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Effect of Nonlinear Processes on the Plasma Heating in Magnetoacoustic Resonance [1)]

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(31. X. 70)

Abstract. In magnetoacoustic resonance, the effect of nonlinear processes on the temperature increase of a dense collision-dominated plasma is investigated. It is shown that the nonlinear processes diminish the theoretical value of the temperature increase even for very small amplitudes of the excited waves.

1. Introduction

The heating of a plasma by means of magnetoacoustic resonance has already been treated in many papers. In the most of them, the heating was investigated in a relatively thin plasma ($n \sim 10^{13}$–10^{14} cm^{-3}), where the transfer of electromagnetic energy is due to a collisionless damping of magnetoacoustic waves (e.g. [1, 2]). On the other hand, Hoegger et al. [3] investigated the resonant absorption of magnetoacoustic waves with small amplitudes in a dense and comparatively cold plasma ($n \sim 10^{15}$–10^{16} cm^{-3}, $T \sim 1$–2 eV). It was shown that the dominant dissipative process which occurs under these conditions is the Joule heating of the plasma, i.e. the collisional damping of the waves. However, the experimental and the theoretical values of the temperature increase, as given in the quoted paper, differ somewhat from each other. In particular, the theoretical values are considerably greater than the experimental ones. In our opinion, one of the reasons of this discrepancy might be the fact that the theoretical values are derived under the assumption that nonlinear electrodynamic effects are negligible for small amplitudes of the waves.

In this work, the effect of the above-mentioned nonlinearities on the energy dissipation of magnetoacoustic waves in a dense plasma is investigated theoretically. It turns out that the nonlinear processes play an important role in the plasma heating even for very small amplitudes of the absorbed waves.

2. Formulation of the Problem and the Basic Equations

We consider a homogeneous fully ionised plasma column immersed in a homogeneous axial magnetic field. The typical values of the plasma and the field parameters are assumed to be: $n_0 \sim 10^{15}$–10^{16} cm^{-3}, $T_0 \sim 1$–2 eV, $B_0 \sim 1$–5 kG. The plasma column is surrounded by conducting walls on which an azimuthal oscillating current is excited with the typical frequencies $\omega \sim 1$–2 MHz. This current induces, inside the

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plasma, radial magnetoacoustic waves. Since the collision frequency ω_{ei} is much greater than ω the waves are considerably damped, a part of their energy is absorbed and consequently the plasma is heated.

Under the stated assumptions the processes which occur in the plasma may be described by means of the following magnetohydrodynamic equations [4]

$$\frac{\partial \varrho}{\partial t} + \operatorname{div}(\varrho \boldsymbol{v}) = 0 \,, \tag{1}$$

$$\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v} \cdot \operatorname{grad})\, \boldsymbol{v} = \frac{1}{\varrho c} (\boldsymbol{j} \times \boldsymbol{B}) \,, \tag{2}$$

$$\operatorname{rot} \boldsymbol{B} = \frac{4\pi}{c} \boldsymbol{j} \,, \tag{3}$$

$$\operatorname{rot} \boldsymbol{E} = -\frac{1}{c} \frac{\partial \boldsymbol{B}}{\partial t} \,, \tag{4}$$

$$\boldsymbol{j} = \overleftrightarrow{\boldsymbol{\sigma}} \cdot \left(\boldsymbol{E} + \frac{1}{c} (\boldsymbol{v} \times \boldsymbol{B}) \right) \,, \tag{5}$$

$$\frac{\partial T}{\partial t} + (\boldsymbol{v} \cdot \operatorname{grad})\, T + \frac{2}{3} T \operatorname{div} \boldsymbol{v} = \frac{1}{3n} Q_{Joule} \,, \tag{6}$$

where all the quantities have their usual meaning, $Q_{Joule} \sim j^2/\sigma$ is the Joule heat.

