Zeitschrift: Helvetica Physica Acta
Band: 40 (1967)
Heft: 5

Artikel: Residual interactions in spherical symmetric nuclei
Autor: Hadermann, J. / Simonius, M.
DOI: https://doi.org/10.5169/seals-113781

Residual Interactions in Spherical Symmetric Nuclei

by **J. Hadermann** and **M. Simonius**[1])
Institute of Theoretical Physics, University of Basel, Basel

(31. I. 67)

Abstract. Introducing a residual two-body interaction in the shell model, the low-lying states are similar to collective vibrations. Energies, state vectors and transition rates have been evaluated in the quasi-boson approximation which is exact for only one pair of nucleons outside the closed shell. Numerical results for some Pb and Ni isotopes are given.

1. Introduction

In order to describe the low-lying levels of even-even nuclei in the shell model, it is necessary to introduce a residual two-body interaction.

We make the assumption that the nucleus consists of closed shells whose excitations are not considered and an even number of protons or neutrons (or holes) outside closed shells. Energies, state vectors and transition rates are evaluated in the quasi-boson approximation (QBA), which is exact for only one pair outside the closed shell. It is seen, that the low-lying states behave like collective vibrations with two fundamental differences;
first, the number of "phonons" excited is limited by the number of pairs outside closed shells.
Secondly, the static quadrupole moment of excited quadrupole states is not zero but has a collective value as for instance in the asymmetric rotator model.

Ratios of transition rates for different nuclei can be given independently of the interaction. Though our method is not restricted to a special residual interaction we consider here only a delta force and assume that the long-range part of the nucleon interactions is included in the mean potential. It can be shown [1] that this force gives the usual parity rule for the one-phonon states of spin J; $(-)^J = (-)^\pi$.

Numerical results for some Pb and Ni isotopes are discussed. While the QBA restricts the validity of the results to nuclei with only a few pairs outside closed shells, it is assumed that the general behaviour of the states holds also for other spherical symmetric nuclei.

2. The Hamiltonian

We can write the Hamiltonian as follows

$$H = H_0 + H_{int}\,, \tag{1}$$

where the diagonal term given by the potential well of the shell model is H_0

$$H_0 = \sum_{jm} \varepsilon_j \, a^+_{jm} \, a_{jm}\,.$$

[1]) Present address: Illinois Institute of Technology, Physics Department, Chicago, Ill.

The single-particle levels are denoted by ε_j and are taken either from experiment or from theoretical considerations [2, 3].

The creation operator a^+_{jm} creates a nucleon in the state (jm). The two-body interaction can be expressed by

$$H_{int} = \sum_{\text{all } jm} \langle j_1\, m_1 \mid \langle j_2\, m_2 \mid V \mid j_1'\, m_1' \rangle \mid j_2'\, m_2' \rangle\, a^+_{j_1 m_1}\, a^+_{j_2 m_2}\, a_{j_2' m_2'}\, a_{j_1' m_1'} ,$$

where V is the residual interaction potential between the two nucleons.

We define the following tensor operators of order JM:

$$B^+_{JM}(j_1\, j_2) = \frac{1}{\sqrt{1+\delta_{j_1 j_2}}} \sum_{m_1 m_2} \langle j_1\, m_1\, j_2\, m_2 \mid JM \rangle\, a^+_{j_1 m_1}\, a^+_{j_2 m_2} , \quad j_1 \geq j_2 ,$$

with the symmetry relation

$$B^+_{JM}(j_1\, j_2) = -\,(-)^{j_1+j_2+J}\, B^+_{JM}(j_2\, j_1) .$$

The interaction can now be expressed by

$$H_{int} = \sum_{JM} \sum_{\text{all } j} \sqrt{(1+\delta_{j_1 j_2})\,(1+\delta_{j_1' j_2'})}\; G_J(j_1\, j_2\, j_1'\, j_2')\; B^+_{JM}(j_1\, j_2)\; B_{JM}(j_1'\, j_2') .$$

