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The Pauli-Kusaka Mixture in Modernized Form[1])

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Summary. Recent studies, in the static strong-coupling theory, of models with SU(3) symmetry, can be extended so as to include vector with pseudoscalar mesons and thereby make the model more realistic.

In the strong-coupling version of *Yukawa's* theory of meson-baryon interactions, for instance in the study of meson binding by baryons (isobar formation), the following approximations are typical: 1) The coupling parameter g is so large that expansions in powers of $1/g$ converge rapidly. 2) The baryons are very heavy; i.e., their recoil velocities are negligible; they act as static source functions extending over a range a (the equivalent meson momentum cut-off is $\sim 1/a$). In this framework, specific model theories are defined through the assumed transformation properties of the fields.

While, in the past, most special models were chosen for study to learn more about the mathematical technique, there has always been a natural desire to work with models 'close to reality', as best 'reality' was known at the time. Twenty-five years ago, before we had any direct experimental knowledge about the *Yukawa* meson, the electric quadrupole moment of the deuteron appeared to favor a mixture of pseudoscalar and vector mesons (*Møller-Rosenfeld, Schwinger*), so it was natural to apply the strong-coupling method to this model (PAULI and KUSAKA [1], WENTZEL [2]). Today, 'reality' is much better defined through the tremendous advances of high-energy physics, in particular the discovery of SU(3) symmetry patterns in the meson and baryon spectra. So, the favored 'realistic' model has recently been a (bare) baryon octet (spin $1/2^+$) strongly interacting with a pseudoscalar (O^-) octet; this difficult problem has been successfully studied by DULLEMOND and VON DER LINDEN [3] (see also BEDNÁŘ and TOLAR [4]). If the f/d ratio occurring in the interaction is chosen within a certain interval, the result is encouraging in that the groundstate of the bound system is an *8*, $1/2^+$, and the first excited state is a *10*, $3/2^+$. For these and higher (still hypothetical) states, a general mass formula has been derived by GOEBEL [5] (he used his own S-matrix method which we shall not discuss here).

We want to point out that it is easy to make the model even more realistic by admitting both pseudoscalar and vector mesons suitably coupled to the bare baryon octet. Indeed, in the now well known meson mass spectrum, the 9 vector mesons (ϱ, K^*, ω and φ) follow the 8 pseudoscalar ones (π, K, η) without a great gap in energy. Admittedly, the vector mass average is higher, but not so much higher as to

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make the neglect of baryon recoils so much more dubious. (Anyway, the maximum recoils are determined by the momentum cut-off rather than the meson rest masses.) So, for reasons other than in 1943, pseudoscalar-vector mixtures again become an object of considerable interest.

In their 1943 paper [1], PAULI and KUSAKA described in detail how to proceed in the case of pseudoscalar (φ_ϱ) and vector ($\boldsymbol{\psi}_\varrho$) fields, each of isospin 1 (the subscript, $\varrho = 1, 2, 3$, indicates the charge-space component), coupled to the (static) bare nucleon (spin and isospin matrices σ_i, τ_ϱ; source density δ_a) through the interaction Hamiltonian

$$\sum_\varrho \tau_\varrho \boldsymbol{\sigma} \cdot \int d^3x \, \delta_a \left(g_{PS} \boldsymbol{\nabla} \varphi_\varrho + g_{V2} \boldsymbol{\nabla} \times \boldsymbol{\psi}_\varrho\right) . \tag{1}$$

The essential step is then to introduce two orthogonal linear mixtures of φ_ϱ and $\boldsymbol{\psi}_\varrho$ field components, as well as their canonically conjugate quantities, such that only one of the two mixtures enters into the bound field problem. The resulting energy spectrum of the bound states depends on only one effective coupling constant

$$g = \sqrt{g_{PS}^2 + 2\, g_{V2}^2} \, , \tag{2}$$

and this g must be large enough for the strong-coupling approximation to be applicable. (If the source radius a is small compared with the Compton wavelengths of both mesons, the condition is $g \gg a$.) We note that this treatment disregards the other possible vectormeson-nucleon (static) interaction

$$g_{V1} \sum_\varrho \tau_\varrho \int d^3x \, \delta_a \, \boldsymbol{\nabla} \cdot \boldsymbol{\pi}_\varrho \tag{3}$$

($\boldsymbol{\pi}_\varrho$ = canonical conjugate to $\boldsymbol{\psi}_\varrho$); one knows, however [6], that this coupling does not contribute to meson binding as long as

$$\left(\frac{\mu \, g_{V1}}{g}\right)^2 < \frac{2}{3}$$

where μ = vectormeson restmass. (We set $\hbar = c = 1$.)

