Zeitschrift: Helvetica Physica Acta
Band: 44 (1971)
Heft: 4

Artikel: On the geometry dependence of nonlinear electrical conduction in intrinsic semiconductors
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DOI: https://doi.org/10.5169/seals-114296

On the Geometry Dependence of Nonlinear Electrical Conduction in Intrinsic Semiconductors

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(23. XII. 70)

Abstract. It is shown that for a cylindrical-shaped specimen the current-voltage characteristic depends in its nonlinear part on the shape and dimensions of the cross-section. It saturates for a large electric field and constant mobility for any nonlinear recombination rate of electrons and holes, whereas for a linear recombination rate it exhibits a superohmic behavior. Neglecting magneto-resistance effects the result can be generalized to field-dependent mobilities. It again yields a saturation of the current for large fields if a linear recombination law and a saturating drift velocity are assumed. The result agrees with experimental data essentially for large dimensions.

1. Introduction

It is a well-known fact that any electrical conduction measurement on a semiconductor of a given shape gives rise to a current-voltage (CV) characteristic $I(V, \Gamma)$ depending trivially on the shape Γ of the specimen, e.g., the linear ohmic resistance $R(\Gamma)$. It has often been assumed up to now that this (trivial) shape dependence can be avoided for cylindrical semiconductors, if the experimental data is reduced to a cross-section averaged current density versus axial field (ACDF) relation $\bar{j}_z(E_z, \Gamma)$. This is certainly true for the linear term, which gives directly the bulk zero-field conductivity σ_0:

$$\sigma(\Gamma) = \sigma_0 \,. \tag{1.1}$$

Contrary to this, the nonlinear deviations of the ACDF relation, in general, display a nontrivial, rather strong shape-dependence.

This property has been observed by Jaggi [1] who sampled the hot electron data by a law

$$\bar{j}_z = \sigma_0 E_z (1 - \varphi(\Gamma) \bar{j}_z^2) \tag{1.2}$$

which seems to fit the conduction measurements quite well at room temperature and up to rather high fields. Experiments have been done on plates and circular cylinders in n-type Si. Jaggi and Weibel [2] find that φ, which is related to the (extrapolated) saturation current, shows a minimum at radius a (or thickness d) $\sim 10^{-4}$ m for extrinsic semiconductors becoming singular for small a ($\lesssim 10^{-5}$ m) and nearly linear for large a ($\gtrsim 10^{-3}$ m). The data for intrinsic material is less extensive, but there is some indication of a similar trend [3].

In order to explain this effect for intrinsic semiconductors, Jaggi [4] points to the importance of radial currents of electrons and holes, which add up to zero and which recombine only at the surface with a velocity s. Another important role is played by the Lorentz-force giving rise to a pinch of electrons and holes. Though Jaggi's explanation makes use of the approximation of a constant electron-hole density (i.e., constant over the cross-section), it gives a qualitatively correct solution to $\varphi(a)$ for the linear range of large a, if a constant electron and hole mobility are assumed.

The purpose of this paper is to give an explanation for $\varphi(d)$ in intrinsic semiconductors for plates on the basis of Jaggi's model. Various recombination effects will be considered and the local mobility can be a function of the field. In order to simplify the analysis convection, diffusion and magneto-resistance terms are neglected. The latter can be estimated to give rise to a negligible correction in the nonlinear conduction (by a factor 10^{-6} smaller than those observed). The diffusion has been discussed by several authors [5–7] in connection with the pinch effect. A numerical discussion of the nonlinear conduction including diffusion will be given by Slonczewski [8]. Preliminary results indicate no qualitative change in the CV characteristic, except that no saturation current will exist any longer.

In Section 2 the physical model will be outlined, in Section 3 the solution given for a plate and Section 4 adds some results for more general cylinders. Finally, a detailed discussion follows in Section 5.

2. Physical Model

Local Ohm's law will be assumed to be of momentum-equation form with a field-dependent momentum relaxation time. Forgetting diffusion and kinetic (convection) terms, it can be written in the form

$$\boldsymbol{v} = -\mu_q(E)\,(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B})\,, \tag{2.1}$$

$$\boldsymbol{w} = \mu_p(E)\,(\boldsymbol{E} + \boldsymbol{w} \times \boldsymbol{B}) \tag{2.2}$$

for isotropic materials, where $\boldsymbol{v}$ and $\boldsymbol{w}$ are the drift velocities, and $\mu_q(E)$, $\mu_p(E)$ the field-dependent mobilities of electrons and holes, respectively. Assuming local neutrality (any space charge would decay with the dielectric relaxation time) the local charge density vanishes and the electron density n equals the hole-density p for intrinsic semiconductors ($n_0 = p_0$ equilibrium densities).

