out}\langle \boldsymbol{p}_1, \ldots, \boldsymbol{p}_n \mid \tilde{T}(\zeta_1, \ldots, \zeta_\alpha) \mid \boldsymbol{q}_1, \ldots, \boldsymbol{q}_m\rangle_{in},\end{aligned} \tag{66'}$$

$$\begin{aligned}&(n!\,m!)^{-1/2}\,(2\pi)^{n+m}\,\tilde{r}_{MS}(\zeta_1, \ldots, -\boldsymbol{q}_m) \\ &\qquad = {}_{in}\langle \boldsymbol{p}_1, \ldots, \boldsymbol{p}_n \mid \tilde{R}(\zeta_1, \ldots, \zeta_\alpha) \mid \boldsymbol{q}_1, \ldots, \boldsymbol{q}_m\rangle_{in}\end{aligned} \tag{66''}$$

up to disconnected terms containing factors $\delta^3(\boldsymbol{p}_i - \boldsymbol{q}_j)$. Hence $\tilde{\tau}_{MS}$ and $\tilde{r}_{MS}$ are concentrated in the essential supports of $\tilde{T}(\zeta_i)$ and $\tilde{R}(\zeta_i)$ respectively, i.e. in a small neighborhood of the origin if T and R are defined with the smooth θ-functions of **I**.

Let us consider, for the moment, the stable A-particles. From **I** we know that the counter $C(0)$ transforms the 1-A-state with wave function $\hat{f}(\boldsymbol{p})$ into a 1-A-state with wave function

$$\hat{g}(\boldsymbol{p}) = \int \delta k \, \hat{\tau}_{MS}(\zeta, \boldsymbol{p}, -\boldsymbol{k}) \, \hat{f}(\boldsymbol{k}) \, . \tag{67}$$

Remember $\zeta = -p + k$. Because of the smoothness of $\hat{\tau}$, $\hat{g}$ will be smooth even if $\hat{f}$ was not. (Note that $\hat{f}$ cannot be arbitrarily wild but must be square integrable.) More generally, application of $C(a)$ to the $\hat{f}$-state yields a state with wave function

$$\hat{g}_a(\boldsymbol{p}) = \exp\{i\,[a_0\,\omega(\boldsymbol{p}) + (\boldsymbol{a}, \boldsymbol{p})]\} \times \int \delta k \, \hat{\tau}_{MS}(\zeta, \boldsymbol{p}, -\boldsymbol{k}) \, \hat{f}(\boldsymbol{k}) \exp\{-i\,[a_0\,\omega(\boldsymbol{k}) + (\boldsymbol{a}, \boldsymbol{k})]\} \, . \tag{68}$$

This is a smooth function (the integral), multiplied by the oscillating factor e^{ipa}. Such a function will be called 'smooth in a'. The corresponding solution of the Klein-Gordon equation in x-space is at the time a_0 concentrated in a neighborhood of $\boldsymbol{a}$. Preparation of a 1-A-state with counters gives, then, a state with a wave function which is smooth in the position of the last counter.

Experimentally one works usually with 1-particle states with 'sharp' momenta. They are states whose wave functions are smooth, but concentrated in a narrow region around a given 4-momentum p'. Such states are prepared with the help of counter telescopes. Ideally a telescope consists of a set of counters $C(x_z)$, $C(x_{z-1})$, ..., $C(x_1)$, $C(x)$, with $x^0 > x_1^0 > \ldots > x_z^0$, arranged such that the relative distances $x - x_1$, ..., $x_{z-1} - x_z$ are all parallel to p'. The telescope is described by the operator

$$C_{p'}(x) = C(x)\, C(x_1) \ldots C(x_z) \, . \tag{69}$$

All variables x, x_i are translated by the same amount if the telescope is translated. The differences $x - x_i$ are internal parameters and are therefore not exhibited. $C_{p'}$ is a counter in the usual sense, except that its region of localization is stretched in the direction p' over a length of many times d_1, and that its sensitivity depends strongly on the momentum of the particle to be observed. The kernel of (67) becomes in this case

$$\hat{\tau}^{p'}_{MS}(\zeta, \boldsymbol{p}, -\boldsymbol{k}) = \int \prod_1^z \{\delta k_i \, e^{i\zeta_i x_i}\} \, \hat{\tau}_{MS}(\zeta, \boldsymbol{p}, -\boldsymbol{k}_1) \times \hat{\tau}_{MS}(\zeta_1, \boldsymbol{k}_1, -\boldsymbol{k}_2) \ldots \hat{\tau}_{MS}(\zeta_z, \boldsymbol{k}_z, -\boldsymbol{k}) \, , \tag{70}$$

