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$\pi^- p \to \varrho^- p$ in the Absorption Model[1])

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Abstract. The absorption model is applied to study the reaction $\pi^- p \to \varrho^- p$ at 4 Gev/c, the reaction being assumed to take place only through pion-exchange. The absorption factor has been taken to be a function of orbital angular momentum only and an exact partial wave summation has been made instead of using an approximate integral expression. The result shows that at small angles the pion-exchange is the dominating factor, but at large angles the contribution from a possible vector-exchange may be important. It is also found that the unitarity limit for partial waves is not violated in contrast to the simple one-meson exchange model where this limit is definitely violated for at least a few low partial waves.

1. Introduction

Absorptive corrections [1–5] have been introduced in the peripheral [6, 7] or the Born term model for strong interactions of elementary particles in the Gev range to explain the experimental features of quasi-two-body inelastic processes of the type

$$a + b \to c + d \qquad \text{(A)}$$

where one or both of the final particles are resonances with extremely short life-time.

The model generally succeeds in fitting [8] the shape of the momentum transfer distribution of production, the decay-correlations and sometimes the absolute value (when coupling constants are known) and energy dependence of cross-sections. But the model fails on the energy dependence of cross-sections for reactions dominated by vector meson exchanges, the momentum transfer dependence of such reactions at the highest energies, the absolute cross-sections for reactions involving double resonance production.

The present work is undertaken to test the sensitivity of the model to certain approximations – such as large j and small angle approximations [4, 9] which allow the use of simplified expressions for the rotation functions and then replace the partial wave summation by an integral expression for the reaction $\pi^- p \to \varrho^- p$ at 4 Gev/c. With this end in view we have restudied the reaction calculating the rotation functions with the help of Reference [10] and then making an exact partial wave summation since only a few partial waves contribute at this energy. In most of references quoted above the absorption factor is taken to be a function of j or of $x - 1/2$, x being related

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to j through $x^2 = (j + 1/2)^2 - 1/4\,(\alpha - 1)^2 - 1/4\,\beta^2$ and $\alpha = \lambda - \mu$, $\beta = \lambda + \mu$, λ and μ being the helicities in the initial and the final states. Here we assume the absorption factor to be a function of the orbital angular momentum only.

The predictions [4] on the spin space density matrix elements of the produced ϱ-mesons are in gross disagreement with the experimental values. Apart from the above mentioned approximations the authors do not consider the possible contribution from vector meson exchange and the instability of the ϱ-meson. So we also calculate the spin space density matrix elements of the ϱ-meson by taking into account all the above mentioned factors. This work will be reported in a later paper. Here we report only the results on differential cross-sections. The arrangement of the paper is as follows: the absorption model is described briefly in Section 2, the $\pi^- p \to \varrho^- p$ reaction is described under the absorption model in Section 3, Section 4 is devoted to a discussion and criticism of the model.

2. Absorption Model

A full account of the theory of the absorption model can be found in References [2, 4, 5]. Here we give a brief description of the model. Let us consider the reaction (A) taking place by the exchange of a pseudo-scalar particle e, this is illustrated in Figure 1, where λ_a, λ_b, λ_c, λ_d, represent the helicities of the particles and the blobs represent the absorptive effects in the initial and the final states. These absorptive effects can be more or less represented by the elastic scatterings in the initial and the final states and since the particles are assumed to be on the mass shell between the blobs the parameters can be taken from the elastic scattering data. As the scattering at forward angles is dominated by non-helicity-flip amplitude it is assumed that the helicities of the particles do not change in the elastic scattering.

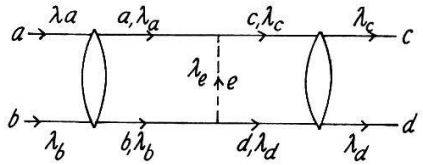

Fig. 1

Diagram indicating absorptive effects in the Peripheral model.