The recombination law for electrons and holes then becomes in the bulk,

$$\operatorname{div} n\,\boldsymbol{v} = \operatorname{div} n\,\boldsymbol{w} = -R(n) + R(n_0) \tag{2.3}$$

and at the surface,

$$n\,v_\perp = n\,w_\perp = S(n) - S(n_0) \tag{2.4}$$

where $R(n)$ is the syntonous volume recombination rate and $S(n)$ the corresponding surface rate. The total current density (q unit charge)

$$\boldsymbol{j} = q\,n\,(\boldsymbol{w} - \boldsymbol{v}) \tag{2.5}$$

obeys the continuity equation $\operatorname{div} \boldsymbol{j} = 0$.

In a very simple picture the recombination rates follow power laws

$$R(n) = r\, n_0 \left(\frac{n}{n_0}\right)^l, \quad S(n) = s\, n_0 \left(\frac{n}{n_0}\right) \tag{2.6}$$

where r is the volume recombination frequency and s the surface recombination velocity. The power $l = 2$ belongs to direct recombinations of electrons and holes, whereas $l = 1$ belongs to a recombination via an impurity level (electron traps). Highers l's can be attributed to complicated more electron-more hole recombination processes. These processes compete with each other and l can be quite different near equilibrium and for high electric fields. Therefore, in general, r, s and even l have to be considered field-dependent.

Together with Maxwell's equations, equations (2.1) to (2.6) give a unique solution to the problem of determining the ACDF relation. Since $l = 1$ and $l \geqslant 2$ behave quite differently, a classification according to l is necessary.

3. Formulation and Solution for a Plate

We consider an infinite plate with middle plane at $x = 0$ and surfaces at $x = \pm a$ (thickness $d = 2a$). By symmetry arguments no quantity depends on y and z, whereas n, v_z and w_z are even and v_x, w_x, B_y and E_x are odd functions in x.

The continuity equation $j_x' = 0$ with boundary condition $j_x = 0$ for $x = \pm a$ immediately gives $j_x \equiv 0$ or $v_x \equiv w_x$. By elimination of E_x from equations (2.1) and (2.2),

$$v_x = -\mu_q \mu_p E_z B_y, \tag{3.1}$$

$$-\frac{v_z}{\mu_q} = \frac{w_z}{\mu_p} = E_z \tag{3.2}$$

are obtained since magneto-resistance corrections $0(B_y^2)$ are neglected. The recombination law gives

$$(n v_x)' = -r n_0 \left[\left(\frac{n}{n_0}\right)^l - 1\right] \tag{3.3}$$

and at the surface $x = a$ holds the boundary condition

$$x = a\colon n v_x = s n_0 \left[\left(\frac{n}{n_0}\right)^l - 1\right] \tag{3.4}$$

whereas for $x = 0$ no partial current should enter (source-free middle plane),

$$x = 0\colon n v_x = 0. \tag{3.5}$$

Maxwell's equations for local charge neutrality are fulfilled if the induction law

$$B_y' = \mu_0 j_z = \mu_0 q (\mu_q + \mu_p) E_z n \tag{3.6}$$

holds (μ_0 absolute induction constant).

It should be noted that in the case of field-dependent quantities μ_q, μ_p, r and s nothing changes, though for isotropic materials $E = (E_x^2 + E_z^2)^{1/2}$ has to be inserted as argument. Since $E_x = 0(E_y)$, $E_x^2 = 0(B_y^2)$ which has already been neglected; i.e., field-dependence is consistently accounted for if field-dependent quantities are replaced by its value at the applied field E_z.