with $\boldsymbol{\zeta}_i = -\boldsymbol{k}_i + \boldsymbol{k}_{i+1}$, $\zeta_0^i = -\,\omega(\boldsymbol{k}_i) + \omega(\boldsymbol{k}_{i+1})$. This function has a small essential support not only in ζ, but also in $\boldsymbol{p}$ and $\boldsymbol{k}$, namely in a small region around $\boldsymbol{p}'$. Application of $C_{p'}(x)$ to a 1-A-state results in a 1-A-state with a wave function which is concentrated around $\boldsymbol{p}'$ and is smooth in x. Of course, this wave function describes the state in question only for times larger than x^0.

In a wide sense a target can also be considered as a telescope, the particles in the target having narrow wave functions around $\boldsymbol{p}' = 0$.

States with two or more A-particles are prepared with the help of several telescopes. There is one difficulty to be considered here. We expect to obtain a 2-A-

state with approximate momenta p', q', by applying the product $C_{p'}(0)\ C_{p'}(x)$, $x^0 = 0$, to an incoming 2-particle-state with total momentum $p' + q'$. However, these two telescopes will in general not satisfy the condition introduced in **I**, that the localization region of $C_{q'}(x)$ lie outside the causal shadow of $C_{p'}(0)$. Assume $C_{p'}$, $C_{q'}$ of the form (69):

$$C_{p'}(0) = C(0)\ C(y_1) \ldots C(y_z)\ ,$$

$$C_{q'}(x) = C(x)\ C(x_1) \ldots C(x_z)\ ,$$

with $x_i^0 = y_i^0$, so that $C(x_i)$ and $C(y_i)$ with the same i are essentially localized in space-like separated regions. Application of the telescopes to $|\boldsymbol{p}, \boldsymbol{q}\rangle_{in}$ gives the state

$$\phi = C(0)\ C(x)\ C(y_1)\ C(x_1) \ldots C(y_z)\ C(x_z)\ |\boldsymbol{p}, \boldsymbol{q}\rangle_{in}\ . \tag{71}$$

By discussing the matrix element in $\langle \boldsymbol{p}'', \boldsymbol{q}'' \mid \phi\rangle$ with the methods of **I** we find that it is essentially different from zero only if $\boldsymbol{p} \sim \boldsymbol{p}' \sim \boldsymbol{p}''$, $\boldsymbol{q} \sim \boldsymbol{q}' \sim \boldsymbol{q}''$ (or another such combination), and is then in a sufficient approximation given by the disconnected term $\langle \boldsymbol{p}'' \mid C(0) \ldots C(y_z) \mid \boldsymbol{p}\rangle \langle \boldsymbol{q}'' \mid C(x) \ldots C(x_z) \mid \boldsymbol{q}\rangle$. But this is also the dominant term in ${}_{in}\langle \boldsymbol{p}'', \boldsymbol{q}'' \mid C_{p'}(0)\ C_{q'}(x) \mid \boldsymbol{p}, \boldsymbol{q}\rangle_{in}$, so that the expected expression

$$\phi = C_{p'}(0)\ C_{q'}(x)\ |\boldsymbol{p}, \boldsymbol{q}\rangle_{in} \tag{72}$$

is correct.

Let us now turn to the preparation of B-states. We shall use the results of the preceding sections, i.e. we shall work in the two lowest non-vanishing orders in g, without showing the order indices explicitly.

A 1-B-state with momentum P' can be prepared similarly to a 1-A-state, by applying a counter telescope $C_{P'}$ to an incoming 2-A-state. In order that the particle character of the B can become manifest it is necessary that its life time be longer than the length of the telescope: we must have $\Gamma^{-1} \gg d_1$, which implies

$$\Gamma \ll d_2\ . \tag{73}$$

We apply $C_{P'}(0)$ to the state

$$\phi_{in} = \int \delta p_1\ \delta p_2\ \hat{f}_1(\boldsymbol{p}_1)\ \hat{f}_2(\boldsymbol{p}_2) \mid \boldsymbol{p}_1, \boldsymbol{p}_2\rangle_{in}\ . \tag{74}$$

