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Autor: Wagner, S. / Winiger, P.

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Calculation of the N-N Phase-Shifts Taking into Account Exited States of the Nucleons[1])

by **S. Wagner**
II. Institut für theoretische Physik der Universität Hamburg
and **P. Winiger**
Physikalisches Institut der Universität Fribourg

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Summary. Elastic N–N phase-shifts are calculated using the strong-coupling potential derived by FIERZ [1] who employed the fixed source PS–PS meson theory of WENTZEL [2]. In the limit of strong coupling, isobaric states of the nucleon appear, and consequently the N–N interaction has to be treated as a multi-channel problem. Using the experimental values of the renormalized pion coupling constant and of the N^*_{33} resonance energy, we introduce a hardcore of about 0.5 fermi as the only phenomenological parameter. The multi-channel Schrödinger equation is solved for positive energies. As a result of the calculations we obtain the correct energy dependence of the two S-wave phase-shifts at intermediate energies. All D-wave phase-shifts check with the experimental values up to 100 MeV; for higher energies the computed phase-shifts are too large. For high values of angular momentum they tend to OPEC phase-shifts; here all phase-shifts with $J \geqslant 3$ are in agreement with experiment. But for most of the intermediate odd phase-shifts there occurs the same kind of disagreement that is known from perturbation theory, when no vector mesons and no spinorbit potential are introduced into the theory.

1. Introduction

Meson field theories in the strong coupling limit were developed between 1940 and 1945 after W. HEISENBERG introduced the idea of spin inertia in 1939 [6]. Following WENTZEL's paper [2] of 1940, in which he developed the concept of strong coupling, SCHWINGER and OPPENHEIMER [9] and PAULI et al. [7, 8] developed different approaches to this concept with common results. Then WENTZEL and his school [2–5] again took up the problem and obtained solution for π–N, N–N and multinucleon interaction in the limit of strong coupling. The main result was the appearance of nucleon isobars for high values of the pion coupling constant. These isobars appear as intermediate states in the N–N interaction. FIERZ [1] obtained a strong coupling N–N potential in matrix form, which is better suited to straightforward numerical calculations than the equivalent operator of SERBER and DANCOFF [8]. The early numerical results failed to agree with the experiments, because too low values of the isobar excita-

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tion energy had been taken, and in addition, unacceptable approximations were introduced[2]).

VILLARS [4] found, that by using an intermediate pion coupling constant and an isobar excitation energy greater than 100 MeV, reasonable N–N low energy results could be obtained; but he used square well potentials for the radial dependence. COESTER [5] came to similar results for the higher nuclei by employing statistical methods to solve the many-body problem. HOURIET et al. [11–14] employed the strong coupling theory to calculate N–N interactions. The FIERZ' representation was used, but only the isobars of low excitation energy were taken into account. Experimental values of the N_{33}^{*} resonance energy and of the renormalized coupling constant were used. Good agreement with experiments was obtained in the calculation of the low energy scattering parameters and the deuteron parameters by HOURIET and HÉRITIER [11]. Following their calculation, the elastic N–N phase-shifts were computed. Some results of these latter calculations are discussed in this paper.

In part 2 FIERZ' strong coupling potential, including the isobaric intermediate states, is reported. In part 3, the way, in which the strong coupling Schrödinger equation can be used, to treat the case of intermediate coupling, is demonstrated. Part 4 contains the description of the adopted numerical methods. The last part contains the discussion of the calculated phase-shifts in comparison with the experimental phase-shift analysis values.

2. N–N Interaction in the Strong Coupling Meson Field Theory

The N–N interaction at sufficiently large distances can be simply described by a static pion field. The Hamiltonian of a pseudoscalar meson field coupled by symmetrical PS-coupling to two fixed nucleons can be written as follows:

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_I$$

$$\mathcal{H}_0 = \frac{1}{2} \sum_{\alpha=1}^{3} \int dV \, \{\Pi_\alpha^2 - \nabla^2 \varphi_\alpha + \mu^2 \varphi_\alpha\}$$

$$\mathcal{H}_I = -\sqrt{4\pi} \frac{f}{\mu} \sum_{\alpha=1}^{3} \sum_{i=1}^{3} \sum_{A=1}^{2} \sigma_i^A \tau_\alpha^A \int dV \, U(r_i) \, \nabla_i^A \varphi_\alpha \tag{1}$$

with $\mu = m_\pi / \hbar c$. φ_α and Π_α are the canonical field operators obeying the familiar boson commutation rules. σ_i and τ_α are the Pauli matrices of spin and isospin respec-