For the special case of a δ-force,

$$V = V_0\, \delta^{(3)}\,(\boldsymbol{r}_1 - \boldsymbol{r}_2) ,$$

it can be shown [1] that

$$G_J(j_1\, j_2\, j_1'\, j_2') = -\,(-)^{j_1+j_1'} \frac{1}{32\,\pi}\; V_0\; R^{n_1 l_1 n_2 l_2}_{n_1' l_1' n_2' l_2'}\; \frac{1}{2}\,(1+(-)^{l_1+l_2+J})$$
$$\times \sqrt{(2\,j_1+1)\,(2\,j_2+1)\,(2\,j_1'+1)\,(2\,j_2'+1)} \begin{pmatrix} j_1 & j_2 & J \\ -\frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix} \begin{pmatrix} j_1' & j_2' & J \\ -\frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix} . \qquad (2)$$

We define the coupling constant G by

$$G = \frac{1}{32\,\pi}\; V_0\; R^{n_1 l_1 n_2 l_2}_{n_1' l_1' n_2' l_2'} ,$$

where the radial part $R^{n_1 l_1 n_2 l_2}_{n_1' l_1' n_2' l_2'}$ of the two-body matrix element is taken as constant. It has been shown [3] that this is a good assumption.

In general it is impossible to diagonalize Hamiltonian, (1), but some qualitative considerations can be made. The interaction H_{JM} induces a strong correlation of the nucleons and depresses the lowest state of spin (JM). Because of the correlation, this state exhibits a collective character, whereas the excited levels of spin (JM) remain nearly undisturbed at the position of the corresponding single-particle levels and show single-particle character.

The parity-selection rule in eq. (2) is a consequence of the short-range force and permits identification of the low-lying levels with those of the vibration model.

3. The Quasi-Boson Approximation (QBA)

In order to diagonalize Hamiltonian, (1), we partially neglect the Pauli principle and consider the operators $B^+_{JM}\,(j_1\, j_2)$ as Boson operators

$$[B_{JM}(j_1\, j_2),\quad B^+_{J'M'}(j_1'\, j_2')] = \delta_{j_1 j_1'}\, \delta_{j_2 j_2'}\, \delta_{J J'}\, \delta_{M M'} \qquad \text{for} \quad j_1 \geq j_2,\; j_1' \geq j_2' .$$

This quasi-boson approximation was first considered for the special case $J = M = 0$ by HØGAASEN-FELDMAN [3]. It permits us to write the Hamiltonian in the form

$$H = \sum_{JM} H_{JM} = \sum_{JM} \sum_{\substack{j_1 \geq j_2 \\ j_1' \geq j_2'}} \{(\varepsilon_{j_1} + \varepsilon_{j_2})\, \delta_{j_1 j_1'}\, \delta_{j_2 j_2'} + 2\sqrt{(2 - \delta_{j_1 j_2})\,(2 - \delta_{j_1' j_2'})}$$
$$\times\, G_J(j_1\, j_2\, j_1'\, j_2')\}\, B^+_{JM}(j_1\, j_2)\, B_{JM}(j_1'\, j_2')\,, \tag{3}$$

where the number operator for the single-particles has been substituted by the number operator for quasi-bosons.

Now every term H_{JM} can be individually diagonalized by the unitary transformation

$$B^+_{JM}(j_1\, j_2) = \sum_K C^J_{j_1 j_2, K}\, \bar{B}^+_{JM}(K),\ j_1 \geq j_2\,.$$

It is emphasized here that the QBA is exact for only one pair of nucleons outside the closed shells.

We write the eigenvectors of the Hamiltonian as follows:

$$|\, \gamma^{JM}_K > = \left(\prod_{\substack{JM \\ K}} \gamma^{JM}_K !\right)^{-1/2} \prod_{\substack{JM \\ K}} \left(\bar{B}^+_{JM}(K)\right)^{\gamma^{JM}_K} |\, 0>\,,$$

where γ^{JM}_K gives the number of quasi-bosons of spin (JM) in the state K

$$\sum_{\substack{JM \\ K}} \gamma^{JM}_K = p\,.$$

The corresponding eigenvalue is

$$E = \sum_{\substack{JM \\ K}} \gamma^{JM}_K\, E^J_K\,,$$

where E^J_K is an eigenvalue of the matrix H_{JM}. The collective level of spin (JM) is labelled by $K = 0$.