The wave functions $\hat{f}_i$ have small supports not containing $\boldsymbol{P}'$, concentrated around $\boldsymbol{p}_i'$, and are smooth in points x_i with $x_i^0 < 0$. The 1-B-state

$$\Psi = C_{P'}(0)\ \phi_{in} \tag{75}$$

exists for times larger than 0. It can be expanded into incoming A-states. Because of the essential support of $\tilde{C}_{P'}$ only 2-A-states contribute non-negligibly:

$$\Psi = \int \delta k_1\ \delta k_2\ \psi(\boldsymbol{k}_1, \boldsymbol{k}_2) \mid \boldsymbol{k}_1, \boldsymbol{k}_2\rangle_{in} \tag{76}$$

with

$$\psi(\boldsymbol{k}_1, \boldsymbol{k}_2) = {}_{in}\langle \boldsymbol{k}_1, \boldsymbol{k}_2 \mid \Psi\rangle\ . \tag{77}$$

With (66) this becomes

$$\psi(k_1, k_2) = \frac{(2\pi)^2}{2} \int \delta p_1\ \delta p_2\ \hat{f}_1(\boldsymbol{p}_1)\ \hat{f}_2(\boldsymbol{p}_2)\ \hat{r}_{MS}^{P'}(\zeta, \boldsymbol{k}_1, \boldsymbol{k}_2, -\boldsymbol{p}_1, -\boldsymbol{p}_2)\ . \tag{78}$$

$\hat{r}^{P'}$ is the retarded function of the telescope field $\tilde{C}_{P'}$ and four A-fields. Disconnected terms in (77) do not contribute because a single A cannot trigger the telescope in view of the support of $\hat{f}_i$.

$\hat{r}^{P'}_{MS}(\ldots)$ is concentrated in ζ around the origin, in $\boldsymbol{p}_1 + \boldsymbol{p}_2$ and in $\boldsymbol{k}_1 + \boldsymbol{k}_2$ around $\boldsymbol{P}'$. According to assumption the $\hat{f}_i$ are smooth in a past point. With the methods of **I** we see that the only relevant term in $\hat{r}^{P'}$ is the pole term

$$2\,\pi\,\frac{\hat{r}^{P'}(\zeta,\,-P,\,k_1,\,k_2)\,\hat{r}\,(P,\,-p_1,\,-p_2)}{P^2 - M^2 + i\,\varepsilon} \tag{79}$$

with $P = p_1 + p_2$. More exactly: The contribution of (79) to (78) is the only one that is possibly large enough to have measurable effects. This may look like a contradiction since this term is of order g^2, while $\hat{r}^{P'}$ also contains terms of order g^0. These lower order terms are, however, strongly damped in our geometry in the way described in **I** and are therefore negligible. They are, of course, larger than the (79)-term for sufficiently small g. But this means only that in this case the latter term is also negligible, so that production of B is not observable at all. We must therefore assume that g is not too small. This slight difficulty is not present in models with strong B-production (see the second paragraph of section 2).

In the region $P^2 \sim M^2$, $K^2 = (k_1 + k_2)^2 \sim M^2$ in which we are interested the factor $\hat{r}^{P'}(\zeta, -P, k_1, k_2)$ in (79) is dominated by the pole contribution

$$2\,\pi\,\frac{\hat{r}^{P'}(\zeta,\,K,\,-P)\,\hat{r}\,(-K,\,k_1,\,k_2)}{K^2 - M^2 - i\,\varepsilon}\,, \tag{80}$$

so that we obtain finally

$$\psi(\boldsymbol{k}_1,\,\boldsymbol{k}_2) = \frac{(2\,\pi)^4}{2}\,|t|^2\,\frac{1}{K^2 - M^2 - i\,\varepsilon}\,F(K)\,, \tag{81}$$

$$F\,(K) = \int \delta p_1\,\delta p_2\,\hat{f}_1(\boldsymbol{p}_1)\,\hat{f}_2(\boldsymbol{p}_2)\,\frac{\hat{r}^{P'}(\zeta,\,K,\,-P)}{P^2 - M^2 + i\,\varepsilon}\,. \tag{82}$$

Remember $P = p_1 + p_2$, $K = k_1 + k_2$, and the equations (10) and (49). F is a smooth function which is essentially non-zero only for $K \sim P'$.

For the norm of Ψ we obtain

$$\|\,\Psi\,\|^2 = \frac{(2\,\pi)^8}{4}\,|t|^4 \int \delta k_1\,\delta k_2\,|\,F(K)\,|^2\,\frac{1}{K^2 - M^2 + i\,\varepsilon}\,\frac{1}{K^2 - M^2 - i\,\varepsilon} \tag{83}$$

We remember that $(K^2 - M^2 \pm i\,\varepsilon)^{-1}$ occurs here as a first approximation to $(K^2 - M^2 \pm i\,M\,\Gamma)^{-1}$ and use (17), (27) and the relation $\delta k_1\,\delta k_2 = d^4K\,\delta_K k_2$:

$$\|\,\Psi\,\|^2 = \frac{(2\,\pi)^4}{2}\,|t|^2 \int \delta K\,|\,F(K)\,|^2\,. \tag{84}$$

We see that Ψ can be interpreted as 1-B-state with the wave function

$$\hat{F}\,(K) = \frac{(2\,\pi)^2}{\sqrt{2}}\,t\,F(K)\,. \tag{85}$$

The fact that $\|\Psi\|^2$ is only of order g^2 despite of ψ being itself of this order, confirms the previous assertion that (80) is the only relevant term. For the terms without a K-pole this reduction of order does not occur.