[2]) After the N_{33}^{*} isobars were found by experiment in 1952, the strong coupling theory was again further developed in different directions (mainly the charged pseudoscalar theory in application to $\pi - N$ interaction). PAIS and SERBER introduced transformations, in which the isobar operators are expressed in the form of integrals of the movement of the total system. LANDOWITZ and MARGOLIS, Wakano (Progr. Theor. Phys. *31*, 879 (1964)), GOEBEL and JAHN [10] extended the strong coupling method to obtain the unstable isobars in $\pi - N$ interaction. JAHN developed approximate procedures to deal with intermediate coupling, starting from the strong coupling limit; contrary to earlier calculations he used canonical commutation rules. He also developed a strong coupling version of the Chew-Low equation and treated the inelastic $\pi - N$ interaction numerically with this method. A detailed review of the literature of this period can be found in Jahn's papers. A new period of strong coupling began with the application of Lie group theory (see footnote [5]).

tively. f is the dimensionless pion coupling constant. $U(r_i)$ is a source function[3]), normalized to 1, describing the meson field near the nucleon centers.

OPPENHEIMER and SCHWINGER [9] have defined a mesonic radius of the nucleon:

$$a^{-1} = \int dV \int dV' \frac{1}{|\boldsymbol{r}-\boldsymbol{r}'|} U(\boldsymbol{r})\, U(\boldsymbol{r}') \,. \tag{2}$$

A potential can be derived from (1) for the two limiting cases of weak coupling and strong coupling. For weak coupling and large sources ($a\,\mu > 1$) no nucleon isobars appear and the familiar OPEC (one pion exchange) potential is obtained, which also follows from first order perturbation theory for point sources. This potential has a physical meaning only for distances $r > 2$ fermi, where the meson clouds are as weak, that only one-pion exchange is probable; in spite of this, the one meson potentials are also applied for small distances in the modern OBEC theories. The strong coupling potentials are derived for small sources ($a\,\mu \ll 1$) under the condition

$$f^2 \gg a\,\mu \,. \tag{3}$$

Here the number of exchanged mesons is not limited. There are two distinct kinds of solutions, one inside and the other outside of the overlapping region of the two source-functions. The far distance solutions were derived by OPPENHEIMER and SCHWINGER [9] for classical field theory and by SERBER and DANCOFF [8] who applied Pauli's strong coupling approach [7]. The same result was also obtained by the approach of WENTZEL [2]. Finally FIERZ [1] developed a matrix representation of this potential. It contains transitions between the ground state of the nucleons and an infinite set of isobaric states, their spin values (equal to the isospin in each state) being half integers[4]). The solutions in the overlapping region of the two source functions, first derived by SERBER and DANCOFF [8] and later by CHUN [15], are of the Wigner type and are spin-isospin independent. The shape of this potential depends only of the shape of the source-function. On the other hand, the above considered solutions contain no other properties of the source-function than its range a. The Schrödinger equation, given in the following, is derived in the region where the source-functions do not overlap. It contains, besides the strong coupling potential, an isobaric excitation term:

$$\sum_{n'} \left[\left(-\frac{d^2}{dx^2} + \frac{L'(L'+1)}{x^2} \right) \delta_{n\,n'} + \varepsilon_{n'}\, \delta_{n\,n'} + V_{n\,n'}(x) \right] F_{n'}^{JTP}(x) = k^2\, F_n^{JTP}(x) \tag{3}$$

[3]) The use of the 'old fashiond' extended source treatment (1) is justified by the following arguments: The pure pionic field with linear coupling is separated from all short range effects of a more complicated interaction type. These effects are: vector meson exchanges (ω, ϱ, φ), pseudoscalar mesons other than pions (η, K, ...) and $\mathrm{N}-\overline{\mathrm{N}}$ pairs. The structure of the function $U(r)$ may be more or less influenced by the exchange of these particles only within a sphere of about 0.7 fermi. The strong coupling solutions contain no explicit properties of the source function except its range. (In the early numerical strong coupling calculations of the radius a, the proton Compton wave-length was chosen, since at this time the form-factor of the nucleon was not yet known; therefore these results do not check with experiments.) If a range greater than the nucleon Compton wave-length is introduced, some effects of the vector mesons are implicitly accounted for. The spherical symmetric form-function has the same effect as an isospin zero vector meson, e.g. the ω.

[4]) The derivation of the strong coupling solutions and their matrix representation is given in the papers of WENTZEL [2] 1943 and of FIERZ [1].

where $k^2 = (E_d\, m_N/2\, m_\pi^2)$, E_d = scattering energy in lab.-system, $x = \mu\, r$ and $r = |\boldsymbol{r}_A - \boldsymbol{r}_B|$.