A Born helicity amplitude for the above process can be written as [5]

$$T_{\lambda\mu}(s, x) = (1 - x/2)^{|\lambda-\mu|/2}\,(1 + x/2)^{|\lambda+\mu|/2}\,A_{\lambda\mu}(s, x)/z - x \qquad (1)$$

where $z - x$ comes from the propagator of the exchanged particle and $A_{\lambda\mu}(s, x)$ is a polynomial in x, and the other quantities are θ = c.m. scattering angle $x = \cos\theta$, $\cos\theta/2 = \sqrt{1 + x/2}$, $\sin\theta/2 = \sqrt{1 - x/2}$,

z = $1/2\,q\,q'\,(m^2 - t + 2\,q\,q'\cos\theta)$

t = negative of the square of four-momentum transfer

m = mass of the exchanged particle

q, q' = c.m. three-momentum of the initial and the final particles respectively.

The expression (1) can further be changed into

$$T_{\lambda\mu}(s, x) = (1 - x/2)^{|\lambda-\mu|/2} (1 + x/2)^{|\lambda+\mu|/2} [A_{\lambda\mu}(s, z)/z - x + B_{\lambda\mu}(s, x)] . \quad (2)$$

Here $A_{\lambda\mu}(s, z)$ is obtained from $A_{\lambda\mu}(s, x)$ by replacing x by z and $B_{\lambda\mu}(s, x)$ is the residual polynomial in x obtained from the quotient of $A_{\lambda\mu}(s, x)$ by $z - x$.

The Born term amplitudes $T_{\lambda\mu}(s, t)$ are normalized in such a way that the differential cross-section for the above process is given by

$$\frac{d\sigma}{d\Omega} = 1/2\, q\, q' \sum_{\lambda\mu} | T_{\lambda\mu} |^2 . \quad (3)$$

In order to take into account the absorption effects one has to expand (2) in the angular momentum states with the help of JACOB-WICK representation of helicity amplitudes [10].

Defining the c^j functions, which are related to the rotation functions of the second kind, as

$$(1 + x/2)^{|\lambda+\mu|/2} (1 - x/2)^{|\lambda-\mu|/2} 1/z - x = \sum_{j_{min}}^{\infty} (2\, j + 1)\, c^j_{\lambda\mu}(z)\, d^j_{\mu\lambda}(x) \quad (4)$$

where $j_{min} = \max (|\lambda|, |\mu|)$ the general expression for the amplitude corrected for absorption is obtained as

$$T^{abs}_{\lambda\mu}(s, x) = A_{\lambda\mu}(s, z) \sum_{j_{min}}^{\infty} (j + 1/2)\, c^j_{\lambda\mu}(z)\, d^j_{\lambda\mu}(x)\, k(j) + \sum_{j_{min}}^{\infty} B^j_{\lambda\mu}(s)\, d^j_{\lambda\mu}(x)\, k(j) \quad (5)$$

where $k(j)$ defines the absorption in the initial and the final states, the rotations functions are defined in Reference [5].

3. $\pi^- p \to \varrho^- p$ at 4 Gev/c

To compare the absorption model predictions and the experimental results [11] for the above reaction let us write the helicity amplitudes explicitly.

Assuming the reaction to proceed through π^0-exchange only and using the couplings and vertex factors given in References [12] and [13], the six independent Born helicity amplitudes are found to be

$$\begin{aligned}
\langle 0 \quad 1/2 \mid B \mid 0 \quad 1/2 \rangle &= 2\, i\, g\, G/m_c\, (q'\, E_a - q\, E_c\, x)\, (1 + x/2)^{1/2}\, 1/2\, q\, q'\, (z - x) \\
\langle 1 \quad 1/2 \mid B \mid 0 \; -1/2 \rangle &= -\, i\, g\, G/4\, \pi \sqrt{2}\, \xi_+ (1 + x/2)^{1/2}\, (1 - x/2)\, 1/z - x \\
\langle 0 \; -1/2 \mid B \mid 0 \quad 1/2 \rangle &= \quad i\, g\, G/4\, \pi\, 2\, \xi_+\, (q'\, E_a - q\, E_c\, x/m_c)\, (1 - x/2)^{1/2}\, 1/q\, (z - x) \\
\langle 1 \quad 1/2 \mid B \mid 0 \quad 1/2 \rangle &= -\, i\, g\, G/4\, \pi \sqrt{2}\, \xi_-\, (1 + x/2)\, (1 - x/2)^{1/2}\, 1/z - x \\
\langle 1 \; -1/2 \mid B \mid 0 \; -1/2 \rangle &= \quad i\, g\, G/4\, \pi \sqrt{2}\, \xi_-\, (1 + x/2)\, (1 - x/2)^{1/2}\, 1/z - x \\
\langle 1 \; -1/2 \mid B \mid 0 \quad 1/2 \rangle &= -\, i\, g\, G/4\, \pi \sqrt{2}\, \xi_+\, (1 + x/2)^{1/2}\, (1 - x/2)\, 1/z - x . \quad (6)
\end{aligned}$$