For a definite spin J, as many equidistant collective levels can be built up as there are pairs of nucleons outside the closed shell. This is in contradiction to the vibration model. But the equal spacing of the collective energy levels (if there is more than one pair of nucleons outside) and the parity $(-)^J$ are predictions which are given also by the vibration model.

4. Matrix Elements of Tensor Operators in the QBA

In the QBA, single-particle operators can be written as two-particle operators (see also eq. (3)).

The matrix elements of a tensor operator T_{JM} can be calculated and expressed by the transformation coefficients $C^J_{j_1 j_2, K}$.

For diagonal matrix elements we get

$$\langle \gamma^{J'M'}_{K'} \mid T_{JM} \mid \gamma^{J'M'}_{K'} \rangle$$
$$= \sum_{\substack{J'M' \\ K}} \gamma^{J'M'}_{K'}\, (-)^{J'-M'}\, \frac{\langle J'M'J'-M' \mid JM\rangle}{\sqrt{2J+1}}\, (-)^J\, (2J'+1)\, T_J \begin{pmatrix} J' & J' \\ K' & K' \end{pmatrix},$$

where

$$T_J\begin{pmatrix} J' & J'' \\ K' & K'' \end{pmatrix} = \sum_{j_1 j_2 j'} (-)^{j_1+j_2} \langle j_1 \| T_J \| j_2 \rangle \sqrt{(1+\delta_{j_1 j'})(1+\delta_{j_2 j'})} \begin{Bmatrix} J'' & J' & J \\ j_2 & j_1 & j' \end{Bmatrix} C^{J''}_{j_1 j', K''} C^{J'}_{j' j_2, K'},$$

and $\langle j_1 \| T_J \| j_2 \rangle$ is the reduced single-particle matrix element. Because of the strong correlation in the state $K = 0$, the static multipole moments are appreciably greater than those of a single-particle calculation. The vibration model in contradiction to this gives zero for multipole moments of excited states.

Off-diagonal matrix elements are given by

$$\langle \cdots, \gamma^{J'M'}_{K'} - 1, \gamma^{J''M''}_{K''} + 1, \ldots \mid T_{JM} \mid \ldots, \gamma^{J'M'}_{K'}, \gamma^{J''M''}_{K''}, \ldots \rangle = (-)^{J'+M'} \frac{\langle J''M'' \; J'-M' \mid JM \rangle}{\sqrt{2J+1}} (-)^J \sqrt{(2J'+1)(2J''+1)} \times \sqrt{\gamma^{J'M'}_{K'}(\gamma^{J''M''}_{K''}+1)} \; T_J\begin{pmatrix} J' & J'' \\ K' & K'' \end{pmatrix}. \tag{4}$$

As an example, the reduced transition probability from the first excited state of spin L to the ground state is given

$$B(T_J, L \to 0^+_0) = \delta_{LJ} \, \mathfrak{p} \left(T_J\begin{pmatrix} L & 0 \\ 0 & 0 \end{pmatrix} \right)^2. \tag{5}$$

As a consequence of the partial neglecting of the Pauli principle in the QBA, the cross-over from collective spin multiplets to the ground state is forbidden.

5. Numerical Calculations

In the following we present some numerical results for the Pb and Ni isotopes. It may be noted again that the QBA is exact for the two isotopes Pb^{206} and Ni^{58}.

5.1. *Lead Isotopes*

The single-particle levels ε_j are taken from experimental data [5] on Pb^{207}. The first five levels are interpreted as single-neutron hole states and have been used in the diagonalization of Hamiltonian (3).