The total momentum J, the total isospin T and the parity P are conserved quantum numbers and therefore any combination (J, T, P) gives a separate system of Equations (3). FIERZ [1] has given a complete decomposition[4]) of the potential $V_{nn'}(x)$ in the irreducible basis

$$n = |\, L, S, j_1, j_2\rangle;\quad n' = |\, L', S', j_1', j_2'\rangle \tag{4}$$

where L and S are the quantum numbers of angular momentum and spin of the N–N system, and

$$j_1 = t_1\,,\quad j_2 = t_2\,, \tag{5}$$

are the quantum numbers of spin and isospin of the single nucleons; they are equal to 1/2 in the ground states and greater than 1/2 in the isobaric states. The term ε_n containing the excitation energy E_r (which is determined by the range a of the source function and by the coupling constant) becomes:

$$\varepsilon_n = \frac{E_r}{3}\left[j_1\,(j_1+1) + j_2\,(j_2+1) - \frac{3}{2}\right]. \tag{6}$$

The potential decomposes as usual into a spin dependent central potential and a tensor potential:

$$\begin{aligned} V_{nn'}(x) = {} & \lambda\,\frac{e^{-x}}{x}\,\langle j_1, j_2\,|\Omega'|\,j_1', j_2'\rangle\,\delta_{SS'}\,\delta_{LL'} \\ & + \lambda\,\frac{e^{-x}}{x}\left(1 + \frac{3}{x} + \frac{3}{x^2}\right)\langle L\,S\,j_1\,j_2\,|\Theta'|\,L'\,S'\,j_1'\,j_2'\rangle \end{aligned} \tag{7}$$

where $\lambda = f_r^2\,(m_N/m_\pi)$ and the Fierz matrices can be obtained from the formulas in [1]:

$$\begin{aligned} \Omega' &= 3\,\Omega(T, S, j_1, j_1', j_2, j_2') \\ \Theta' &= 9\left[T(J, T, S, S', L, L', j_1, j_1', j_2, j_2') - \frac{1}{3}\,\Omega\right]. \end{aligned} \tag{8}$$

In the limit of very strong coupling Ω' and Θ' are infinite quadratic matrices, whose elements are all of the order of magnitude 1. These matrices connect the isobaric states

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[5]) Recently the Lie group properties of Wentzel's strong coupling $\pi - N$ solutions were studied in some papers [16]. The original Hamiltonian (1) transforms according to the rotation group for the spin and for the isospin. In the limit of strong coupling the interaction part of the transformed Hamiltonian has the Lie group symmetry SU(2) × SU(2) × T_9. The matrix representation of FIERZ [1] adapted here, is equivalent to these results, as mentioned by JAHN [10]. But there is an essential difference in the two approaches. The term (6) containing the excitation energy is directly derived in strong coupling theories; it is not allowed in the case of the pseudoscalar symmetrical theory that this term vanishes (which would mean, that the range of the source function were zero). On the other hand, an equivalent mass breaking term can be introduced ad hoc in group theories in the usual way, which has be done by NE'EMAN [16]. The so called strong coupling group was also extended by GOEBEL and by DULLEMOND [16] to deal with the octet representation, containing the strange particles; the lowest excited multiplet, which is obtained in this case, is the decouplet Δ_{33}. Deshpanade derived the same group from Chew's reciprocal bootstrap dynamics.

with the scattering states $j_1 = j_2 = 1/2$. The radial dependence is the same as that of OPEC potentials, but here the renormalized coupling constant

$$f_r^2 = \frac{1}{9} f^2 \tag{9}$$

occurs, because the greater part of the mesonic field is coupled to the spin of the bound meson clouds.

There are some limitations for the set of irreducible bases according to the conditions:

$$\begin{aligned} &|j_1 - j_2| \leqslant S, \; T \leqslant j_1 + j_2 , \\ &|S - L| \leqslant J \leqslant S + L , \\ &\Delta L, \Delta S = 0 \text{ or } 2; \; \Delta j_1, \Delta j_2 = 0 \text{ or } 1 . \end{aligned} \tag{10}$$

Symmetrized eigenfunctions in j_1 and j_2 can be used. These functions then have to be totally antisymmetric according to Pauli's principle, which demands:

$$L + S + T = \text{odd} . \tag{11}$$

Thus the strong coupling Schrödinger equation turns out to be an infinite system of coupled differential equations of second order. The potential (aside from usual meaning of the matrix elements with $j_1 = j_2 = 1/2$ in the oscillatory differential equations) couples the differential equations of the different irreducible bases. These solutions of the field theory are of some interest, since the physical case can thus be restrained between the two limits of weak and strong coupling, where explicite solutions are possible. Since the system is infinite, the question of convergence arises. Obviously the contribution of higher isobars decreases, because the large centrifugal terms $L\ (L + 1)/x^2$ occurs in their differential equations. This is so, because the restrictions (10) cause the large values of $S = j_1 + j_2$ to be connected with the higher isobars, and consequently also the large values of L.