We now adopt a cylindrical coordinate system r, φ, z and assume the length ol the plasma column to be infinite. Hence we can consider only the purely radia, oscillations of the plasma, writing $\partial/\partial\varphi \equiv \partial/\partial z \equiv 0$ in equations (1) to (6). Furthermoref we write

$$\varrho = \varrho_0 + \tilde{\varrho} \,, \quad \boldsymbol{B} = \boldsymbol{B}_0 + \tilde{\boldsymbol{B}} \,, \quad \boldsymbol{v} \equiv \tilde{\boldsymbol{v}} \,, \quad \boldsymbol{j} \equiv \tilde{\boldsymbol{j}} \,, \quad \boldsymbol{E} \equiv \tilde{\boldsymbol{E}} \,, \quad T = T_0 + \tilde{T} \,,$$
$$\overleftrightarrow{\boldsymbol{\sigma}} = \overleftrightarrow{\boldsymbol{\sigma}}_0 + \overleftrightarrow{\tilde{\boldsymbol{\sigma}}} \,, \qquad \boldsymbol{B}_0 \equiv (0, 0, B_0) \,,$$

where the quantities with the subscript 0 are static and those with the symbol $\sim$ represent the perturbations caused by the excitation.

On taking into account all these assumptions and the boundary condition

$$\tilde{\boldsymbol{B}}\,(r = R, t) = (0, 0, B_{ex} \cos\omega t) \,, \tag{7}$$

and eliminating the quantities $\tilde{\boldsymbol{j}}$ and $\tilde{\boldsymbol{E}}$ by virtue of equations (3) and (5) we can reduce the given set of equations to

$$\frac{\partial \tilde{\varrho}}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} \left(r \tilde{v} \; (\varrho_0 + \tilde{\varrho}) \right) = 0 \,, \tag{8}$$

$$\frac{\partial \tilde{v}}{\partial t} + \tilde{v} \frac{\partial \tilde{v}}{\partial r} + \frac{1}{\varrho_0 + \tilde{\varrho}} \cdot \frac{1}{4\pi} (B_0 + \tilde{B}) \frac{\partial \tilde{B}}{\partial r} = 0 \,, \tag{9}$$

$$\frac{\partial \tilde{B}}{\partial t} - \frac{c^2}{4\pi} \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \tilde{B}}{\partial r} (\xi_0 + \tilde{\xi}) \right) + \frac{1}{r} \frac{\partial}{\partial r} \left(r \tilde{v} \, (B_0 + \tilde{B}) \right) = 0 \,, \tag{10}$$

$$\frac{\partial \tilde{T}}{\partial t} + \tilde{v} \frac{\partial \tilde{T}}{\partial r} + \frac{2}{3} (T_0 + \tilde{T}) \frac{1}{r} \frac{\partial}{\partial r} (r \tilde{v}) - \frac{1}{3} \left(\frac{c}{4\pi} \right)^2 \frac{\xi_0 + \tilde{\xi}}{n_0 + \tilde{n}} \left(\frac{\partial \tilde{B}}{\partial r} \right)^2 = 0 \,, \tag{11}$$

where $\tilde{v} \equiv \tilde{v}_r$, $B \equiv \tilde{B}_z$, $\xi_0 + \tilde{\xi} \equiv (\sigma_{0\varphi\varphi} + \tilde{\sigma}_{\varphi\varphi})^{-1}$.

In what follows it is convenient to introduce the dimensionless variables

$$r' = \frac{r}{R}\,,\quad t' = \frac{c_A}{R}\,t$$

and the dimensionless functions

$$\varrho' = \frac{\tilde{\varrho}}{\varrho_0}\,,\quad v' = \frac{\tilde{v}}{c_A}\,,\quad B' = \frac{\tilde{B}}{B_0}\,,\quad T' = \frac{\tilde{T}}{T_0}\,,\quad \xi' = \frac{\tilde{\xi}}{\xi_0}$$

where $c_A = B_0/\sqrt{4\pi\varrho_0}$ is the Alfvén velocity. If we omit the prime (′) and introduce the differential operators

$$M \equiv \frac{\partial}{\partial r}\,,\quad L \equiv \frac{1}{r}\frac{\partial}{\partial r}r\,,\quad N \equiv \frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial}{\partial r}\,,$$

equations (8) to (11) and the boundary condition (7) may be rewritten as follows

$$\frac{\partial\varrho}{\partial t} + L\,v = -L(\varrho\,v)\,, \tag{12}$$

$$\frac{\partial v}{\partial t} + M\,B = -\,\varrho\,\frac{\partial v}{\partial t} - (1+\varrho)\,\frac{1}{2}\,M\,v^2 - \frac{1}{2}\,M\,B^2\,, \tag{13}$$

$$\frac{\partial B}{\partial t} - a\,N\,B + L\,v = L\,(a\,\xi\,M\,B - v\,B)\,, \tag{14}$$