Figure 1 shows the level scheme of Pb^{206} calculated in this way compared to the experimental data [7]. The coupling constant $G = -0.040$ has been fitted by the energy difference between the ground state and the first excited state. The calculated level sequence agrees very well with the experimental data.

We have investigated whether G is reasonably well independent of J (a statement implied by the δ-force). For $J = 0$ we have fitted G by the energy difference $0^+_0 - 0^+_1$ and for $J = 2$ by the energy difference $2^+_0 - 2^+_1$. The state 3^+ determines the zero point of the energy scale. Figure 2 shows the corresponding level scheme which is almost the same as fig. 1. The two coupling constants are the same to within 10%.

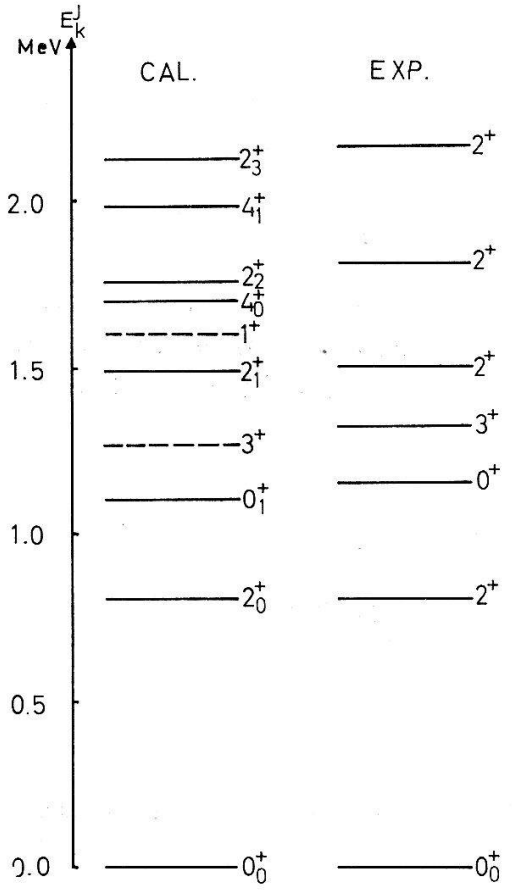

Figure 1

The experimental [7] level scheme of Pb^{206} is compared to the calculated level sequence. The coupling constant is $G = -0.040$ (see also sect. 5). The calculated levels are marked with spin and parity assignement, the index gives the number K.

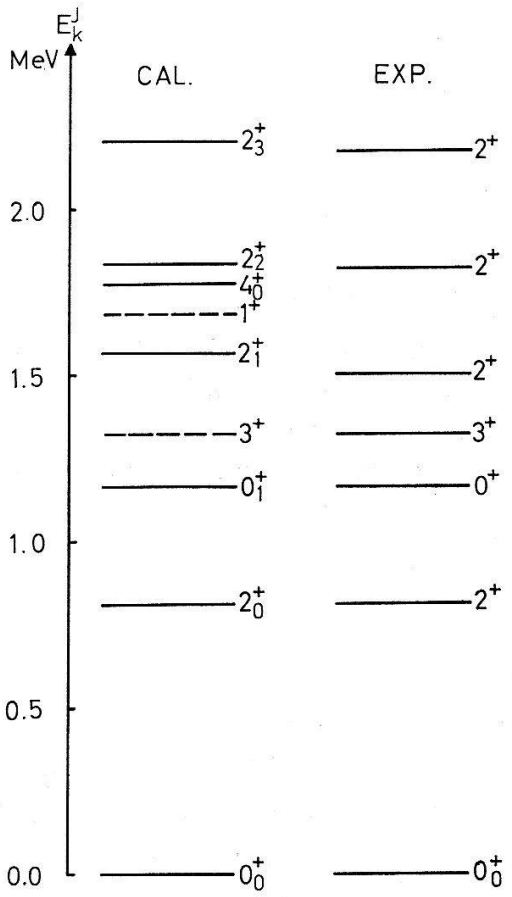

Figure 2

The calculated level scheme of Pb^{206} is compared to experiment [7]. For spin $J = 0$ the coupling constant is $G = -0.042$, for spin $J = 2$, $G = -0.045$.