The helicities of the ϱ, the outgoing proton, the incoming pion and the incoming proton are specified from left to right. The other six helicity amplitudes are related to these by space reflection through

$$\langle -\lambda_\varrho - \lambda_p \mid B \mid -\lambda_\pi - \lambda_p \rangle = (-1)^{-\lambda_\varrho + \lambda_p + \lambda_p} \langle \lambda_\varrho\, \lambda_p \mid B \mid \lambda_\pi\, \lambda_p \rangle .$$

The quantities used in these expressions are

$$z = (m_\pi^2 + q^2 + q'^2 + m_\varrho^2 - m_{\pi^0}^2/4\,s)/2\,q\,q' ,$$

g, $G = \pi\,\pi\,\varrho$ and $p\,p\,\pi$ coupling constants ,

$$\xi_\pm = q\,(E_2 + M/E_1 + M)^{1/2} \pm q'\,(E_1 + M/E_2 + M)^{1/2} ,$$

E_1, E_2 = energies of the initial and the final protons respectively,

M = their mass ,

$$s = m_\pi^2 + M^2 + 2\,M\sqrt{p_L^2 + m_\pi^2} ,$$

$$t = -\,q^2 - q'^2 + 2\,q\,q'\cos\theta + (E_\pi - E_\varrho)^2 .$$

It is evident from a comparison of the angular factors in the above expressions with those which must be present in Equation (2), that all the matrix elements except two contain exceptional terms of varying degrees of importance. Let us consider, for example, the term $\langle 1^1/_2 \mid B \mid 0\ -\ ^1/_2\rangle$: the angular factor which must be present on general grounds is $(1 + x)^{1/2}/z - x$. However, the unmodified amplitude contains an additional factor $(1 - x)$ and may be decomposed in the following way.

$$(1 + x)^{1/2} - (z - x)^{-1}\,(z - 1)\,(1 + x)^{1/2} .$$

The first term contributes only to the $j = ^1/_2$ amplitudes and is termed 'exceptional'. The remaining term is of the normal form given in Equation (2) and contributes to all partial wave amplitudes. Since $z - 1$ is small at high energies, the normal matrix elements are much smaller than the exceptional term. The Born approximation for this amplitude vanishes in the forward direction, $x = 1$, because of the destructive interference of the exceptional term and remainder of the amplitude: but since we do not consider the short-range part of the interaction due to the absorption of the low partial waves, this modified amplitude actually do not vanish and there is a sizeable contribution in the forward direction [5].

So separating the matrix elements into normal and exceptional terms and making use of the Equation (4) we get the following matrix elements