$$\frac{\partial T}{\partial t} + \frac{2}{3}\,L\,v = -\,v\,M\,T - \frac{2}{3}\,T\,L\,v + \frac{2}{3}\,\frac{a}{\beta}\,\frac{1+\xi}{1+\varrho}\,(M\,B)^2\,, \tag{15}$$

$$B\,(r=1,t) = b\cos\omega_0\,t\,, \tag{16}$$

where

$$a = \frac{c^2\,\xi_0}{4\pi\,R\,c_A}\,,\quad \beta = \frac{8\pi\,n_0\,T_0}{B_0^2}\,,\quad b = \frac{B_{ex}}{B_0}\,,\quad \omega_0 = \frac{R\,\omega}{c_A}\,.$$

This set of equations must be completed by an explicit expression for the dimensionless resistivity ξ. The general expression for the resistivity, as given in [4], is too complicated in the region of the considered plasma parameters, where $\omega_{ce} \sim \omega_{ei}$ (ω_{ce} is the electron cyclotron frequency). For our purpose, however, it is sufficient to use the following approximate formula

$$\xi = (1+\delta\,T)\,(1+T)^{-3/2} - 1\,, \tag{17}$$

where

$$\delta = \frac{\xi_0(2\,T_0)\,2^{3/2}}{\xi_0(T_0) - 1} - 1\,,$$

and $\xi_0(T_0)$ is given by the formula (4.34) in [4].

3. Solution of the Equations and the Temperature Increase

In order to solve the set of equations derived in the foregoing section we use a perturbation method. In particular, we seek the solution in power series with the parameter $b \ll 1$, i.e. we write

$$B = B^{(1)} + B^{(2)} + \cdots ,$$

where $B^{(1)} \sim b$, $B^{(2)} \sim b^2$ and analogously for ϱ, v and T.

In the first (linear) approximation the solution is easily obtained by neglecting the right sides of equations (12) to (15). If we write $B^{(1)}$ and analogously the other quantities in the form

$$B^{(1)}(r, t) \equiv B_1^{(1)}(r)\, e^{-i\omega_0 t} + \overline{B}_1^{(1)}(r)\, e^{i\omega_0 t}$$

(the bar designates the complex conjugated quantity) and make use of the boundary condition (16) we find

$$B_1^{(1)} = J\,, \quad v_1^{(1)} = \frac{1}{i\,\omega_0} M\, J\,, \quad \varrho_1^{(1)} = \left(\frac{K_1}{\omega_0}\right)^2 J\,, \quad T_1^{(1)} = \frac{2}{3}\left(\frac{K_1}{\omega_0}\right)^2 J\,, \tag{18}$$

where

$$J = \frac{b}{2\,D_1} J_0(K_1\, r)\,, \quad D_1 = J_0(K_1)\,, \quad K_1 = \omega_0\,(1 - i\,a\,\omega_0)^{-1/2}\,,$$

and J_0 is the Bessel function.

Having found all the quantities in the linear approximation we may proceed to higher approximations. The equations (12) to (15) for the second order quantities are

$$\frac{\partial \varrho^{(2)}}{\partial t} + L\, v^{(2)} = -\, L(\varrho^{(1)}\, v^{(1)})\,, \tag{19}$$

$$\frac{\partial v^{(2)}}{\partial t} + M\, B^{(2)} = -\, \varrho^{(1)} \frac{\partial v^{(1)}}{\partial t} - \frac{1}{2} M\, v^{(1)\,2} - \frac{1}{2} M\, B^{(1)\,2}\,, \tag{20}$$

$$\frac{\partial B^{(2)}}{\partial t} - a\, N\, B^{(2)} + L\, v^{(2)} = L\,(a\, \xi^{(1)}\, M B^{(1)} - v^{(1)}\, B^{(1)})\,, \tag{21}$$

$$\frac{\partial T^{(2)}}{\partial t} + \frac{2}{3} L\, v^{(2)} + v^{(1)}\, M\, T^{(1)} + \frac{2}{3} T^{(1)}\, L\, v^{(1)} = \frac{2}{3}\, \frac{a}{\beta}\, (M\, B^{(1)})^2\,, \tag{22}$$

with the boundary condition $B^{(2)}\,(r = 1, t) = 0$.