Scattering states of two B-particles can be prepared with two B-telescopes. Both are applied to 2-A-systems converging on their entrances. (That such a simultaneous production of two converging unstable particles hardly ever occurs in practice need not concern us here.) In this way we obtain the product of two states of the form (76):

$$\Psi_2 = \frac{(2\pi)^8}{4} |t|^4 \int \delta k_1 \, \delta k_2 \, \delta k_1' \, \delta k_2' \frac{F_1(K)}{K^2 - M^2 - i\,\varepsilon} \times \frac{F_2(K')}{K'^2 - M^2 - i\,\varepsilon} |\boldsymbol{k}_1, \ldots, \boldsymbol{k}_2'\rangle_{in}\,, \tag{86}$$

with $K = k_1 + k_2$, $K' = k_1' + k_2'$. F_i is smooth in the exit of the corresponding telescope. The form (86) is valid for sufficiently large times.

We interprete Ψ_2 as a state $|\hat{F}_1, \hat{F}_2\rangle$ with two incoming B-particles with wave functions $\hat{F}_i$. In order that this interpretation is consistent we must have

$$\langle \hat{F}_1, \hat{F}_2 \mid \hat{F}_1, \hat{F}_2\rangle = \langle \hat{F}_1 \mid \hat{F}_1\rangle \langle \hat{F}_2 \mid \hat{F}_2\rangle + \langle \hat{F}_1 \mid \hat{F}_2\rangle \langle \hat{F}_2 \mid \hat{F}_1\rangle\,. \tag{87}$$

This scalar product can be computed with (64). On the one hand, we obtain terms in which only variables occurring in the same $\hat{F}_i$ are paired. These terms sum up to the desired result (87). On the other hand, there are cross terms of the general form

$$|t|^8 \int \delta k_1 \ldots \delta k_2' \frac{F_1^* \,(k_1 + k_2')}{(k_1 + k_2')^2 - M^2 + i\,\varepsilon} \frac{F_2^* \,(k_1' + k_2)}{(k_1' + k_2)^2 - M^2 + i\,\varepsilon} \times \frac{F_1 \,(k_1 + k_2)}{(k_1 + k_2)^2 - M^2 - i\,\varepsilon} \frac{F_2 \,(k_1' + k_2')}{(k_1' + k_2')^2 - M^2 - i\,\varepsilon}\,.$$

It is easy to see that this integral exists. Hence the wrong terms are of order g^8 and are negligible with respect to the terms of order g^4 occuring in (87).

Analogously we can construct states with any number of incoming A and B.

The outgoing particles, after scattering has taken place, are again analysed with telescopes. A telescope with entrance in x_z will react mainly to particles with appropriate momentum, whose wave functions are smooth in x_z. This is so because C_p acting to the left on a state ${}_{out}\langle \ldots |$ gives a state with a wave function smooth in x_z, which is practically orthogonal to wave functions oscillating strongly in x_z.

A scattering process will, then, look like this: any number of A- and B-particles are directed with telescopes to a given small region in Minkowski space. The outgoing particles after scattering are analysed by further telescopes. The interaction region, i.e. the common target region of the *in* telescopes, is of a macroscopic size. This means that its dimensions are large compared to the Compton wave length $1/m$ of the A. We assume, however, that the distances involved are small with respect to $1/\Gamma$, so that the B-beams suffer no noticeable attenuation through decay during the experiment. In a $2\,A \to 2\,A$ scattering experiment the processes proceeding through formation of an intermediate B will then not be observed. But these events are of order

g^2 and are negligible with respect to direct scattering which is of order g^0. The indirect events can, of course, be observed in a different counter geometry. We need not discuss this further, since such a process can be treated as a sequence of two independent simpler events: the creation and the decay of a B.

The probability that all the telescopes of such an arrangement are triggered in an event can be calculated with the methods of **I**. Let us first consider the simplest possible process, the decay $B \to 2\,A$. In this process we can study the novel features introduced by the presence of unstable particles.