$$\begin{aligned}
\langle 0 \quad 1/2 \mid B^j \mid 0 \quad 1/2\rangle &= i\,a\,\xi_-\,h\,\Sigma\,(j + 1/2)\,c^j_{1/2\,1/2}\,d^j_{1/2\,1/2} + i\,a/2\,E_\varrho/m_\varrho\,\xi_-\,\Sigma\,d^j_{1/2\,1/2} \\
\langle 1 \quad 1/2 \mid B \mid 0\ -\ 1/2\rangle &= i\,a\sqrt{2}\,\xi_+\Sigma\,(j + 1/2)\,c^j_{1/2\,1/2}\,d^j_{1/2\,1/2} - i\,a/\sqrt{2}\,\xi_+\,\Sigma\,d^j_{1/2\,1/2} \\
\langle 0\ -\ 1/2 \mid B \mid 0 \quad 1/2\rangle &= i\,a\,\xi_+\,h\,\Sigma\,(j + 1/2)\,c^j_{-1/2\,1/2}\,d^j_{-1/2\,1/2} + i\,a/2\,\xi_+\,E_\varrho/m_\varrho\,\Sigma\,d^j_{-1/2\,1/2} \\
\langle 1 \quad 1/2 \mid B \mid 0 \quad 1/2\rangle &= -\,i\,a\sqrt{2}\,\xi_-\,\Sigma\,(j + 1/2)\,c^j_{-1/2\,1/2}\,d^j_{-1/2\,1/2} + i\,a/2\sqrt{2}\,\xi_-\,\Sigma\,d^j_{-1/2\,1/2} \\
\langle 1\ -\ 1/2 \mid B \mid 0\ -\ 1/2\rangle &= i\,a\,\xi_-\,2^{1/2}\,\Sigma\,(j + 1/2)\,c^j_{1/2\,3/2}\,d^j_{1/2\,3/2} \\
\langle 1\ -\ 1/2 \mid B \mid 0 \quad 1/2\rangle &= i\,a\sqrt{2}\,\xi_+\,\Sigma\,(j + 1/2)\,c^j_{-1/2\,3/2}\,d^j_{-1/2\,3/2} .
\end{aligned} \tag{7}$$

Each of the terms in the expansion has to be multiplied by the absorption factor k. The following quantities have been used in the above equation

$$a = g\,G/4\,\pi , \qquad h = (q'\,E_\pi - q\,E_\varrho\,z)/q\,m_\varrho$$

$c^j_{\lambda\mu}$ and $d^j_{\lambda\mu}$ functions are calculated with the help of the relations given in the Reference [10].

The absorption factor is given by the expression

$$k(l) = e^{i\delta_i(l)}\, e^{i\delta_f(l)}$$

where δ_i and δ_f are the complex elastic scattering phase-shifts for the pion-proton and rho-proton processes respectively. δ_i is found from the experimental data and since the elastic scattering in the final state is not known it is supposed to be the same as in the initial state. Following the method of Reference [4] the absorption factor is well represented by

$$e^{2i\delta(l)} = 1 - C\, e^{-\gamma l(l+1)}$$

where $C = \sigma_{\text{tot.}}/4\pi A$ and $\gamma = 2q^2 A$, $A = 8.33 \pm 0.49$ (Gev)$^{-2}$
For consistency $C \leqslant 1$ and from fits to various data C is found to be of the order 0.7–1.0 corresponding to complete or almost complete absorption of the s-wave. Here the value used is 1 and using the results of Reference [6] for pion-proton scattering the value of γ is found to be 0.036.