3. Modification of the Strong Coupling Equations for an Intermediate Coupling Calculation

The TOMONAGA-approximation enables an intermediate coupling calculation of the π–N system without limiting the number of exchanged mesons. M. FRIEDMANN, T. D. LEE and R. CHRISTIAN [17] have shown by this, that the physical case is very close to the strong coupling limit of quantum field theory. An equivalent TOMONAGA-calculation of the strong coupling equations, which shall be used in this paper, resembles very much such a more realistic case. In this case it makes no sense to take all the higher isobars into account. For finite coupling constant the splitting of the bound and unbound meson states for high angular momentum is no longer strict. Only the isobars with the lowest excitation energy should therefore be treated in the strong coupling manner, while the extremely unstable higher isobars can be added to the continuum. Then the system of Equations (3) is finite and containes only the following channels:

$$\begin{aligned} &(N + N) \text{ and } (N + N_{33}^*) \quad \text{for } T = 1 , \\ &(N + N) \text{ and } (N_{33}^* + N_{33}^*) \text{ for } T = 0 . \end{aligned} \tag{12}$$

The $(N^*_{33} + N^*_{33})$-channel needs twice the excitation energy than that of the $(N + N^*_{33})$-channel. The latter is therefore less important, but neither is neglectable. Below the inelastic threshold only the channel $(N + N)$ gives solutions of an oscillatory type with finite amplitudes at infinite distances. The isobaric eigenfunctions are nonzero only within the range of the potential. But they contribute to the phase-shifts of the asymptotic wave-functions.

No explicit use shall be made of the source function $U(\boldsymbol{r})$, but it is implicitly assumed, that it is of spherical symmetry. Two effects depending on $U(|\boldsymbol{r}|)$ are treated phenomenologically: the isobar excitation energy and the core. In both cases only the range is of importance and not its shape.

The excitation energy of the N^*_{33} and the renormalized coupling constant are fixed by their experimental values:

$$E_r = 300 \text{ MeV} ,$$
$$f_r^2 = 0.07 \text{ to } 0.08 \quad \text{or} \quad g^2 = 12 \text{ to } 13 . \tag{13}$$

The second consequence of the source function is the potential in the inner region, where both source functions overlap. Here the solutions of the previous chapter are no longer valid; the interaction depends essentially of the shape of $U(\boldsymbol{r})$. While the strong coupling potential (7) for a large separation of the sources depends of the renormalized coupling constant $f^2/9$, the latter is proportional to the unrenomalized coupling constant f^2. If a suitable shape of $U(\boldsymbol{r})$ is taken, there will be no singularity of this potential for $r \to 0$, as would be the case in weak coupling potentials. Here as in [11] this potential is idealized by a hard core, independent of the state:

$$V(|\boldsymbol{r}|) = \begin{cases} \infty & \text{for } r = r_c \\ V_{nn'}(x) & \text{for } r > r_c . \end{cases} \tag{14}$$

The core radius r_c is the only phenomenological parameter of this theory. In this approximation scheme the core radius r_c and the excitation energy E_r are treated as independent variables. But actually they both depend on the same source function and coupling constant.

With these modifications, the strong coupling Schrödinger equation can be solved exactly, both for the bound states of light nuclei and the positive energy stattering states. These solutions may be significant for as high energies as the dynamic of the nucleon can be treated non-relativistically and recoil effects are negligible.

4. The Numerical Methods[6])

For obtaining the phase-shifts, the solutions of the coupled system of Schrödinger equations are worked out. Schwinger's method of variation-iteration (which was successfully adapted by HOURIET and HÉRITIER [11] in the case of low energy scattering and bound state wave-functions) failed to converge in most of the cases con-

[6]) The computer calculations were performed on IBM 7090 at CERN (Geneva), on UNIVAC III of the Centre Electronique (Fribourg), on IBM 7040 of the Ecole Polytechnique (Lausanne), on IBM 7094 of Deutsches Rechenzentrum (Darmstadt); preliminary calculations were done on IBM 1620 of Gebr. Sulzer AG (Winterthur).

sidered here. The methods described here are sufficient for solving the scattering problem with the used potentials and within the treated range of energy. In the case of the phase shifts of high angular momentum, where the precision is not sufficient, and in the cases of strong spin-orbit potentials, these methods were extended later by ACHOUR [12], ADE [13] and WAGNER [14].