The level scheme of Pb^{204} can be obtained from figure 1 provided the nuclear parameters vary only slowly with mass, that is, we take G to be -0.040. The level scheme of Pb^{204} is given in figure 3. Note that there exists a collective spin triplet state at 1.60 MeV.

In table 1, some calculated reduced transition probabilities in Pb^{206} are given and compared to existing experiments.

All results have been obtained by assuming an effective charge of one electron. The magnitude of the transition $2_0^+ \to 0_0^+$ is a consequence of the correlation of the nucleons in these two states.

There exists an experimental [7] branching ratio of 56/44 between the transitions $2_1^+ \to 2_0^+$ and $2_1^+ \to 0_0^+$. If the former is M1, we get the ratio 18/100. The E2 transition is still 7.5 times slower. As far as our model and a radiative transition is concerned, we cannot account for this discrepancy.

The corresponding quantities of Pb^{204} can easily been deduced from the foregoing by means of formula (4).

Table I

Some reduced electric quadrupole and magnetic dipole transition probabilities in Pb^{206} are compared to existing experiments

Electric quadrupole transition	Theory	Experiment[a])
$2_0^+ \rightarrow 0_0^+$	$6.40 \cdot 10^{-1}$ b^2	$(6.5 \pm 2.5) \cdot 10^{-1}$ b^2
$0_1^+ \rightarrow 2_0^+$	$2.05 \cdot 10^{-2}$ b^2	
$2_1^+ \rightarrow 0_0^+$	$1.78 \cdot 10^{-2}$ b^2	
$2_1^+ \rightarrow 2_0^+$	$1.41 \cdot 10^{-2}$ b^2	
Magnetic dipole transition		
$2_1^+ \rightarrow 2_0^+$	$2.095 \cdot 10^{-2}$ $(e\,\hbar/2\,M\,c)^2$	

[a]) Ref. [6].

We give here only one result, which can be compared to experiment. If the parameters are the same as for Pb^{206}, the reduced transition probability for $2_0^+ \rightarrow 0_0^+$ should be twice that of Pb^{206}, i.e. 1.28 b^2. The experimental value [6] is 1.10 ± 0.45 b^2.

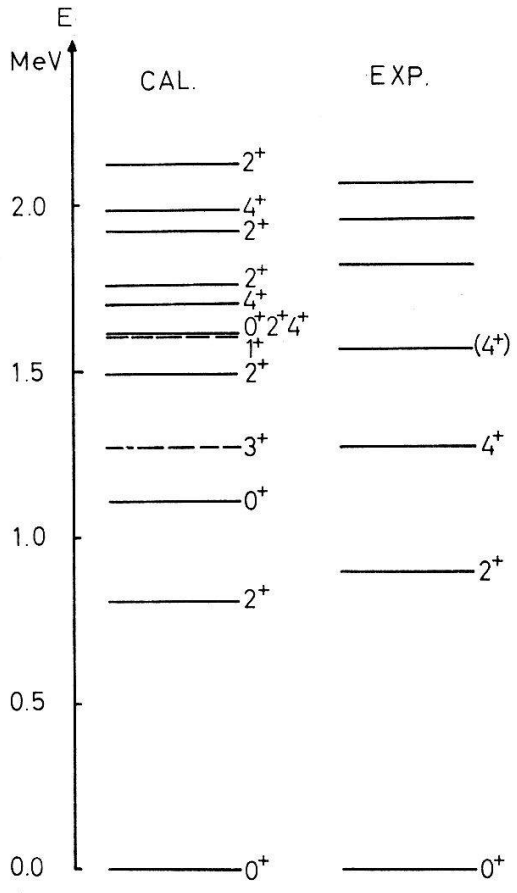

Figure 3

The theoretical level scheme of Pb^{204} obtained from figure 1 is compared to the experimental data [7] (see sect. 5).