Take the 1-B-state Ψ defined by (76), (81), (82), for $\boldsymbol{P}' = 0$. We analyse this state with two A-telescopes $C_p(a_1)$ and $C_q(a_2)$ which are aimed at a region around $b = (b^0, \mathbf{0})$ with $1/m \ll b^0 \ll 1/\Gamma$. The entrances of the telescopes shall be later than b^0, and $a_i - b$ shall be parallel to p and q respectively. These telescopes can register B-decays into two A with momenta p and q occurring near b. The probability that both telescopes are triggered is

$$W = \frac{(\Psi, C_p^*(a_1)\, C_2^*(a_2)\, C_q(a_2)\, C_p(a_1)\, \Psi)}{\int \delta K\, |\hat{F}(\boldsymbol{K})|^2}\,. \tag{88}$$

The numerator N of this expression can be calculated by summing over a complete set of *out*-states as intermediate states. Only two-particle states contribute essentially:

$$N = \int \delta k_1\, \delta k_2\, \left|{}_{out}\langle \boldsymbol{k}_1, \boldsymbol{k}_2 \,|\, C_q(a_2)\, C_p(a_1) \,|\, \Psi\rangle\right|^2 . \tag{89}$$

The matrix element ${}_{out}\langle \boldsymbol{k}_1, \boldsymbol{k}_2 \,|\, C_q\, C_p \,|\, \boldsymbol{k}_1', \boldsymbol{k}_2'\rangle_{in}$ in this expression is dominated by its disconnected part $1/2\langle \boldsymbol{k}_1 \,|\, C_q \,|\, \boldsymbol{k}_1'\rangle\, \langle \boldsymbol{k}_2 \,|\, C_p \,|\, \boldsymbol{k}_2'\rangle + \ldots$, so that

$$\begin{aligned} N = {} & (2\,\pi)^8\, |t|^4 \int \delta k_1\, \delta k_2\, \delta k_1'\, \delta k_2'\, \delta k_1''\, \delta k_2''\, \hat{\tau}^p(\zeta_1, k_1, -k_1')\, \hat{\tau}^q(\zeta_2, k_2, -k_2') \\ & \times \hat{\tau}^{p*}(\zeta_3, k_1, -k_1'')\, \hat{\tau}^{q*}(\zeta_4, k_2, -k_2'')\, \frac{F(K')}{K'^2 - M^2 - i\,\varepsilon}\, \frac{F^*(K'')}{K''^2 - M^2 + i\,\varepsilon} \\ & \times \exp\left[i\, a_1\, (k_1'' - k_1') + i\, a_2\, (k_2'' - k_2')\right]. \end{aligned} \tag{90}$$

The exponentials come in like in (68), and as usual $K' = k_1' + k_2'$, $K'' = k_1'' + k_2''$. Under our assumptions on a_i we can replace the factors $(K'^2 - M^2 - i\,\varepsilon)^{-1}$ and $(K''^2 - M^2 + i\,\varepsilon)^{-1}$ by $2\,\pi\, i\, \delta_+(K')$ and $-2\,\pi\, i\, \delta_+(K'')$ respectively, since the expression obtained from (90) by changing one of the ε-signs decreases rapidly for $a_i^0 \to \infty$ and is already negligible for our a_i. Hence

$$\begin{aligned} N = {} & 2\,(2\,\pi)^6 \int \delta k_1'\, \delta k_2'\, \delta k_1''\, \delta k_2''\, \delta_+(K')\, \delta_+(K'')\, \hat{F}(K')\, \hat{F}^*(K'') \\ & \times \hat{\tau}\,(k_1', k_2', -K')\, \hat{\tau}^*(k_1'', k_2'', -K'')\, \exp\left[i\, a_1\, (k_1'' - k_1') + i\, a_2\, (k_2'' - k_2')\right] \\ & \times L_p(\boldsymbol{k}_1', \boldsymbol{k}_1'')\, L_q(\boldsymbol{k}_2', \boldsymbol{k}_2'') \end{aligned} \tag{91}$$

with

$$L_p(\boldsymbol{k}', \boldsymbol{k}'') = \int \delta k\, \hat{\tau}_{MS}^{p*}(\zeta_1, \boldsymbol{k}, -\boldsymbol{k}')\, \hat{\tau}_{MS}^{p*}(\zeta_2, \boldsymbol{k}, -\boldsymbol{k}'')\,. \tag{92}$$

The function L_p characterizes the efficiency of the telescope C_p. It is essentially different from zero only for $\boldsymbol{k}' \sim \boldsymbol{k}'' \sim \boldsymbol{p}$. Because of this and the support of $\hat{F}$, N is not negligible only for $(p+q)^2 \sim M^2$, as was to be expected.

