

Zeitschrift: Helvetica Physica Acta

Band: 44 (1971)

Heft: 4

Artikel: Influence of the chemical environment on β -decays

Autor: Alder, K. / Baur, G. / Raff, U.

DOI: <https://doi.org/10.5169/seals-114297>

Nutzungsbedingungen

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften auf E-Periodica. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Das Veröffentlichen von Bildern in Print- und Online-Publikationen sowie auf Social Media-Kanälen oder Webseiten ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. [Mehr erfahren](#)

Conditions d'utilisation

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. La reproduction d'images dans des publications imprimées ou en ligne ainsi que sur des canaux de médias sociaux ou des sites web n'est autorisée qu'avec l'accord préalable des détenteurs des droits. [En savoir plus](#)

Terms of use

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. Publishing images in print and online publications, as well as on social media channels or websites, is only permitted with the prior consent of the rights holders. [Find out more](#)

Download PDF: 19.01.2026

ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>

Influence of the Chemical Environment on β -Decays

by **K. Alder, G. Baur and U. Raff**

Institute for Theoretical Physics, University of Basel, Switzerland

(26. XII. 70)

Abstract. The influence of a change of the electron screening on β^- -decay is discussed. The results are used to calculate the change of the half-life of various β -decays in different chemical environments.

I. Introduction

It has been claimed that in various measurements an influence of the chemical environment on the half-life of β^- -decays could be proved. Quite strong effects, in some cases even changes up to 4%, were reported [1, 2]. In this paper we want to show by an approximate calculation that such a change of the half-life can only be of the order of magnitude of 10^{-4} to 10^{-5} . A possible exception is the very low energy transition ${}^3\text{H} \rightarrow {}^3\text{He}$ where the effect may be of the order of 10^{-3} .

We assume that the decay rate changes due to the change of the Fermi function $F(Z, W)$ caused by different values of the screening potential in different chemical environments. The Fermi function is computed by a modified WKB method and the results are compared with a non-relativistic calculation using a Hulthen potential.

II. Beta Decay Theory

(a) Relativistic Approximation

The ft -value of a beta transition is defined as the product of the statistical function $f(Z, W_0)$ and the half-life t_H

$$f \cdot t_H = \frac{f \cdot \ln 2}{A} = \text{const.} \quad (1)$$

It depends only on nuclear matrix elements and on the β -decay constants and is therefore not changed by the chemical environment. The transition probability is denoted by A , while the statistical function $f(Z, W_0)$ is defined through the endpoint energy W_0 of the beta particle by

$$f(Z, W_0) = \int_1^{W_0} p W (W_0 - W)^2 F^\pm(Z, W) dW. \quad (2)$$

The momentum and energy of the emitted electron are p and W , while $F^\pm(Z, W)$ is the Fermi function in the screened potential. This function $F_c^\pm(Z, W)$ for a pure Coulomb field is given by

$$F_c^\pm(Z, W) = 2(1 + \gamma) [\Gamma(2\gamma + 1)]^{-2} (2pR)^{2(\gamma-1)} e^{\mp\xi\frac{W}{p}} \left| \Gamma\left(\gamma + i\alpha Z \frac{W}{p}\right) \right|^2, \quad (3)$$

where γ and ξ are connected with the fine structure constant by $\gamma = (1 - \alpha^2 Z^2)^{1/2}$ and $\xi = \pi\alpha Z$. The charge number of the nucleus is Z , while R is the nuclear radius for which we assume $R = 1.2 \cdot A^{1/3}$ fm. We denote the difference between the screened electrostatic potential and the pure Coulomb potential at the nucleus by V^s and call it screening potential. The quantity W^s is defined by $W^s = W - V^s$. The modified WKB approximation¹⁾, according to Rose [3] and Good, Jr. [4], replaces the pure Coulomb Fermi function $F_c^\pm(Z, W)$ by

$$F^\pm(Z, W) \simeq \frac{p^s W^s}{p W} F_c^\pm(Z, W^s). \quad (4)$$

The screened statistical function $f^{scr}(Z, W_0)$ is then obtained by numerical integration.

For the change of the decay rate we then obtain

$$\frac{\Delta\Lambda}{\Lambda} = \frac{f_I - f_{II}}{f_I}, \quad (5)$$

where the subscripts I and II denote the different chemical environments which are manifested in a change of the screening potential V^s .