If we insert the expressions (18) in the right sides of these equations we can see that the resulting set of inhomogeneous equations is rather complicated. In order to simplify the whole problem we assume, in what follows, that

$$a \ll 1\,, \qquad \beta \ll 1\,. \tag{23}$$

We now let

$$B^{(2)}(r, t) = B_2^{(2)}(r)\, e^{-2i\omega_0 t} + \text{c.c.} + B_0^{(2)}(r);\ \text{analogously for } v^{(2)}\,,$$

and

$$\varrho^{(2)}(r, t) = \varrho_2^{(2)}(r)\, e^{-2i\omega_0 t} + \text{c.c.} + \varrho_{0s}^{(2)}(r)\, t;\ \text{analogously for } T^{(2)}\,.$$

Then the quantities of the second order are found, after some simple rearrangements, in the form

$$B_2^{(2)} = \varrho_2^{(2)} = P(2\,\omega_0) \left\{ M\, J \left(\frac{M^2\, J}{\omega_0^2} - 2\, J \right) \right\}, \tag{24}$$

$$B_0^{(2)} = \frac{1}{\omega_0} \{ |M\, J(1)|^2 - |M\, J|^2 \}\,, \tag{25}$$

$$v_2^{(2)} = \frac{1}{2\,i\omega_0}\left\{Q\,(2\,\omega_0)\left[M\;J\left(\frac{M^2\,J}{\omega_0^2} - 2\;J\right)\right] - 2\;J\;M\;J\right\}, \tag{26}$$

$$T_2^{(2)} = \frac{1}{3}\left\{2\;P(2\,\omega_0)\left[M\;J\left(\frac{M^2\,J}{\omega_0^2} - 2\,J\right)\right] - \frac{1}{3}\;J^2 + \frac{i\;a}{\omega_0\,\beta}\,(M\;J)^2\right\}, \tag{27}$$

$$T_{0s}^{(2)} = \frac{4}{3}\,\frac{a}{\beta}\,|M\;J|^2\,, \tag{28}$$

where $M\;J(1) \equiv M\;J\,|_{r=1}$ and $\varrho_{0s}^{(2)} \sim v_0^{(2)} \sim O(a)$. The quantities P and Q are the integral operators defined by the following relations

$$\begin{aligned} P(2\,\omega_0)\;\psi = \pi\,\omega_0\,D_2^{-1}\Big\{J_0(2\,\omega_0)\Big[N_0(2\;\omega_0\,r)\int_0^r J_1(2\;\omega_0\,\xi)\;\psi(\xi)\;\xi\;d\xi\; + \\ + J_0(2\;\omega_0\,r)\int_r^1 N_1(2\,\omega_0\,\xi)\;\psi(\xi)\;\xi\;d\xi\Big] - N_0(2\,\omega_0)\;J_0(2\omega_0 r)\int_0^1 J_1(2\;\omega_0\;\xi)\;\psi(\xi)\;\xi\;d\xi\Big\}\,, \end{aligned} \tag{29}$$

$$Q(2\,\omega_0)\;\psi = (M\;P(2\,\omega_0) - 1)\;\psi\,, \tag{30}$$

where $D_2 = J_0(K_2)$, $K_2 = 2\,\omega_0\,(1 - 2\,i\,a\,\omega_0)^{-1/2}$, J_1 is the Bessel function and N_0, N_1 are the Neumann functions.

Essentially, we are interested in the temperature increase. From the time dependence of the quantity $T^{(2)}$ it is immediately seen that the time average value of the temperature increase $\langle \partial T^{(2)}/\partial t\rangle$ per a time unit is given by the expression (28), i.e.

$$\left\langle \frac{\partial T^{(2)}}{\partial t} \right\rangle = T_{0s}^{(2)}\;. \tag{31}$$

This result has been found in [3].

It should be noted that owing to the assumptions (23) the quantity $T_{0s}^{(2)}$ is determined just by the right side of equation (22) which includes the magnetic field in the first order only. That means, the temperature increase in the second approximation is not influenced by nonlinear electrodynamic processes. In order to show how these processes are involved in the temperature increase we must find the solution of equations (12) to (15) in higher approximations that the second one. Before doing that, however, it is useful to determine, by means of a preliminary analysis, up to which order the equations must be solved.