The system of Equations (3) contains two types of differential equations: one or two wave-functions of oscillatory type $v(x) = F^{JTB}_{LS\frac{11}{22}}(x)$, and several wave-functions of isobaric type, the latter with the boundary condition to be zero at infinity. For the cases of one oscillatory type differential equation in the system, the following iterative procedure has been developed: The wave-function $v(x)$ is obtained by parabolic approximation and continuation step by step. In a first iteration stage all isobaric wave-functions are neglected in Equation (3):

$$\left[\frac{d^2}{dx^2} - \frac{L\,(L+1)}{x^2} + k^2\right] v(x) = V_{11}(x)\, v(x)\,. \tag{15}$$

The Taylor series solutions are constructed by

$$\begin{aligned} v\,(x+\Delta x) &= v(x) + v'(x)\,\Delta x + v''(x)\,(\Delta x^2/2)\,, \\ v'\,(x+\Delta x) &= v'(x) + v''(x)\cdot\Delta x\,. \end{aligned} \tag{16}$$

Starting at the core x_c, $v(x_c)$ has to be zero. The value of the derivative $v'(x_c)$ may be chosen arbitrary, affecting only the amplitude of $v(x)$. The solution $v(x)$ is then calculated point by point according to Equation (16).

The differential equations of the isobaric components $F^{JTP}_n(x)$, which are coupled to the oscillatory function $v(x)$ by the system (3) are transformed into integral equations by their corresponding Green's functions,

$$F^{JTP}_n(x) = \int_{x_c}^{\infty} G^L(x, x') \left[\sum_{n'} F^{JTP}_{n'}(x)\, V_{nn'}(x)\right] dx \tag{17}$$

with the conditions $G^L(x_c, x) = 0$ and $G^L(\infty, x') = 0$. The Green's functions are constructed as follows:

$$\begin{aligned} x < x'\colon\ G^L(x, x') &= \frac{1}{2\,\eta}\,[p^L_1(x) - p^L_2(x)\; p^L_1(x_c)/p^L_2(x_c)]\; p^L_2(x') \\ x > x'\colon\ G^L(x, x') &= \frac{1}{2\,\eta}\,[p^L_1(x') - p^L_2(x')\; p^L_1(x_c)/p^L_2(x_c)]\; p^L_2(x)\,, \end{aligned} \tag{18}$$

p^L_1 and p^L_2 are the two fundamental solutions of the homogeneous equation

$$\left[\frac{d^2}{dx^2} - \frac{L\,(L+1)}{x^2} - \eta^2\right] p^L(x) = 0;\ \eta^2 = \varepsilon_n \frac{m_N}{2\,\hbar\, m^2_\pi} - k^2 \tag{19}$$

and may be calculated by the recursion formulas

$$\begin{aligned} p^L_1(\eta\, x) &= -\,p^{L-1}_1(\eta\, x)\; 2\left(L - \frac{1}{2}\right)/\eta\, x + p^{L-2}_1(\eta\, x)\,, \\ p^L_2(\eta\, x) &= \quad p^{L-1}_2(\eta\, x)\; 2\left(L - \frac{1}{2}\right)/\eta\, x + p^{L-2}_2(\eta\, x)\,. \end{aligned} \tag{20}$$

The fundamental solutions of lowest order are:

$$p_1^0(\eta\, x) = e^{\eta x}\ ;\ p_1^1(\eta\, x) = e^{\eta x}\left[1 - \frac{1}{\eta\, x}\right]$$

$$p_2^0(\eta\, x) = e^{-\eta x};\ p_2^1(\eta\, x) = e^{-\eta x}\left[1 + \frac{1}{\eta\, x}\right]. \tag{21}$$

The approximate solution for $v(x)$ of Equation (15) is used in (17), where in the first step the isobaric wave-functions are set equal to zero on the right-hand side. In the following iteration step the solutions of (17) are introduced into the completed differential Equation (15):

$$\left[\frac{d^2}{dx^2} - \frac{L\,(L+1)}{x^2} + k^2\right] v(x) = V_{11}(x)\, v(x) + \sum_{n'=2}^{N} F_{n'}^{JTP}(x)\, V_{nn'}(x)\,. \tag{22}$$

This solutions and the wave-functions of the isobaric type previously obtained, are then again introduced into (17). This procedure of iteration converges in several steps to the exact solution of Equation (3).