The spectroscopic quadrupole moment [8] of the state 2_0^+ is

$$Q = -0.505 \text{ b},$$

and of state 2_1^+ is

$$Q = -0.095 \text{ b}.$$

The same trend as in the transition probabilities is noted here. The quadrupole moment of the collective state is enhanced by a factor 5 due to the correlation of the nucleons in this state.

5.2. *Nickel Isotopes*

Since the single-particle levels cannot be taken from experiment those of KISSLINGER and SORENSEN [9] are used.

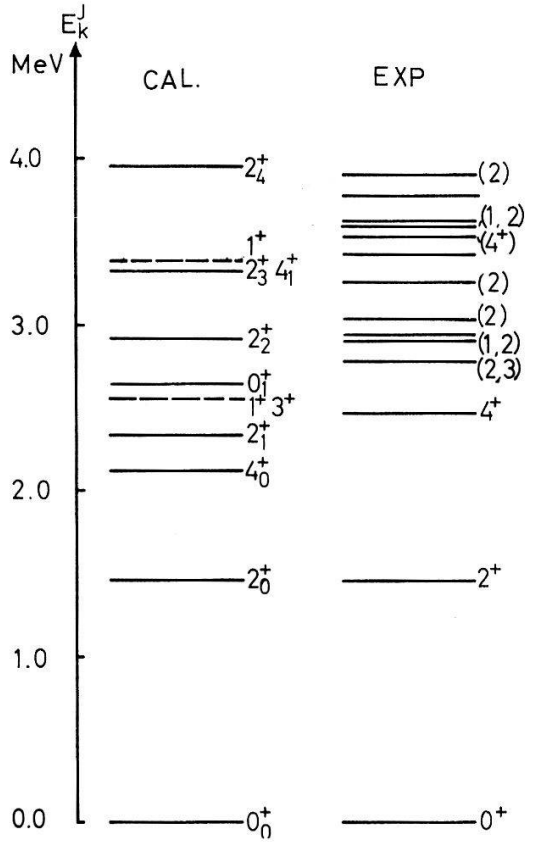

Figure 4

The calculated energy levels of Ni^{58} are compared to experiment [10]. The coupling constant is $G = -0.094$.

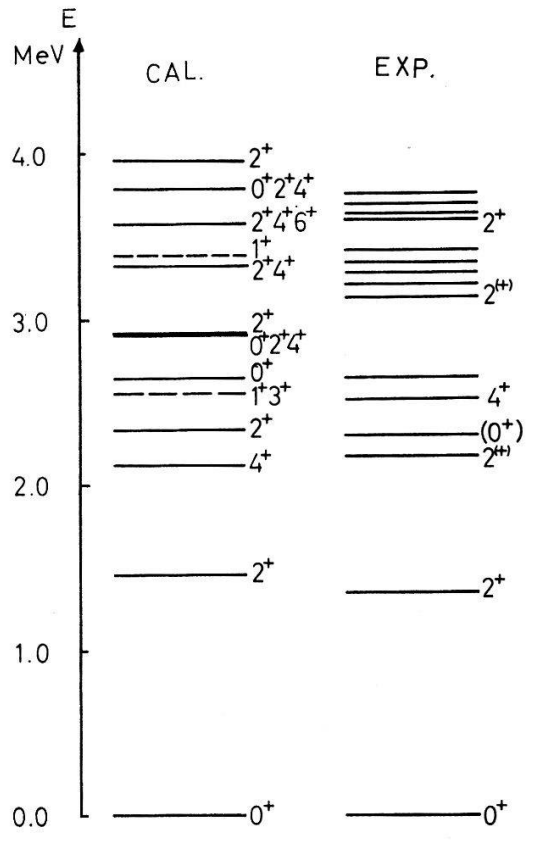

Figure 5

The theoretical level scheme of Ni^{60} obtained from figure 4 is compared to experimental data [7] (see sect. 5).