(b) Non-relativistic Approximation

A Hulthen potential $V_H(r)$ with variable screening parameter λ is assumed to act on the electron

$$V_H(r) = \frac{Z\alpha\lambda e^{-\lambda r}}{1 - e^{-\lambda r}} \xrightarrow[r \rightarrow 0]{} \frac{Z\alpha}{r} - \frac{Z\alpha\lambda}{2} + \dots \quad (6)$$

For small r the potential takes Coulombian form with an energy shift of $1/2 Z\alpha\lambda = V^s$.

The Schrödinger equation can be solved analytically for $l = 0$ which leads to the explicit Fermi function $F_s(Z, W)$ [5]

$$F_s(Z, W) = \left| \frac{\Gamma(1 + i\nu) \Gamma(1 - i\xi)}{\Gamma(1 - 2i\kappa)} \right|^2 \quad (7)$$

with

$$\nu = \frac{(p^2 + a^2)^{1/2}}{\lambda} - \frac{p}{\lambda} \quad \xi = \frac{(p^2 + a^2)^{1/2}}{\lambda} + \frac{p}{\lambda}$$

and

$$\kappa = \frac{p}{\lambda} \quad a = 2Z\alpha\lambda \quad p = (W^2 - 1)^{1/2}.$$

¹⁾ The WKB results were compared to a solution of the Dirac equation in a self-consistent field by Matese and Johnson [6]. The agreement of these detailed calculations with the simple WKB approximation is very good. It is well worth noticing that the WKB method depends only on the well determined constant V^s .

III. Results and Discussion

For various atoms and various states of ionicity²⁾ the screening potential V^s can be computed by means of the Hartree-Fock method yielding the approximate result $V^s \simeq 1.45 Z^{4/3} \alpha^2$. Such calculations show also that the change of V^s is always smaller than $\alpha^2 = 27.21$ eV if an electron is added to the atom. We denote the difference of the screening potential V^s in two chemical environments by $\Delta I \alpha^2$. Thus we get for the relative change of the transition probability

$$\frac{\Delta A}{A} = \frac{\Delta I}{1.45 Z^{4/3}} \frac{f^{scr} - f^{unscr}}{f^{unscr}} . \quad (8)$$

In the non-relativistic approximation the screening potential V^s is connected with the parameter λ of the Hulthen potential by $V^s = Z \alpha \lambda/2$.

In the following we try to estimate the characteristic parameter ΔI for the two different chemical states, i.e. for solid NaCl and for an aqueous solution of NaCl. The solid state forms a ionic crystal with Na^+ and Cl^- at the lattice points. The total screening potential is the sum of the potential $V^s(\text{Na}^+)$ from the Na^+ ion and $V^s(\text{latt})$ from the surrounding ions. The value $V^s(\text{Na}^+)$ can be computed by a Hartree-Fock program while $V^s(\text{latt})$ according to [8] is given by

$$V^s(\text{latt}) = \alpha_M \frac{e^2}{a} \quad (9)$$

where $\alpha_M = 1.747$ is the Madelung constant and $a = 2.814 \text{ \AA}$ is the lattice constant. The contribution $V^s(\text{latt})$ is of the order of 8.9 eV. In the aqueous solution, the Na^+ ion is surrounded by six water molecules which form a dipole layer around the Na^+ ion.

The centers of the positive and negative charge of the water molecule are r_+ and r_- respectively. It is convenient to write for the water dipole moment $\delta (r_+ - r_-)$, where the charge δ is of the order of the electron charge. The contribution of the six water dipoles to the screening potential is thus given by

$$V^s(\text{dip}) = 6 e \delta \left(\frac{1}{r_-} - \frac{1}{r_+} \right) . \quad (10)$$

From the experimental value of the dipole moment of 1.86×10^{-18} esu and from $r_- = 1.05 \text{ \AA}$ and $r_+ = 1.60 \text{ \AA}$ we obtain for the dipole screening potential a value of 20 eV. In the difference of the screening potentials between liquid and solid states, the explicit value of $V^s(\text{Na}^+)$ cancels. We get

$$V^s(L) - V^s(S) = 11 \text{ eV} = 0.4 \alpha^2 \quad (11)$$

which leads finally to the relative change in the transition probability

$$\frac{A_L - A_S}{A_L} = - 0.03^0/00 . \quad (12)$$

We also have investigated the dependence of the relative change $\Delta A/A$ as a function of Z and the energy W_0 , both for electron and positron emitters, which is shown in Figures 1 and 2. The parameter ΔI is assumed to be unity. It is noted that the effects

²⁾ For example for Na^+ and Na^0 we found $\Delta = 0.4 \alpha^2$.

become largest for low Z and low W_0 in the case of electron emission, and for high Z and low W_0 for positron emission. In the latter case, the electron capture might be dominant and must also be considered.