If we now want to go over to a higher internal symmetry [e.g., isospin group $SU(2) \to SU(3)$], the main point is to realize that the construction of the appropriate mixtures is quite independent of how the matrices τ_ϱ in (1) are defined, provided they are the *same* in the pseudoscalar and vector coupling terms. In particular, in the place of the nucleon, we may now have an SU(3)-octet baryon (spin 1/2), and instead of the isospin 1 mesons, we will have SU(3)-octets of 0^- and 1^- mesons [SU(3)-singlets may be added later]. The interaction (1), with $\varrho = 1 \ldots 8$, can formally be taken over, with the τ_ϱ re-interpreted as the appropriate eight 8×8 matrices

$$\tau_\varrho = \alpha \, F_\varrho + (1 - \alpha) \, D_\varrho$$

involving the arbitrary parameter α ($\alpha/1 - \alpha$ = 'f/d ratio'). We then expect the PAULI-KUSAKA analysis again to be applicable provided α has the same value in the pseudoscalar and vector coupling terms in (1). This we assume.

Two reasons may be cited to support this assumption. 1) The higher symmetry SU(6) (static) may be invoked as it has proved successful in several respects. Then, the pseudoscalar and vector mesons are members of the same SU(6) multiplet (a *35*), and α is constant within this multiplet. 2) GOEBEL [5], whose S-matrix method involves renormalized quantities, including α, finds a unique value for this α_{ren}, in conjunction with his simple (and correct) mass spectrum. This uniqueness must again obtain in the mixture case and, as to the unrenormalized α's, no generality can be lost by choosing them equal for the O^- and 1^- mesons.

If this is accepted, the introduction of the PAULI-KUSAKA mixtures (see Ref. [1], Equations (23)–(26), with the octet index now replacing the isospin index) does indeed reduce the bound-field Hamiltonian to one equivalent to the pure pseudoscalar case: the coupling constants g_{PS} and g_{V2} enter in the combination (2) only. For a proof, one may simply follow PAULI's and KUSAKA's argument, just discarding or replacing equations of insufficient generality. See the Appendix for indications how to do this [7].

After having thus established the mathematical equivalence of the Hamiltonians of the mixture and pure pseudoscalar bound fields, one may go on and isolate the 'rotational part' of the bound-field Hamiltonian which determines the lowest bound states of the system; i.e., the isobar spectrum. With the help of symmetry arguments it is easy to see that this 'rotational' Hamiltonian is, except for a constant factor, the same as the one studied by DULLEMOND and VON DER LINDEN [3] (for the pseudoscalar model) with the simplifying assumption of a large source radius ($a\ \mu \gg 1$). Hence, we know the isobar spectrum, also for the generalized model.

So far, we have ignored an interaction of type (3) ($\varrho = 1\ldots 8$). It is believed that, in reality, such coupling exists, and that it is 'pure f' ($\tau_\varrho = F_\varrho$, $\alpha = 1$) [8]. As in the isospin 1 case, the interactions (1), (3) involve different (orthogonal) field components (transverse and longitudinal polarizations, respectively), and an analysis similar to Ref. [6] (§ 11) is indicated. Unfortunately, instead of a 4×4 matrix (see Equation (11.3) in Ref. 6), we would then have to examine a 16×16 matrix and find its lowest eigen-value, as it depends on the coupling ratio ($\mu\, g_{V1}/g$). No attempt to solve this problem was made. So much is clear, however, that as long as this ratio remains below a certain critical value (which depends on α), the interaction (3) cannot contribute to vector-meson binding, and the isobar spectrum remains unaffected. On the other hand, if the coupling ratio exceeds the critical value, the mass spectrum must (as in Ref. [6], §§ 12, 13) change radically [9], and the experimental knowledge would presumably rule out this possibility.