In figure 4 the calculated level scheme of Ni^{58} is compared to the experimental data [10].

The fitted coupling constant is $G = 0.094$. As the spin sequence of the levels is rather good, it is possible to obtain better agreement with experiment by shifting upwards the relatively uncertain single-particle energy $\varepsilon_{f_{5/2}}$ so that the energies of the 4_0^+, 2_1^+ and 0_1^+ levels also increase.

Figure 5 shows the level scheme of Ni^{60} obtained from figure 4. Data are from ref. [7]. Because the multiplicity of the single-particle levels is small, the QBA becomes less adequate.

This can also be seen in the transition probabilities. The only free parameter, the effective charge of the neutrons, is assumed to be unity.

Some reduced transition probabilities are compared with experiment in table II.

Table II

Some reduced electric quadrupole and magnetic dipole transition probabilities in Ni^{58} are compared to existing experiments

Electric quadrupole transition	Theory	Experiment
$2_0^+ \rightarrow 0_0^+$	$3.48 \cdot 10^{-1}$ b^2	$(3.60 \pm 0.35) \cdot 10^{-1}$ b^2 [a] $(3.55 \pm 0.70) \cdot 10^{-1}$ b^2 [b]
$2_1^+ \rightarrow 2_0^+$	$1.34 \cdot 10^{-3}$ b^2	
Magnetic dipole transition		
$2_1^+ \rightarrow 2_0^+$	$1.33 \cdot 10^{-1}$ $(e\,\hbar/2\,M\,c)^2$	

[a]) Ref. [11]. [b]) Ref. [12].

That the QBA is less accurate in Ni than in Pb is also shown by the reduced transition probability B $(E2,\ 2_0^+ \rightarrow 0_0^+)$ in Ni^{60}. It should be $6.96 \cdot 10^{-1}$ b^2. Experimental data [11, 12] give $(4.55 \pm 0.40) \cdot 10^{-1}$ b^2, and $(6.00 \pm 0.60) \cdot 10^{-1}$ b^2, respectively. For Ni^{62}, the situation is worse; the experimental value B $(E2, 2_0^+ \rightarrow 0_0^+)$ is $(4.15 \pm 0.40) \cdot 10^{-1}$ b^2, whereas the theory gives 1.04 b^2.

As the term H_{00} in Hamiltonian (3) corresponds to the usual pairing force, we can compare our coupling constant G to that of ref. [9], denoted by G_{KS}. The relation is $G/G_{KS} = -0.5$. For Pb we obtain for this ratio -0.364 and for Ni -0.294.

A remark must be made about the lighter elements. In this region of the periodic table, neutron and proton shells are not filled independently, and the two-particle interaction must be handled with isospin formalism. Our calculations with O^{18} obtained no reasonable energy levels.

We wish to thank Professor K. Alder for helpful comments and his interest in this work.

References

[1] U. Götz, private communication.
[2] S. G. Nilsson, Mat. Fys. Medd. Dan. Vid. Selsk. *29*, No. 16 (1955).
[3] W. W. True and K. W. Ford, Phys. Rev. *109*, 1675 (1958).
[4] J. Høgaasen-Feldman, Nucl. Phys. *28*, 258 (1961).
[5] D. E. Alburger and A. W. Sunyar, Phys. Rev. *99*, 632 (1955).
[6] O. Nathan, Nucl. Phys. *30*, 332 (1962).
[7] Nuclear Data Sheets.
[8] A. R. Edmonds, *Angular Momentum in Quantum Mechanics*, Princeton University Press, 1957, p. 78.
[9] L. S. Kisslinger and R. A. Sorensen, Mat. Fys. Medd. Dan. Vid Selsk. *32*, No. 9 (1960).
[10] P. F. Hinrichsen, G. T. Wood and S. M. Shafroth, Nucl. Phys. *81*, 449 (1966).
[11] P. H. Stelson et al., Nucl. Phys. *32*, 652 (1962).
[12] D. S. Andreyev et al., Nucl. Phys. *19*, 411 (1960).