The average current density through the plate follows from equation (3.6) by integration,

$$\bar{j}_z = \frac{B_y(a)}{\mu_0 a} . \tag{3.7}$$

Introducing the units

$$E_0 = \left(\frac{r}{\mu_0 q n_0 (\mu_q + \mu_p) \mu_q \mu_p} \right)^{1/2}, \quad j_0 = \sigma_0 E_0 \tag{3.8}$$

and the dimensionless quantities

$$\xi = \frac{x}{a} , \quad y(\xi) = \frac{B_y(x)}{\mu_0 a j_0} , \quad \varepsilon = \frac{E_z}{E_0}$$

$$\iota = \frac{\bar{j}_z}{j_0} = y(1) , \quad \lambda = \frac{r a}{s} \tag{3.9}$$

the differential equation for $y(\xi)$ becomes ($\cdot = d/d\xi$)

$$(y \dot{y})^{\cdot} = \left(\frac{\dot{y}}{\varepsilon} \right)^{l} - 1 \tag{3.10}$$

and the boundary conditions are

$$\xi = 0 \qquad y \dot{y} = 0 , \tag{3.11}$$

$$\xi = 1 \qquad \lambda y \dot{y} = 1 - \left(\frac{\dot{y}}{\varepsilon} \right)^{l} . \tag{3.12}$$

Case $l = 1$:

By direct integration of equation (3.10) using equation (3.11) it follows that

$$y \dot{y} = \frac{y}{\varepsilon} - \xi ; \tag{3.13}$$

applied to $\xi = 1$ a second equation besides equation (3.12) results for $y(1)$ and $y(1)$, allowing the determination of ι,

$$\iota = \varepsilon \frac{\lambda + 1 - 1/\varepsilon^2 + \left((\lambda + 1)^2 + 2 (\lambda - 1)/\varepsilon^2 + 1/\varepsilon^4 \right)^{1/2}}{2 \lambda} \tag{3.14}$$

which for small field gives

$$\iota = \varepsilon (1 + \varepsilon^2 + \ldots) \tag{3.15}$$

and for high fields

$$\iota = \varepsilon \frac{\lambda + 1}{\lambda} \left(1 + \frac{\lambda - 1}{(\lambda + 1)^2} \cdot \frac{1}{\varepsilon^2} + \ldots \right) . \tag{3.16}$$

Case $l = 2$:

Substituting $\dot{y} = p(y)$, an exact solution of (3.10) is possible. It becomes

$$C \xi = \int_0^{Cy} \frac{dy'}{\bar{p}(y')} \tag{3.17}$$

where

$$\bar{p}(y) = \varepsilon \left(\frac{1 - y^{2\frac{1-\varepsilon^2}{\varepsilon^2}}}{1 - \varepsilon^2} \right)^{1/2} . \tag{3.18}$$

The integration constant C follows from

$$C = \psi_\varepsilon(z) , \qquad C = \lambda \phi_\varepsilon(z) \tag{3.19}$$

where

$$\psi_\varepsilon(z) = \int_0^z \frac{dy}{\bar{p}(y)} \tag{3.20}$$

$$\phi_\varepsilon(z) = \frac{z\bar{p}\,(z)}{1 - \bar{p}^2(z)/\varepsilon^2} \tag{3.21}$$

and the current

$$\iota = z/C . \tag{3.22}$$

In the limit $\varepsilon \to \infty$ $(\lambda > 0)$,

$$\bar{p}(z) = (1 - z^2)^{1/2}/z , \ \psi_\infty(z) = 1 - (1 - z^2)^{1/2}, \ \phi_\infty(z) = (1 - z^2)^{1/2} \tag{3.23}$$

exist and therefore the saturation current

$$\iota_\infty = \left(1 + \frac{2}{\lambda} \right)^{1/2} . \tag{3.24}$$

For warm electrons and $\lambda < \infty$ we have the expansion

$$\bar{p}(y) = \frac{\varepsilon}{(1 - \varepsilon^2)^{1/2}} \tag{3.25}$$

up to small nonanalytic terms in ε, therefore the current is

$$\iota = \varepsilon \left(1 + \frac{\varepsilon^2}{2} + \ldots \right) \tag{3.26}$$

For $\lambda = \infty$ this expansion becomes

$$\iota = \varepsilon \left(1 - (\ln 2 - 1/2)\, \varepsilon^2 + \ldots \right) \tag{3.27}$$

showing a discontinuity in the cubic-field term.

The high-field result equation (3.24) follows direct from the differential equation (see Section 4), but whereas equation (3.11) is fulfilled for finite fields, it breaks down in the limit $\varepsilon \to \infty$; this means that the electron current has a discontinuity at $x = 0$ or the middle plane acts like a sink of electrons. The saturation formula (3.24) shows that $\lambda = 0$ has a specific asymptotic behavior giving no saturation current, but

$$\iota = 2\,\varepsilon \left(1 - \frac{1}{4\,\varepsilon^2} + \ldots \right) \tag{3.28}$$

i.e., it becomes linear again with a high-field conductivity, which is twice the low-field conductivity. For $l = 2$ it has been shown that the saturation current is always approached from below for all $\lambda > 0$. For $l = 2$ also a monotonicity law for $\iota(\varepsilon, \lambda)$ holds stating

$$\iota(\varepsilon, \lambda_1) > \iota(\varepsilon, \lambda_2) \text{ if } \lambda_1 < \lambda_2 \tag{3.29}$$

or if for a given field E_z, a is increased, the average current density $\bar{j}_z$ decreases.