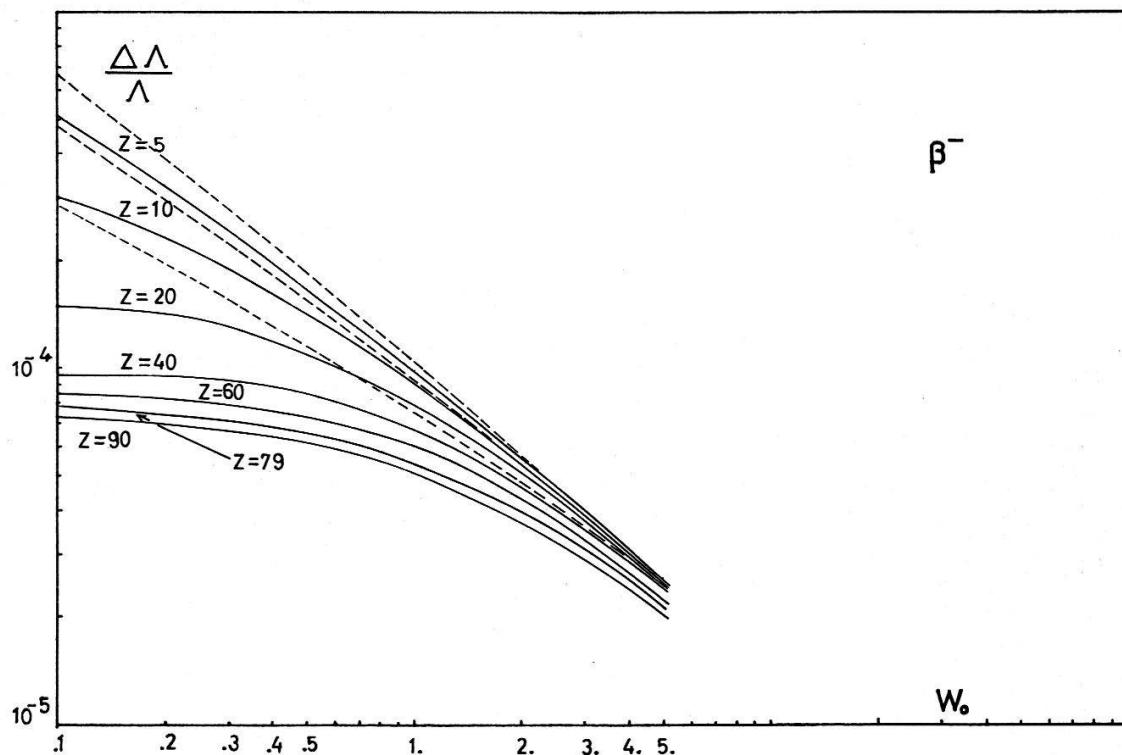


Figure 1

The change $\Delta\Lambda/\Lambda$ is shown as a function of Z and W_0 for β^- -decays for $\Delta I = 1$. The dashed lines are calculated with the approximation for low Z given by D. H. Wilkinson [8]. These lines correspond to $Z = 5, 10$ and 20 .

In Table 1 we collected the results of some β^- -decays. Especially for low Z and low W_0 , the simple non-relativistic calculation shows a remarkable agreement with the relativistic one. From the above considerations it seems to us that the reported changes of the half-life [1, 2] in the β^- -decay of ^{24}Na and ^{131}I are not correct. The conditions seem to be more favourable for the low energy transition $^{35}\text{S} \rightarrow ^{35}\text{Cl}$, especially since ^{35}S can be produced in different valence states (+6 to -2). Also the very low energy (18 keV) transition $^3\text{H} \rightarrow ^3\text{He}$ should give rather large effects in different chemical surroundings.