A modification of this method is needed in the triplet states with $J \neq L$, where the system (3) contains two differential equations of oscillatory type. Both asymptotic wave-functions $u(x)$ with $L = J - 1$ and $w(x)$ with $L = J + 1$ ought to have the same phase-shift, according to the definition of the eigenphase-shift solutions of BLATT and BIEDENHARN [18]. Besides the phase-shifts, the quotient of the asymptotic wave-function amplitudes has to be determined. There are two solutions α and β with negative reciprocal quotients of amplitudes:

$$\delta_\alpha = \delta_u = \delta_w;\ \delta_\beta = \delta_u = \delta_w$$

$$\operatorname{tg}\varepsilon = [w_\alpha(x)/u_\alpha(x)] = -\,[u_\beta(x)/w_\beta(x)] \text{ for } k\,x \gg L + 2\,. \tag{23}$$

Again, as in the above procedure, the wave-functions of the isobars are obtained from integral equations, and the wave-functions of oscillatory type are constructed by parabolic approximation. Several solutions with different initial derivative quotients have to be found in order to interpolate the parameters of the eigen-phase-shift solutions [18].

5. Discussion of the Calculated Phase-shifts

In Figure 1 the calculated $^1D_2$ wave-function solutions of 200 MeV are shown. The shape of oscillatory wave-functions with and without coupled isobars is not very different, even in the case of $^1D_2$, where the major contribution to the phase-shift is due to the isobaric channels. All isobaric functions are of the type illustrated and vanish within the first arc of the oscillatory function. Their amplitudes are about two orders of magnitude smaller than the asymptotic scattering amplitude. The parameters f_r^2, E_r and r_c have to be fixed. Those values are chosen (see Fig. 3), which are compatible with the low energy data according to HOURIET and HÉRITIER [11]. In Figure 2 the dependence of the two most important phase-shifts on these parameters is shown. This dependence is reciprocal for f_r^2 and r_c. Therefore, the coupling constant can be fixed at the experimental value, allowing only r_c to be determined.

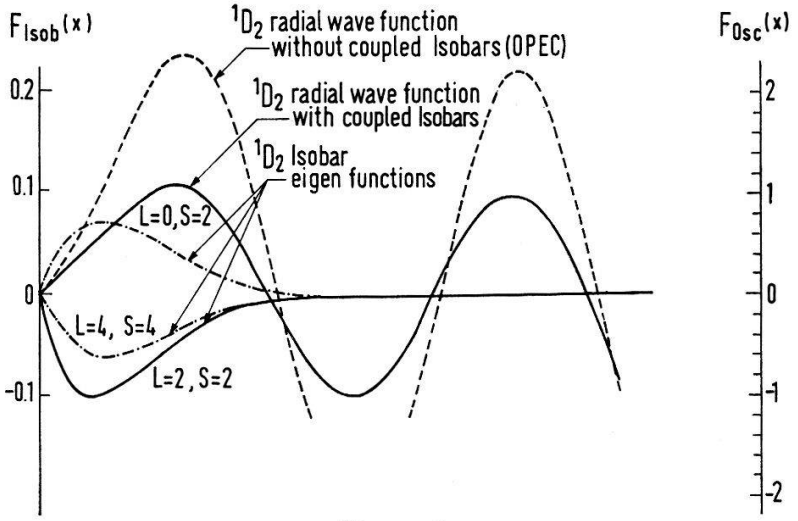

Figure 1

$^1D_2$ eigen functions for 200 MeV, Set A.

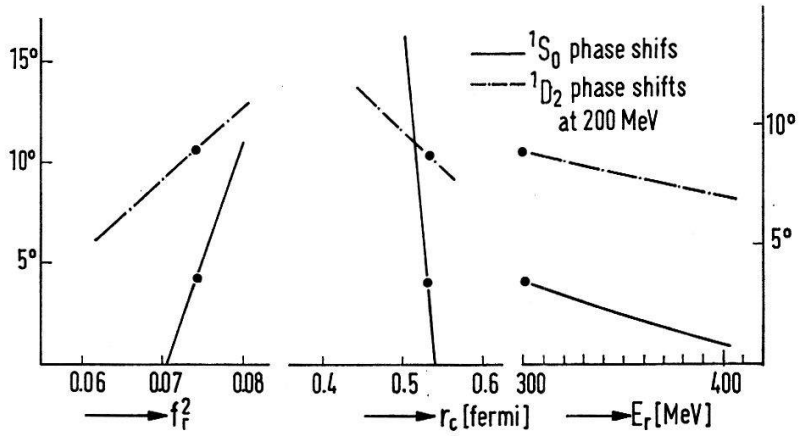

Figure 2

Dependence of the phase shifts $^1S_0$ and $^1D_2$ on the parameters f_r^2, r_c and E_r (for scattering energy 200 MeV).