On taking into account the time dependence of the quantities of the first and second orders one can easily see from equations (12) to (15) that any quantity of the third order has the time dependence of the following form

$$\psi^{(3)}(r,\,t) = \psi_3^{(3)}(r)\;e^{-3i\omega_0 t} + (\psi_1^{(3)}(r) + \psi_{0s}^{(3)}(r)\;t)\;e^{-i\omega_0 t} + \text{c.c.}$$

Thus, in the third approximation the considered equations would yield only an oscillating solution. At the first glance, it might seem that one should seek the solution up to the fourth order. Fortunately, it turns out that owing to the assumptions (23) the time average value of the temperature increase in the fourth approximation is determined only by the quantities of the first, second and third orders. In particular, according to equation (15) the following relation holds

$$\left\langle \frac{\partial T^{(4)}}{\partial t} \right\rangle = \left\langle -\left(v^{(1)} M\, T^{(3)} + \frac{2}{3}\, T^{(3)}\, L\, v^{(1)} + v^{(2)} M\, T^{(2)} + \right.\right.$$

$$\left. + \frac{2}{3}\, T^{(2)}\, L\, v^{(2)} \right) + \frac{2}{3}\, \frac{a}{\beta} \left\{ (\varrho^{(1)2} - \xi^{(1)} \varrho^{(1)} - \varrho^{(2)} + \xi^{(2)})\, (M\, B^{(1)})^2 + \right. \tag{32}$$

$$\left.\left. + 2\, (\xi^{(1)} - \varrho^{(1)})\, M\, B^{(1)}\, M\, B^{(2)} + (M\, B^{(2)})^2 + 2\, M\, B^{(1)}\, M\, B^{(3)} \right\} \right\rangle .$$

Hence we see that the only quantities we must still find are $B^{(3)}$ and $T^{(3)}$. The latter may obtained in the same way as the quantities of the second order. Since the explicit expressions for $B^{(3)}$ and $T^{(3)}$ are very cumbersome we do not write them. On inserting all the necessary quantities in the right side of the expression (32) we finally obtain, after rather tedious rearrangements, the following formula

$$\left\langle \frac{\partial T^{(4)}}{\partial t} \right\rangle = \frac{2}{3}\, \frac{a\, \omega_0^2}{\beta |D_1|^4} \left(\frac{b}{2} \right)^4 \left\{ C_1(r)\, J_1^2(\omega_0 r) + \omega_0^2 \left[2\, \omega_0^2 \frac{X^2(r)}{|D_2|^2} + \right.\right.$$

$$\left.\left. + J_1(\omega_0 r)\, (C_2(r) - 2\, \omega_0^2\, C_3(r)) \right] + \frac{8}{3}\, \frac{a\, \omega_0^2 \gamma}{\beta}\, J_1(\omega_0\, r)\, C_4(r)\, t \right\} , \tag{33}$$

where $\gamma = \delta - 3/2$,

$$C_1(r) = 2 \left(2\, \gamma + \frac{15}{(\omega_0\, r)^2} - 6 - \frac{10}{3}\, \frac{(a\, \omega_0)^2\, \gamma}{\beta} \right) J_1^2(\omega_0 r) - 8\, J_1^2(\omega_0) -$$

$$- 84\, \frac{J_0(\omega_0\, r)\, J_1(\omega_0\, r)}{\omega_0\, r} + 3\, (31 - 7\, \gamma) J_0^2(\omega_0\, r) ,$$

$$C_2(r) = \left\{ \left[\frac{2}{3}\, (\gamma - 1) - 3 \right] J_1(\omega_0\, r)\, W(r) + \left(12\, J_0(\omega_0\, r) - \frac{9}{2}\, \frac{J_1(\omega_0 r)}{\omega_0\, r} \right) X(r) \right\} \times$$

$$\times\, (D_2^{-1} + \overline{D}_2^{-1}) + 2\, (D_1^{-1} + \overline{D}_1^{-1}) \left[\frac{5}{3}\, \frac{(a\, \omega_0)^2\, \gamma}{\beta}\, Z(r) - F(r) \right] ,$$

$$C_3(r) = (D_1^{-1}\, D_2^{-1} + \overline{D}_1^{-1}\, \overline{D}_2^{-1})\, G(r) + \frac{8}{3}\, \frac{(a\, \omega_0)^2\, \gamma}{\beta}\, Y(r)\, (D_1^{-2} + \overline{D}_1^{-2}) ,$$