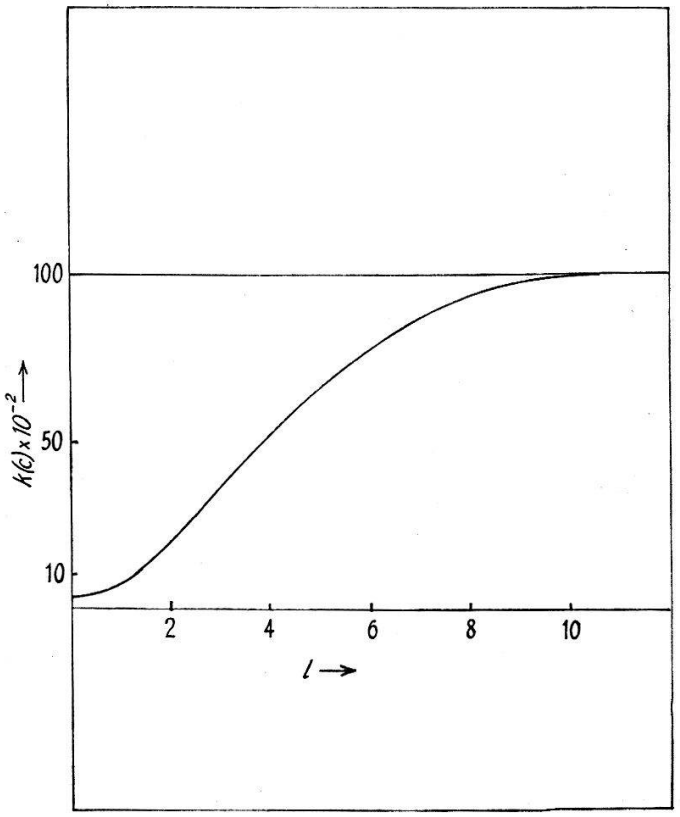

Fig. 2

Elastic scattering phase factor $e^{2i\delta(l)}$ as a function of angular momentum.

A curve is drawn (Figure 2) for $e^{2i\delta(l)}$ as a function of l. It is found that the absorption is the greatest for a few low partial waves.

The coupling constants are often known from other experiments. For π-exchange the coupling constants are $G^2_{\pi p p}/4\pi \approx 14.5$ and g^2 is proportional to the width of the decay:

$$\Gamma_{\varrho \to \pi\pi} = 2/3\, g^2_{\pi\pi\varrho}/4\pi\, p^3/m^2_\varrho$$

where p is the three-momentum of each of the decay products in the rest frame of ϱ. Taking $\Gamma_\varrho \approx 100$ Mev,

$$g^2_{\pi\pi\varrho}/4\pi \approx 2.0.$$

It is found that at the energies under consideration only a few partial waves ($\leqq 10$) contribute to the process in question, hence it is worthwhile to directly sum the exact partial wave projections rather than making large j and small angle approximations in order to use the simplified integral expression [4, 5]. The theoretical and experimental differential cross-sections are compared in Figure 3.

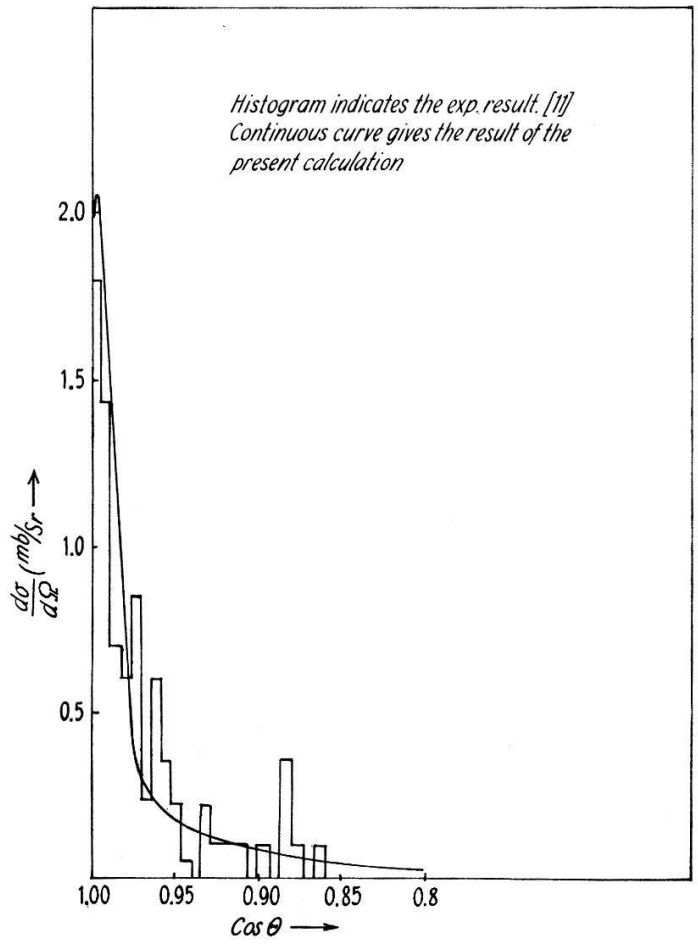

Fig. 3

Comparison of theoretical and experimental angular distribution of differential cross-section for production.