The reciprocal dependence of f_r^2 and r_c for the phase-shifts $^1S_0$ and $^1D_2$ (which depend essentially on the core) is in accordance with the theoretical results of Wentzel, namely

$$E_r = \frac{a\,\mu}{f^2} \approx \text{const.}\,\frac{r_c}{g^2}\,.$$

The dependence on the resonance energy E_r of the N_{33}^* isobars is very weak. Therefore, the neglected resonance broadening is no serious weakness of the theory.

The calculated phase-shifts are compared with the values of the last published phase-shift analysis by ARNDT and MACGREGOR [19][7]), which contains all experimental data until 1966. The coupled phase-shifts of their analysis were transformed into the parametrization of the BLATT and BIEDENHARN convention [18], using Reference [20].

The energy dependence of the calculated phase-shifts are shown in Figures 3 and 4. The inclusion of the isobaric intermediate states in this calculation of the wave functions

[7]) The last published ε_1 values of ARNDT et al. [21, 22] for 23 MeV and 50 MeV are nearly zero.

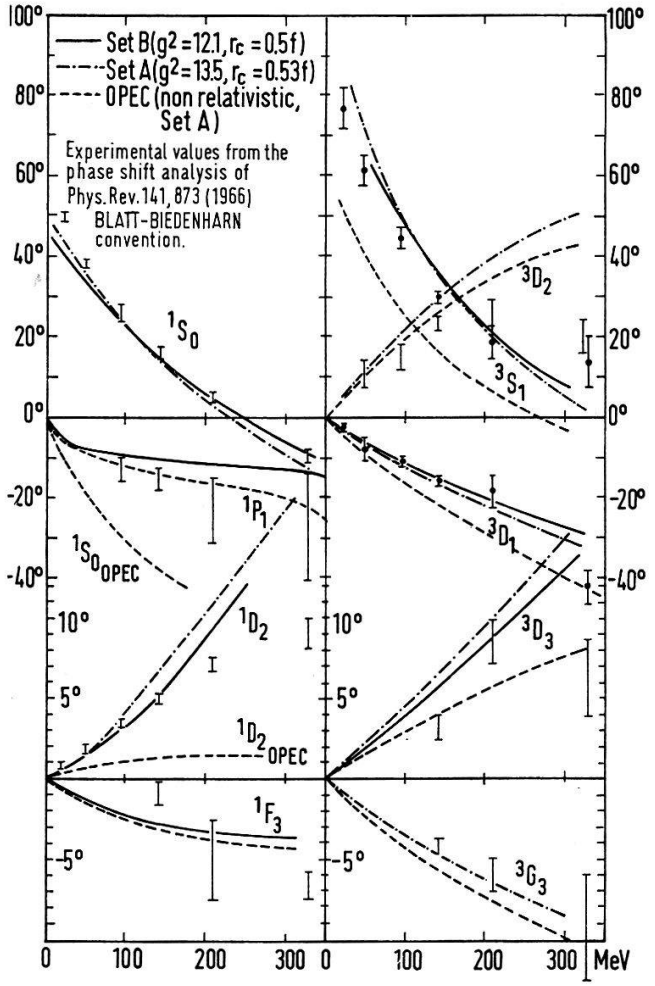

Figure 3 Figure 4

Singlet phase shifts. Triplet even phase shifts.

causes the phase-shifts to rise. This contribution to the phase-shifts increases mostly with energy, except for S-waves, where only a parallel displacement occours above 10 MeV. The isobaric contribution, which is quite different in the treated states, will now be discussed in detail:

1) The correct energy dependence is obtained for both S-waves, which are also in good agreement for very low energies, according to [11]. The deviation from OPEC phase-shifts is considerably great for the S-waves, expecially for $^1S_0$, where the lower channel $(N + N^*_{33})$ contributes. The $^1S_0$ phase-shift, which is negative at all energies in OPEC-approximation, is displaced to about 50° by the N^*_{33} intermediate state. The equivalent displacement of the $^3S_1$ phase-shift is less, because only the 600 MeV niveau $(N^*_{33} + N^*_{33})$ contributes. The splitting of both S-wave phase-shifts, which would be zero for unbroken SU(4) symmetry [23, 24], is of correct sign, but slightly small. Below 50 MeV the slope is smaller than the experimental one. The mixing parameter ε_1 has a quite different behaviour in the weak and strong coupling case. The latest published phase-shift analysis values seem to agree only with the latter case. The deviation of the $^3S_1 - {}^3D_1$ and ε_1 phase-shifts above 200 MeV cannot be explained by any kind of a static potential, including the strong coupling potential. Perhaps, the nucleon recoil may account for.