Table 1

The change $\Delta\Lambda/\Lambda$ for some selected β^- -decays for $\Delta I = 1$. The entries show the transition, the charge number of the daughter nucleus, the transition energy, the relativistic and unrelativistic statistical factor. The last few entries indicated the change of the transition probability according to the relativistic and the unrelativistic approximation

Transition	Z	W ($m_e c^2$)	f (rel.)	f (n. rel.)	$\Delta\Lambda/\Lambda$ % ₀₀ rel.	$\Delta\Lambda/\Lambda$ % ₀₀ n. rel.
$^3\text{H} \rightarrow ^3\text{He}$	2	1.036	2.603 (- 6)	2.85 (- 6)	2.75	3.0
$^{24}\text{Na} \rightarrow ^{24}\text{Mg}$	12	3.724	2.453 (+ 1)	2.02 (+ 1)	0.07	0.03
$^{35}\text{S} \rightarrow ^{35}\text{Cl}$	17	1.326	1.190 (- 2)	1.15 (- 2)	0.11	0.1
$^{131}\text{I} \rightarrow ^{131}\text{Xe}$	54	2.186	5.906 (0)	1.91 (0)	0.07	0.008

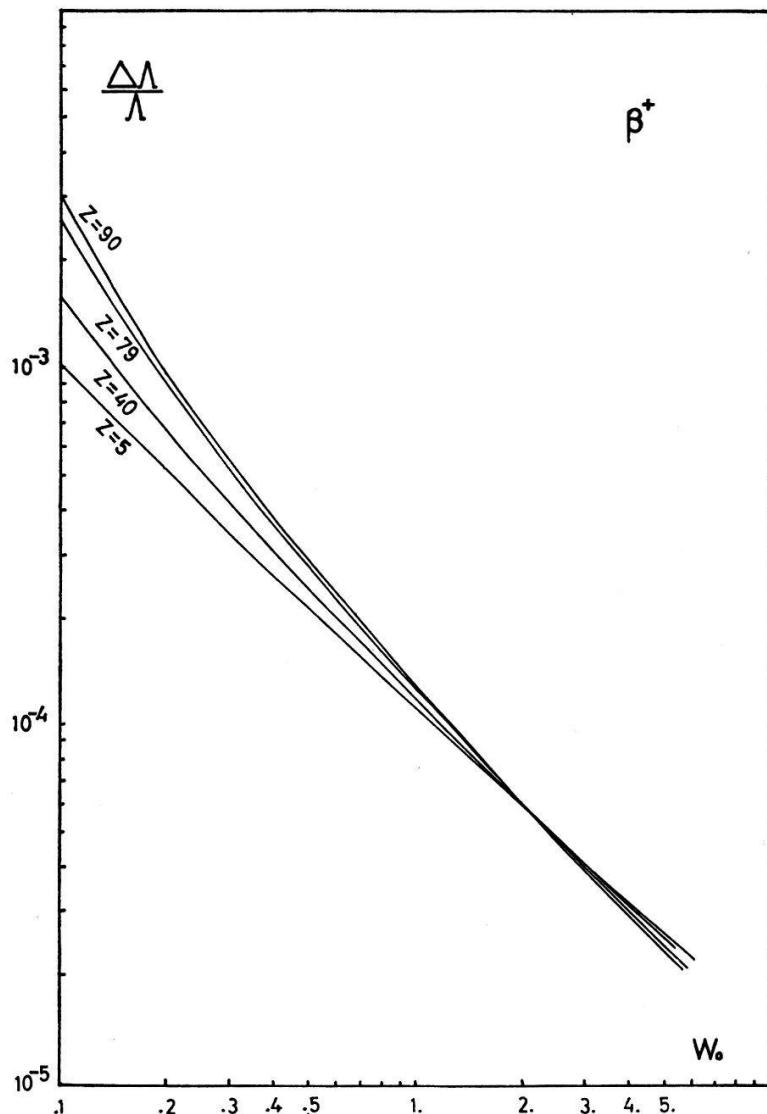


Figure 2
The change in half-life
 $\Delta\Lambda/\Lambda$ is shown as a function
of Z and W_0 for β^+ -decays
for $\Delta I = 1$.

REFERENCES

- [1] P. KEMÉNY, Radiochem. Radioanal. Lett. 2/3, 119 (1969).
- [2] P. KEMÉNY, Radiochem. Radioanal. Lett. 2/1, 41 (1969).
- [3] M. E. ROSE, Phys. Rev. 49, 727 (1936).
- [4] R. H. GOOD, JR., Phys. Rev. 90, 131 (1953); 94, 931 (1954).
- [5] L. DURAND III, Phys. Rev. 135, B310 (1964).
- [6] J. J. MATESE and W. R. JOHNSON, Phys. Rev. 150, 846 (1966).
- [7] J. C. SLATER, *Introduction to Chemical Physics* (McGraw-Hill Book Comp. Inc., New York and London 1939), p. 388.
- [8] D. H. WILKINSON, Nucl. Phys. A 150, 478 (1970).