(91) shows that $t = \hat{\tau}(k_1, k_2, -K)$, all variables on their mass shells, can be interpreted as S-matrix element for the decay of a B with momentum K into two A with momenta k_1, k_2. If $L_{p,q}$ factorized:

$$L_p(\boldsymbol{k}', \boldsymbol{k}'') = \hat{f}_p^*(\boldsymbol{k}')\, \hat{f}_p(\boldsymbol{k}'') ,$$

(91) would be the familiar S-matrix element for decay of a B with wave function $\hat{F}$ into two A with wave functions $\hat{f}_p$ and $\hat{f}_q$. Since the telescopes $C_{p,q}$ do not actually determine wave functions but analyse states in a more complicated way we obtain the more complicated expression (91).

If b^0 is increased to something of the order $1/\Gamma$ we expect a factor $exp(-b^0\,\Gamma)$ in W. Such a factor does not appear directly in our approximation since $exp(-b^0\,\Gamma) = 1 + O(g^2)$. Indirectly it can be inferred from (27) and the physical interpretation of t as S-matrix element for decay, which tells us that the attenuation dW/db^0 is proportional to $|t|^2$. As has been shown in **I** the exponential factor does appear if we work with the exact Green's functions, assuming the existence of the pole (5).

In the same way as decay we can treat scattering. We can, for instance, set up a $2\,B$ scattering process by preparing a $2\,B$-*in*-state according to (86) and analysing the final state with two telescopes $C_P(a_1)$, $C_Q(a_2)$ in a suitable geometrical arrangement. For normalized wave functions $\hat{F}_{1,2}$ we obtain for the probability that both counters are triggered:

$$\begin{aligned} W = {} & (2\pi)^{16} \int \delta K_1\, \delta K_2\, \delta K_1'\, \delta K_2'\, \delta P'\, \delta Q'\, \delta P''\, \delta Q''\, \tilde{\tau}(K_1, K_2, -P', -Q') \\ & \times \tilde{\tau}^*(K_1', K_2', -P'', -Q'')\, \hat{F_1}(K_1)\, \hat{F_1^*}(K_1')\, \hat{F_2}(K_2)\, \hat{F_2^*}(K_2') \\ & \times \exp[i\, a_1 (P' - P'') + i\, a_2 (Q' - Q'')]\, L_P(\boldsymbol{P}', \boldsymbol{P}'')\, L_Q(\boldsymbol{Q}', \boldsymbol{Q}'') . \end{aligned} \tag{93}$$

L_P is defined in analogy to (92). We see that $\tilde{\tau}(K_1, K_2, -P, -Q)$ can be interpreted as S-matrix element for elastic $2\,B$-scattering. If $\hat{\tau}$ is essentially constant in the supports of $\hat{F}_i$ and $L_{P,Q}$ we can draw the factor $|\hat{\tau}(\overline{K}_1, \overline{K}_2, -P, -Q)|^2$, $\overline{K}_i$ a point in $supp\ \hat{F}_i$, in front of the integral. The remaining integral is an experimental factor depending on counter efficiency and geometry.

6. Asymptotic Conditions

In this section we shall indicate briefly how an S-matrix for B-particles could be defined in a more conventional way, using an approximate asymptotic condition.

We introduce a quasilocal field for B-particles:

$$\tilde{B}(Q) = \int d^4k\, \chi(k)\, [\tilde{T}^*(\tfrac{1}{2}Q - k, \tfrac{1}{2}Q + k) - \tilde{\tau}^*(\tfrac{1}{2}Q - k, \tfrac{1}{2}Q + k)] . \tag{94}$$

$\chi(k)$ is a smooth C^∞-function which decreases at ∞ sufficiently fast to make the integral (94) converge. Remember that $\tilde{T}$ and $\tilde{\tau}$ are amputated.

Choose $\tilde{f}(Q)$ in the space S of tempered test functions, with support in $4\,m^2 < Q^2 < 9\,m^2$, $Q_0 > 0$. Define

$$\hat{f}(\boldsymbol{Q}) = \tilde{f}(Q)\Big|_{Q_0 = (\boldsymbol{Q}^2 + M^2)^{1/2}} \tag{95}$$

We introduce the 'creation operator'

$$B_f^*(t) = \int d^4Q\, e^{itQ^-}\, \tilde{f}(Q)\, \tilde{B}^*(Q)\,, \tag{96}$$

$Q^- = Q_0 - (\boldsymbol{Q}^2 + M^2)^{1/2}$. We wish to obtain information on the behaviour of B_f^* for large $|t|$.