Finally, to enlarge octets into nonets, one may want to add SU(3)-singlets ($\varrho = 1$ only, $\tau_1 = 1$; $\omega - \varphi$ mixing is irrelevant). The complication so introduced is similar in type to the one discussed in the preceding paragraph. The closest analogon in old-style work is the study by RAMACHANDRAN [10] of the coupling of both π and η mesons (O^-, isospin 1 and O, respectively) with a nucleon. Again, one encounters a critical value for the pertinent coupling ratio below which the singlet does not contribute to the bound field, whereas at higher values the mass spectrum is drastically altered.

In conclusion, we would like to emphasize again that, when we chose for re-study pseudoscalar-vector mixtures, this special choice was motivated by what we empirically know about the meson spectrum. Other coherent mixtures, as allowed by the transformation character of the fields, may be appropriate topics for further exploration. Under the aspect of 'realism', axial vector [11] admixtures (*8*, 1^+) may merit attention, perhaps with an appeal to broader universality assumptions to provide the needed f/d value (remark by P. G. O. Freund).

Appendix

Here we adopt the notation of Ref. [1]. No generalization is needed in the definitions and equations leading up to the Hamiltonian (18) except the re-interpretation of the superscript α (= our ϱ) as an octet index, with the understanding that the matrices τ^α (= our τ_ϱ) are newly defined. At this point one conveniently introduces the mixtures by the substitutions (24) and (26) of Ref. [1]. The 'potential energy' (19) becomes:

$$E^0 = \sum_{\alpha, i} \left[\frac{1}{2\,I} \{ (\Phi_i^{0\alpha})^2 + (\Psi_i^{0\alpha})^2 \} + (f^2 + 2\,g^2)^{1/2}\, \Phi_i^{0\alpha}\, \sigma_i\, \tau^\alpha \right],$$

and realizing that the $\Psi_i^{0\alpha}$ field is uncoupled and may be dropped from the bound-field Hamiltonian, the equivalence with the pseudoscalar E^0 part is obvious. The then following steps in the strong-coupling approximation (diagonalizing and minimizing E^0) are well described in the literature [12]. As to the 'kinetic energy part', equations like (32) are not general enough but their use can be avoided by going back to (18) and substituting (26) (with $\Omega_i^{0\alpha} \to 0$); the resulting Hamiltonian is given by (35) provided one writes

$$\Pi_i^{0\alpha} \text{ instead of } (2\,D)^{-1} \sum_\beta L^{00\alpha\beta}\, e_i^\beta\,.$$

Making the same substitution in the transformation formulas (36)[2]), the equivalence with the pseudoscalar model is easily verified: $(f^2 + 2\,g^2)^{1/2}$ assumes the role of f.

References

[1] W. Pauli, S. Kusaka, Phys. Rev. *63*, 400 (1943).
[2] G. Wentzel, Helv. phys. Acta *16*, 222 (1943). This is a short report on results only, without derivation. See this paper for quotations of earlier work.
[3] C. Dullemond, F. J. M. von der Linden, Ann. Phys. *41*, 372 (1967); F. J. M. von der Linden, doctoral dissertation, Nijmegen (1968).
[4] M. Bednář, J. Tolar, Nucl. Phys. *B5*, 255 (1968).
[5] C. J. Goebel, Phys. Rev. Lett. *16*, 1130 (1966).
[6] G. Wentzel, Helv. phys. Acta *16*, 551 (1943).
[7] One should be aware of the approximations tacitly introduced. For a fuller discussion, see W. Pauli, S. M. Dancoff, Phys. Rev. *62*, 85 (1942), p. 96–97 ($g_V = 0$), or Ref. [6], p. 568 ($g_{PS} = 0$).
[8] *Conserved vector current* hypothesis; see e.g., F. Gürsey, A. Pais, L. A. Radicati, Phys. Rev. Lett. *13*, 299 (1964).
[9] For instance, an *8*, $3/2^+$ or a *10*, $1/2^+$ are likely to appear among the lowest states.
[10] R. Ramachandran, Phys. Rev. *139*, B110 (1965).
[11] A model with 1^+ mesons of isospin 1 has been studied by K. Rüdenberg, Helv. phys. Acta *24*, 89 (1951).
[12] Most instructive is the paper by Bednář and Tolar, Ref. [4].

[2]) Note that this redefinition of the 'free fields' implies a change in their zeropoint energies which must be counted as part of the energy of the bound system.