The electron density shows in all cases a pinch effect, i.e., $n(x)$ decreases and $B_y(x)$ increases with increasing x (>0). This means that electrons and holes, because of $n(a) < n_0$, are generated at the surface planes and move inward according to the Lorentz force. Though there is volume recombination for $|x| < x_1 < a$ $(n(x_1) = n_0)$, a strong accumulation remains at the center plane in the stationary state; it will be attained when the electron and hole density in the center region is high enough for it to be compensated by volume recombination.

Case $l = 3, 4 \ldots$:

Cases with higher recombination power l behave qualitatively, like case $l = 2$; the larger l, the smaller the cubic term in the ACDF relation, whereas the saturation current is independent of l ($\geqslant 2$) as will be proven in the next section.

4. The Current Saturation in a Nonlinear Recombination Model with Constant Mobilities

The problem discussed in Section 2 will be generalized here to arbitrary cross sections γ. In order to avoid non-existing saturation currents, we restrict ourselves to $l \geqslant 2$.

In analogy to the plate a consistent solution to the equation (2.1) to (2.6) and the Maxwell equations follows from

$$j_x \equiv 0 \qquad j_y \equiv 0 \tag{4.1}$$

since $0(\boldsymbol{B}^2)$ terms are disregarded. If $\operatorname{div} \boldsymbol{B} = 0$ is satisfied by introducing the potential $\psi(x, y)$ in the x, y plane through

$$\boldsymbol{B} = \left(-\frac{\partial \psi}{\partial y}, \frac{\partial \psi}{\partial x} \right), \tag{4.2}$$

the partial currents perpendicular to the z-axis follow from

$$\boldsymbol{v} = \mu_q \, \mu_p \, E_z \frac{\partial \psi}{\partial \boldsymbol{x}} . \tag{4.3}$$

The induction equation

$$\Delta \psi = \frac{\mu_0 \, \sigma_0}{n_0} E_z n \tag{4.4}$$

and the recombination equation

$$\nabla (n \, \boldsymbol{v}) = -r \left[\left(\frac{n}{n_0} \right)^l - 1 \right] \tag{4.5}$$

with the boundary condition (2.4), (2.6).

Finally, an equation for ψ in γ results, which has a well-defined l-independent limiting form for large fields,

$$\nabla \cdot (\Delta\psi \nabla\psi) = -\frac{\mu_0 \sigma_0 r}{\mu_q \mu_p}$$

$$\Delta\psi \frac{\partial\psi}{\partial\nu} = \frac{\mu_0 \sigma_0 s}{\mu_q \mu_p} . \tag{4.6}$$

On the outside of γ $\operatorname{rot} \boldsymbol{B} = 0$, $B_\perp$ continuous implies

$$\Delta\psi = 0 \qquad \psi \text{ continuous.} \tag{4.7}$$

The averaged saturation current density becomes from equation (4.4),

$$\bar{j}_z = \frac{1}{\mu_0 Q} \int_{(Q)} d^2x \, \Delta\psi \tag{4.8}$$

where Q is the area of the cross-section γ.

The result is independent of l, since according to equation (4.4) $n/n_0 = \mathrm{O}\,(1/E_z)$ and the power term in the recombination equations vanishes. Though it seems that $l = 1$ would also be included in this result, it can be shown that in this case only the assumption of unphysical sources will give a saturation of the current (e.g., for a plate, $n\, v_x$ is discontinuous for $x = 0$ meaning the center plane acts like an electron-hole sink).