Under our assumption that the decay products A are strongly interacting we cannot expect any convergence in the strong Haag-Ruelle sense, because the time-independent quantity $\|\dot{B}_f^*(t)\,\Omega\|^2$ does not vanish, in contrast to the stable case. It is of order g^0, hence not negligible.

With respect to the LSZ asymptotic condition we are in a better position. Let $\tilde{\varphi}(p_1, \dots, p_n) \in S$ be a smooth test function with support in $p_i \in V_+$, so that $\varphi(x_1, \dots, x_n)$ is concentrated around the origin. Let $|\hat{g}_1, \dots, \hat{g}_m\rangle$ be a m-A-particle *in*-state with smooth wave functions $\hat{g}_i$. (Remember that our definition of smoothness excludes strong oscillations.) The $\hat{g}_i$ shall not overlap with $\hat{f}$ and among themselves. For simplicity we assume also that no partial sum of $Q \in supp\,\hat{f}$ and $p_i \in supp\,\tilde{g}_i$, with $\tilde{g}_i$ a continuation of $\hat{g}_i$ in a small neighborhood of the mass shell, lies on a threshold. We are then not forced to know anything on the nature of the threshold singularities.

Consider

$$M(t) = \int dp_i\, \delta q_i\, dQ\, dk\, \tilde{\varphi}(p_1, \dots, p_n)\, \Pi\, \hat{g}_i(q_i)\, \tilde{f}(Q)\, e^{itQ^-}\, \chi(k)$$
$$\times \langle \Omega\,|\, \widetilde{T^n(p_1, \dots, p_n)\; T\,(\tfrac{1}{2}Q - k, \tfrac{1}{2}Q + k)}\,|\, \boldsymbol{q}_1, \dots, \boldsymbol{q}_m\rangle_{in}\,. \tag{97}$$

T^n is the unamputated T-product. According to a result of **II** we can replace the matrix element in (97) for sufficiently large $|t|$ ($|t| \gg 1/m$) in a good approximation by $\tilde{\tau}^n(p_1, \dots, p_n, \frac{1}{2}Q - k, \frac{1}{2}Q + k, -q_1, \dots, -q_m)$. Here the p_i are not amputated but the other variables are.

We develop $\tilde{\tau}^n$ in a cluster sum. The terms containing the two Q-variables in different factors decrease rapidly for $t \to -\infty$ because of our assumptions of smoothness. In the remaining terms both Q-variables will occur in a factor which is again of the form (97), but with a truncated $\tilde{\tau}^n$. The terms of order g^0 and g^1 in this $\tilde{\tau}^{nT}$ are smooth in the relevant region, so that the corresponding terms in M decrease strongly. In 2nd order the pole

$$\frac{\tilde{\tau}^T(\frac{1}{2}Q - k, \frac{1}{1}Q + k, -Q)\; \tilde{\tau}^{nT}(p_1, \dots, p_n, Q, -q_1, \dots, -q_m)}{Q^2 - M^2 + i\,\varepsilon}$$

is present. Its contribution to $M(t)$ converges for $t \to -\infty$ toward the same expression with $(Q^2 - M^2 + i\,\varepsilon)^{-1}$ replaced by $-2\,\pi\,i\,\delta_+(Q)$. This is not negligible if g is not too small (see the remark after (79)). This limit agrees with the LSZ limit of the stable case.

In higher orders in g occur multiple poles $(Q^2 - M^2 + i\,\varepsilon)^{-\sigma}$. They generate terms in $M(t)$ behaving like $|t|^{\sigma-1}$, which are not small compared to the g^2-terms for large $|t|$. The convergence in 2nd order does thus not constitute a genuine asymptotic condition but only an approximate one: the 2nd order limit is a good approximation to $M(t)$ for large enough but not too large $|t|$. This becomes clear if we introduce the exact pole $(Q^2 - M^2 + i\,M\,\Gamma)^{-1}$ in place of the approximate $i\,\varepsilon$-pole. $M(t)$ decreases then like $e^{-\Gamma|t|}$: it converges to 0 for $t \to -\infty$. For $|t| \ll 1/\Gamma$ this exponential factor is, however, not distinguishable from 1.

In the same way we can discuss the pseudo-asymptotic behaviour of the destruction operator $B_f(t)$. The result is that $B_f^*(t)$, $B_f(t)$ can, for $-1/m \gg t \gg -1/\Gamma$, be replaced in a good approximation by the creation and annihilation operators $B_f^{in\,(*)}$ of a free field with mass M. With them we can form incoming B-states. Outgoing states are constructed analogously. An S-matrix can then be defined as usual, and reduction formulae can be derived.