Up to now no other theory allowed to obtain the correct S-wave phase-shifts without the addition of several phenomenological parameters or hypothetical mesons. This may be explained by the fact, that in the usual field-theoretical or S-matrix calculations only the one- and two-meson states are contained. But the S-waves depend also on multi-meson exchanges.

2) The D-wave phase-shifts are in agreement with experiments below 100 MeV, but get too large for higher energies. Essential is the enlargement of the $^1D_2$ phase-shift as compared with the OPEC phase-shift. At higher energies nearly the whole phase-shift is effected by the $(N + N^*_{33})$-channels. Relativistic effects might lower the D-wave phase-shifts at higher energies, so that they are close to the experimental values [14].

3) In contrary to the considerably good agreement of the above discussed even phase-shifts with experiment, there occours the same kind of discrepancy of the most odd phase-shifts as in the well known case of OPEC-calculations. But here the discrepancy is even enlarged. The $^3P_1$ phase-shift equals OPEC-values and is in good agreement with experiments. But all $J \neq L$ odd phase-shifts ($^3P_0$, $^3P_2 - {}^3F_2$ and ε_2, and less $^3F_4 - {}^3H_4$ and ε_4) differ considerably from their experimental determinations[8]).

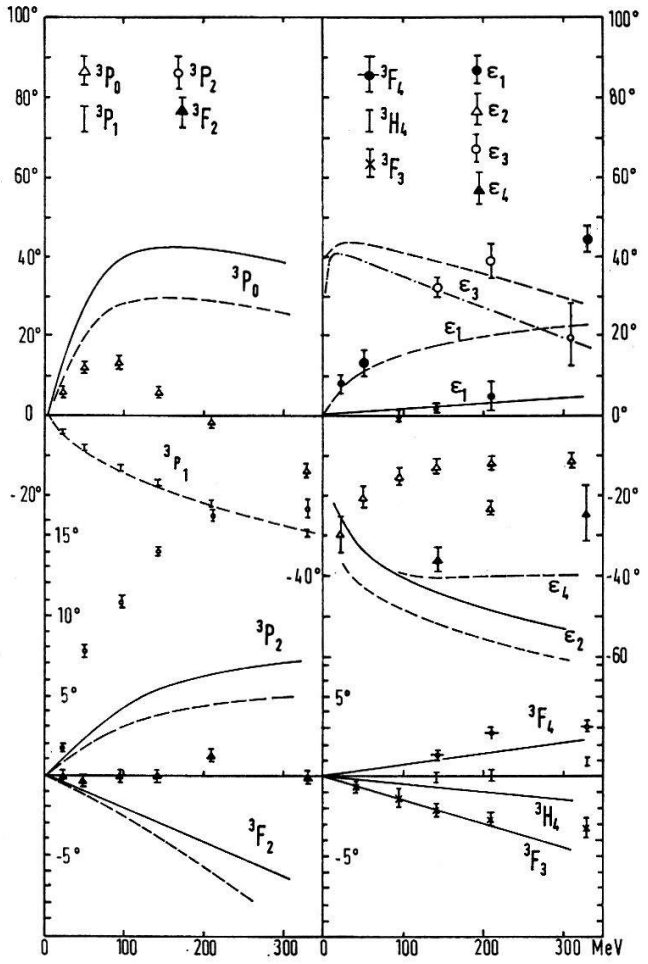

Figure 5 Figure 6

Triplet odd phase-shifts. Mixing parameters, F-phase shifts.

[8]) The results of the $^3F_4$–$^3H_4$ phase-shifts are published with the kind permission of F. Ade.

This may be explained by the lack of a spin-orbit potential in this theorie. Only the above phase-shifts are very sensible to such potentials. In addition, other short range effects, which are idealized by the state-independent hard core, might have a certain influence of these phase-shifts.

4) All phase-shifts $J \geqq 4$, which are not shown here, are in agreement with experiments; they tend all closely to OPEC-values. This has been shown by ACHOUR [12]. This OPEC-condition is consistent with this theoretical assumption in the 'modified phase-shift analysis'.

6. Conclusions

In the weak coupling field theories in Born's approximation (OPEC) only the higher phase-shifts with $J > 4$ are in agreement with experiments; this is also the case in the strong coupling theory. Yet in the latter case, there is also a remarkable agreement to the S-wave phase-shifts at intermediate energies. The D-wave phase-shifts at not too high energies are also in good agreement. But greater discrepancies occour in both cases of the odd $J \neq L$ phase-shifts, and they are even enlarged in strong coupling theory. These phase-shifts are very sensitive to the inner part of the potential and to a spin-orbit potential, which is not considered in this work.

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