$$C_4(r) = J_1^3(\omega_0\, r) + i\, (\overline{D}_1^{-1} - D_1^{-1})\, a\, \omega_0^3\, Z(r) ,$$

$$W(r) = \frac{D_2}{\omega_0}\, P(2\, \omega_0) \left[J_1(\omega_0\, r) \left(3\, J_0(\omega_0\, r) - \frac{J_1(\omega_0\, r)}{\omega_0\, r} \right) \right] ,$$

$$X(r) = \frac{D_2}{\omega_0^2}\, Q\, (2\, \omega_0) \left[J_1(\omega_0\, r) \left(3\, J_0(\omega_0\, r) - \frac{J_1(\omega_0\, r)}{\omega_0\, r} \right) \right] ,$$

$$Z(r) = - \frac{D_1}{\omega_0^2}\, Q(\omega_0)\, J_1^3(\omega_0\, r) ,$$

$$Y(r) = \frac{\pi}{2} \left\{ J_0(\omega_0) \left[N_1(\omega_0\, r) \int_0^r J_0(\omega_0\, \xi)\, y(\xi)\, \xi\, d\xi + J_1(\omega_0\, r) \int_r^1 N_0(\omega_0\, \xi)\, y(\xi)\, \xi\, d\xi \right] - \right.$$

$$- N_0(\omega_0)\, J_1(\omega_0 r) \int_0^1 J_0(\omega_0 \xi)\, y(\xi)\, \xi\, d\xi \Bigg\} ,$$

$$y(r) = \frac{D_1}{\omega_0} P(\omega_0)\, J_1^3(\omega_0 r) ,$$

$$G(r) = -\frac{D_1}{\omega_0^1} Q(\omega_0) \left[J_1(\omega_0 r) \left(3\ W(r) + \frac{X(r)}{\omega_0 r} \right) \right] ,$$

$$F(r) = -\frac{D_1}{\omega_0^2} Q(\omega_0)$$

$$\times \left[J_1(\omega_0 r) \left(\frac{2\ J_0(\omega_0 r)\, J_1(\omega_0 r)}{\omega_0 r} + 3\, J_1^2(\omega_0 r) - J_0^2(\omega_0 r) - J_1^2(\omega_0) \right) \right] .$$

4. Discussion of the Results and Conclusions

It is obvious that in the considered approximation the complete temperature increase ΔT per the time interval Δt is given by the superposition of the expressions (31) and (33), i.e.

$$\Delta T = \int_0^{\Delta t} \left(\left\langle \frac{\partial T^{(2)}}{\partial t} \right\rangle + \left\langle \frac{\partial T^{(4)}}{\partial t} \right\rangle \right) dt \quad . \tag{34}$$

This quantity was computed for some typical values of the parameters n_0, B_0, T_0, b and $\Delta t = 2\pi/\omega_0$ as a function of the frequency ω_0 and the radius r.

The results are given in Figures 1 to 4, where the dashed lines represent the quantity ΔT computed in the second approximation. The maximal value of the temperature increase ΔT_{max} with respect to the radius r is given as a function of the frequency ω_0 in Figures 1 and 2, the resonant value of the temperature increase ΔT_{res} is given as a function of the radius r in Figures 3 and 4. From these curves it is immediately seen that the nonlinear processes diminish the value of the temperature increase even for very small values of the parameter b. For higher values of this parameter the quantity ΔT, as given by the expression (34), cannot be computed since the fourth order term becomes comparable with the second order one and it would be necessary to take into account higher order terms.

The results may be explained by the following argument. The Joule heat is proportional to the plasma resistivity which decreases as the plasma temperature increases. This effect, however, appears in the formula for the temperature increase only in higher approximations than the second one. Thus, it is clear that the formula (34) gives lower values of the temperature increase than that derived in the second approximation.

Furthermore, from Figures 3 and 4 it is seen that the nonlinear processes change also the radial distribution of the quantity ΔT_{res}. Namely, the maximal value is shifted more to the centre of the plasma column.