A form of the unitary restriction on the amplitude for a single inelastic channel follows from the requirement that the single channel partial wave cross-section be less than the total inelastic partial-wave cross-section as determined from elastic scattering. It has been known for a long time that the one meson exchange diagrams violate unitarity in the lowest partial waves. Therefore to see the effect of absorption on the unitarity limit the ratio of the partial wave cross-section to the absolute upper bound $(2j + 1)\pi/q^2$, is plotted against 1 in Figure 4.

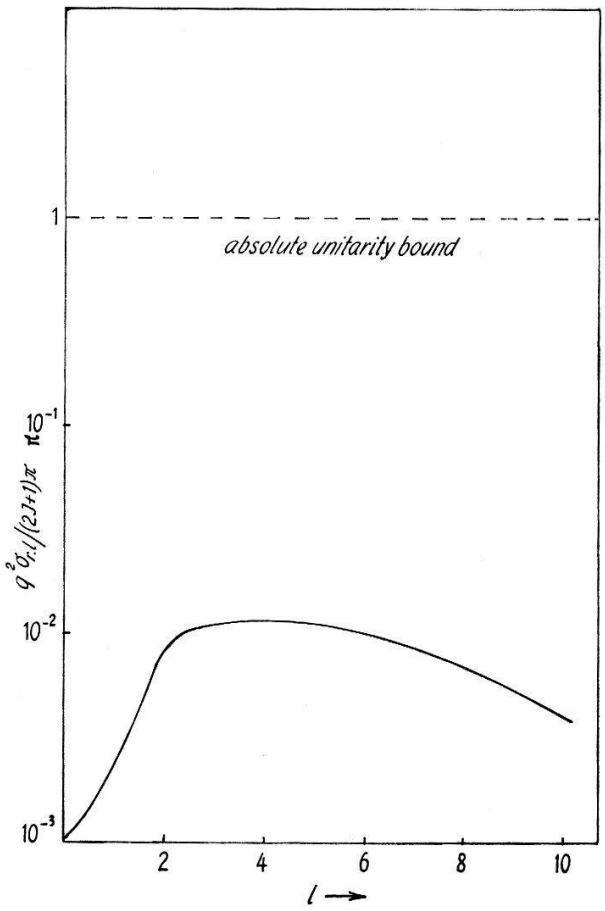

Fig. 4

Partial wave cross-section divided by the absolute upper bound $(2j+1)\,\pi/q^2$ plotted against angular momentum.

4. Discussions and Criticism

The predicted cross-sections seem to agree well with the experimental values [11]. The values are slightly smaller than those of References [4] and [5] which may be attributed to the different choice of the absorption factor. Besides, there is a peak dominating at very near forward angles in our work. This peak may be due to the effect of $\lambda - \mu > 0$ amplitudes at small angles.

The cross-section goes down at large angles indicating that the vector-meson (ω)-exchange may be considerable at such angles and this may modify the predictions of the pion-exchange model on the spin-space density matrix elements of the produced ϱ-meson.

The present calculations have been done on the extreme assumption of point couplings i.e. no form factor at the vertices are taken into account although some such structure undoubtedly exists. The fit to the experimental data [11] has been obtained by others [11] by using the Selleri-modification [15] at the vertices. However, in view of the absorption model it may be remarked that the form factors are not as rapidly varying as it was thought [6, 7]. The form factors may play a more important role in the vector-meson exchange processes.

The present calculations involve certain drawbacks: we have no knowledge of the final state elastic scattering; the elastic amplitude has been taken to be purely

imaginary though there is some evidence [16] of a sizeable (30–40% of the imaginary part) real part; the contributions from the helicity-flip amplitudes to the absorptive corrections have not been taken into account; the way the particular form of the absorption factor has been introduced is open to criticism. Because of the good fit obtained it may be said that the factors mentioned above have very negligible effect on the present reaction but may have to be considered for some other processes where these factors could play a significant role.

The spin-space density matrix elements are important in determining the nature of the exchange. So the calculation of these matrix elements should also throw light on the validity of these model for the types of reactions discussed. This work on density matrix is in progress and will be reported in a later paper.

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