The equation (4.6), (4.7) can be solved for a few simple cases. Of practical interest is the circular cylinder and the double plate:

a) Circular cylinder radius a

$$\bar{j}_z^{sat} = \left(\frac{\sigma_0 r}{\mu_0 \mu_q \mu_p}\right)^{1/2} \left(1 + \frac{4 s}{r a}\right)^{1/2} . \tag{4.9}$$

b) Parallel double plates, thickness $d = a - b$, distance of inner surfaces 2b (outer surfaces 2a)

$$\bar{j}_z^{sat} = \left(\frac{\sigma_0 r}{\mu_0 \mu_q \mu_p}\right)^{1/2} \left(1 + \frac{2 s}{r d}\right)^{1/2} ; \tag{4.10}$$

the result is independent of the inner distance 2b of the plates, in analogy to the magnetic field of an infinite current sheet which is independent of the distance from the sheet.

c) One plate of thickness 2a [see equation (3.24)] follows from b) for $b = 0$

$$\bar{j}_z^{sat} = \left(\frac{\sigma_0 r}{\mu_0 \mu_q \mu_p}\right)^{1/2} \left(1 + \frac{2 s}{r a}\right)^{1\,2} . \tag{4.11}$$

These three cases can be expressed by one formula,

$$\bar{j}_z^{sat} = \left(\frac{\varrho_0 r}{\mu_0 \mu_q \mu_p}\right)^{1/2} \left(1 + \frac{4 s}{r a_\gamma}\right)^{1/2} \tag{4.12}$$

where a_γ is some mean radius of the cross-section γ defined by

$$a_\gamma = \frac{2 Q}{U} \tag{4.13}$$

where U is the circumference of γ. It has not yet been investigated whether this result can be proven quite generally, eventually as an asymptotic approximation for large a_γ.

5. Discussion

For $l = 2, 3 \ldots$ the final formula (4.12), (4.13) agrees with the result of Jaggi [4], if one puts $r = 0$ (no volume recombination processes) up to a factor 2. This difference comes from Jaggi's approach $n = \text{const}$, which does not admit any pinching. For very thick cross-sections, the saturation current density becomes independent of the shape of the circumference. The typical length for the influence of the surface is the surface recombination length

$$l_s = \frac{s}{r} \; ; \tag{5.1}$$

it describes the range where surface recombination overrides the volume recombination effects. For typical semiconductors it is of the order of some 10^{-3} m, and it is outside the range of Jaggi's experiments. Therefore a linear dependence of $\varphi(a)$ is observed, since $a \ll l_s$ still. But $\varphi(a)$ should become constant for very large $a \gg l_s$, in agreement with physical intuition. Our result gives a linear a-dependence even for very small a's and does not explain the increase of $\varphi(a)$ for small a's observed in experiments, primarily done on extrinsic material.

The ACDF relation exhibits a super-ohmic behavior for small fields [see equation (3.26)] contrary to the observation. In order to fit the experimental data, field-dependent mobilities have to be assumed, which are sub-linear at room temperature and may well compensate the super-linear behavior in the ACDF relation. But it is then inconsistent to use the constancy of the mobilities to explain the saturation. This criticism is very severe, since applied fields go up to 10^6 V/m.

The field-dependence of mobilities can be deduced from time-of-flight measurements [9]. Assuming that for isotropic material $\boldsymbol{v}(\boldsymbol{E})$ obeys Jaggi's formula, the local (a fortiori geometry-independent) mobilities become

$$\mu_{p,q}(E) = \frac{2\,\mu^0_{p,q}}{1 + \left[1 + \left(\dfrac{2\,\mu^0_{p,q}\,E}{v^\infty_{p,q}}\right)^2\right] 1/2} \; , \tag{5.2}$$

where $\mu^0_{p,q}$ is the zero-field mobility, and $v^\infty_{p,q}$ the saturation drift of holes and electrons. For large fields $\mu(E)_{p,q}$ behaves like $1/E$. If the same behavior also holds for the recombination quantities, a saturation of the current density will result in the case $l = 1$ (recombination via traps) from equation (3.14).

The saturation current again depends on the geometry, and $\varphi(a)$ behaves for small a like a^2 and becomes constant for $a \gg l_s$. Near the point of inflection it shows a linear behavior. This field-dependent version again does not explain the increase of $\varphi(a)$ for small a.

The current density for small field will become sublinear if the cubic correction in equation (3.26) is smaller than the local cubic-field term derived from equation (5.2). A detailed calculation is only possible if the functions $\mu_q(E)$, $\mu_p(E)$, $r(E)$ and $s(E)$ are explicitly known in the range of fields where nonlinear conduction measurements can be done.

For an interpretation of the data in the form of equation (1.2) it is sufficient to have a model with some quasi-saturation, i.e., $r(E)$, $s(E)$, have only to decrease in some manner, but the $1/E$-law is not necessary in that sense.

The author wishes to thank Dr. R. Jaggi for many interesting discussions.

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