In conclusion we can summarize the results of our investigation in the following way. In a dense collision-dominated plasma the energy dissipation of resonant magnetoacoustic waves is considerably influenced by nonlinear processes even for

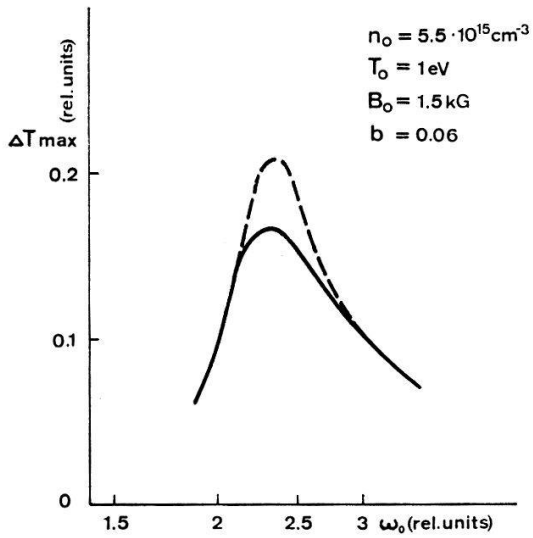

Figure 1
Variation of temperature increase with frequency in hydrogen plasma.

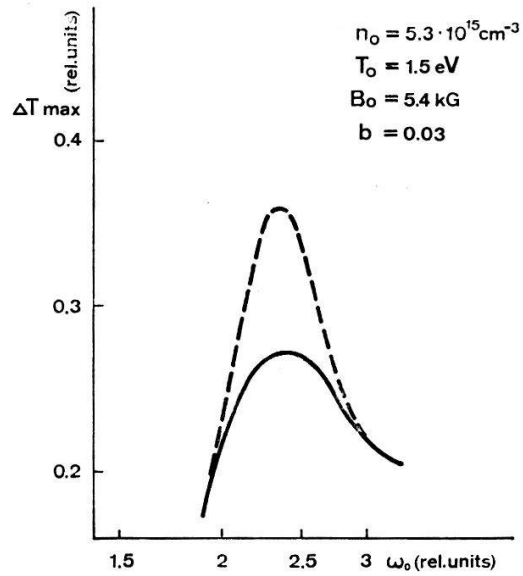

Figure 2
Variation of temperature increase with frequency in argon plasma.

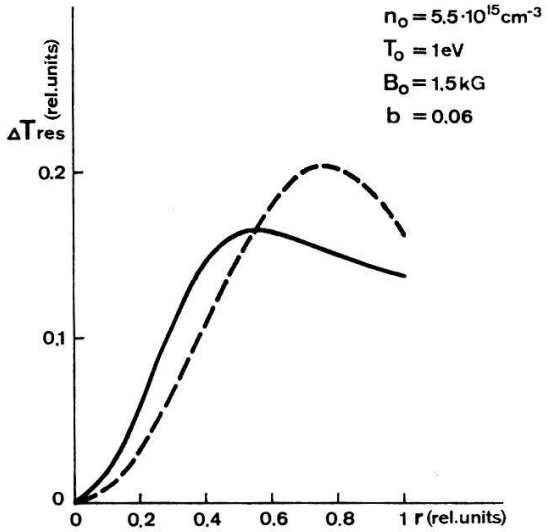

Figure 3
Variation of temperature increase with radius in hydrogen plasma.

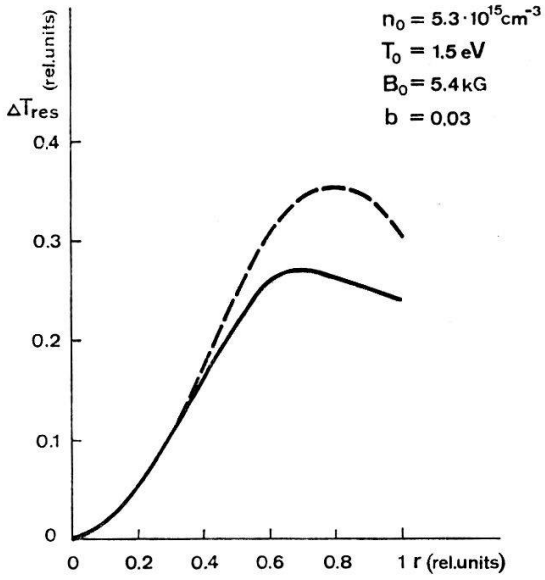

Figure 4
Variation of temperature increase with radius in argon plasma.

very small wave amplitudes. The problem of a plasma heating by means of magneto-acoustic waves with great amplitudes is not to be solved within the framework of the perturbation theory and consequently, it is necessary to solve equations (12) to (15) by a numerical computation method.

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