This procedure for the definition of S is formally simpler than the one used in the preceding section. Nevertheless, we prefer the latter because it is physically more transparent. The physical meaning of the mathematical constructions is clear there, which can hardly be said for the considerations of the present section.

7. Is There a Local B-Field?

An obvious question to be asked is whether a scalar local field $B(x)$, local relative to $A(x)$, can be associated with the unstable B-particles, at least to some low order in g. We do not know the answer. This section will only contain a more explicit statement of the problem.

A quasilocal field B without simple covariance properties has been introduced in (94). Formally this field becomes local for the choice $\chi \equiv 1$, because we have then in x-space

$$B(x) = T(x, x) - \tau(x, x)\,. \tag{98}$$

After multiplication with a suitable renormalization constant this field gives indeed the desired on-mass-shell Green's functions $\tilde{\tau}(P_1, \ldots, P_n, p_1, \ldots, p_m)$. However, we have every reason to expect this renormalization constant to be zero, i.e. the right-hand side of (98) does in general not exist as it stands. It is not known at present how to make sense of an expression like $T(x, x)$ outside of perturbation theory.

Another possible approach proceeds via the Haag expansion (see **II**). If the desired field $\tilde{B}(P)$ exist in $\mathfrak{H}$ then it can be expanded as

$$\tilde{B}(P) = \sum_{l=2}^{\infty} \frac{(2\,\pi)^l}{l!} \int dp_1 \ldots dp_l \, \frac{\tilde{r}(P, p_1, \ldots, p_l)}{P^2 - M^2 - i\,\varepsilon\,P_0}$$
$$\times :\tilde{A}_{in}(-p_1) \ldots \tilde{A}_{in}(-p_l):\,, \tag{99}$$

where $\tilde{r}(P, p_1, \ldots, p_l)$ is the amputated retarded function of 1 B-field and l A-fields. B is scalar if the $\tilde{r}$ are Lorentz invariant. B is local relative to A if $r(x, y_1, \ldots, y_l)$ is retarded in the variables $x - y_i$, i.e. if $\hat{r}(P, p_1, \ldots, p_l)$ is analytic in $Im\, p_i \in V_-$ for all i.

In Section 4 we have defined $\hat{r}(P, p_1, \ldots, p_l)$ up to order g^2 for $P^2 = M^2$. Nothing has been said about the analyticity properties of these objects, for the good reason that there is not much that we can say about it.

The problem is simplest for the functions $\tilde{r}(p_1, \ldots, p_n \updownarrow q_1, \ldots, q_m)$ which were the primary tools of our study of unitarity. They can be written as functions of the variables $p_2, \ldots, p_n, q_2, \ldots, q_m$ and $Q = \Sigma\, q_i$. They are then analytic [7] in the domain

$$Im\, p_i \in V_-,\ Im\, q_i \in V_-,\ Im\, Q \in V_+ \text{ for } \downarrow,\ Im\, Q \in V_- \text{ for } \uparrow .$$

It is reasonable to demand the same analyticity for the numerator $b(.. \updownarrow ..)$ in (50). The restriction of b to real Q, $Q^2 = M^2$, is then still analytic in the remaining variables p_i, q_i. In particular, $\hat{r}(p_1, \ldots, p_n \updownarrow Q)$, $Q^2 = M^2$, is in $p_2, \ldots, p_n$ analytic if $Im\, p_i \in V_-$, as desired. The question is whether this function can be extended to arbitrary Q in such a way that the result is also analytic in Q for $Im\, Q \in V_\pm$ and satisfies the correct unitarity equations. The question is not solved at present.

The problem is still harder for the functions $\hat{r}(Q, p_1, \ldots, p_n)$ which occur in (99) and are therefore of the greatest interest to us. They should be analytic in $Im\, p_i \in V_-$ for all i. Because of $Q + \Sigma\, p_i = 0$ we are forced to make all the p_i real if we want to approach a point with real Q from this domain. This means that $\hat{r}(Q, p_1, \ldots, p_n)$, with Q restricted to its mass shell, will not show any analyticity, not even in a lower number of variables[3]). It is not known under what conditions the $\hat{r}(Q, p_1, \ldots, p_n)$ defined earlier for $Q^2 = M^2$ is the restriction to the mass shell of a function with the correct analyticity and unitarity properties.

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[3]) Strictly speaking, this negative verdict is not quite true: under our assumptions we can, e.g. continue $r(p_1, p_2 \uparrow q_1, q_2)$ across the $(q_1 + q_2)$-cut onto the second sheet [8], and this will yield some p_2-analyticity of $\hat{r}(Q, p_1, p_2)$ in $Q^2 = M^2$, but not nearly as much as we need.