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Three-body Photonic Decay of Hyperons and Algebra of Currents[1])

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Abstract. The three-body photonic decay of hyperons is studied in the framework of quantum field theory under the hypotheses of the algebra of currents and the soft pion limit. The transition amplitude under consideration can be expressed by the two-body hadronic and photonic decay ones. The main contribution arises from the p-wave hadronic as well as the corresponding photonic decay amplitudes which allows us to examine these amplitudes in comparison with the experimental information on three-body photonic decays. For this purpose the relative as well as the absolute rate of the charged and the neutral pion associated decays and the decay asymmetry parameter are calculated.

The method presented will give a tool for clarifying the present experimental and theoretical ambiguities of two-body hadronic decay amplitudes mentioned by many authors.

1. Introduction

Three-body photonic decay of hyperons was investigated from time to time for understanding the properties of the relevant particles and the nature of the associated interactions [1–5]. The recent development of the symmetry physics of elementary particles based on the algebra of currents [6] and quantum field theory [7] may give a further insight in that problem as was pointed out by one of us (S. I.) [8].

The purpose of this paper is to pursue this problem explicitly in the three-body photonic decay of hyperons in the light of these theories. We start our discussion from the effective Hamiltonian derived before [8] from the Cabibbo current-current type weak Hamiltonian [9, 10] and the electromagnetic interaction. Making use of the reduction technique for the pion, the partially-conserved axial-vector current (PCAC) hypothesis [11], the soft pion limit [12] and finally the chiral $SU(6) \otimes SU(6)$ subgroup algebra of $SU(12)$ [13], we arrive at the desired expression. We find that the decay amplitude under consideration can, in a proper approximation, be expressed in terms of the amplitudes for the two-body hadronic and for the two-body photonic decays.

It is known that the s-wave amplitudes of the two-body hadronic decay of hyperons are nicely explained [14, 15] by Suzuki-Sugawara Hamiltonian [9, 10] but the p-wave solution is not unambiguous from the theoretical point of view [16] as well as from its experimental fits (relative size of the amplitudes). We are not concerned here with the derivation of the amplitudes, so we shall normalize our p-wave amplitudes to the experimental (two-body) value [17] and compare them with the (three-body) experimental [18] and the theoretical estimates [14, 15] (as we shall see the two-body s-wave amplitudes can be neglected in the three-body decays under consideration).

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As far as two-body photonic decay of hyperon is concerned, the experimental situation is not satisfactory and the present information may not be very conclusive. Even if we confine ourselves to the only reported $\Sigma^+ \to p\,\gamma$ decay, the branching probability does not seem to be numerically established (numbers are changing from time to time). We shall therefore use the relative amplitude based on the theoretical estimate [8] and normalize its absolute value to the most recent data [18] on $\Sigma^+ \to p\,\gamma$ decay rate.

The three-body photonic decay amplitude calculated by the procedure stated above will be compared with the available experimental decay rate [18]. It gives a clue for the understanding of the p-wave Σ_+^+ amplitude. We also calculate the relative rate of the neutral and the charged pion associated three-body photonic decays and the polarization of the decay baryon in these decays.

Sec. 2 is dedicated to the formalism of our theory, Sec. 3 summarizes the formulae suitable for application and in Sec. 4 they are evaluated numerically. Sec. 5 summarises our results.

2. Formalism

The effective Hamiltonian for the three-body photonic decay of hyperons is given by

$$\langle \alpha\,\pi^i\,\gamma_{out} \mid \beta_{in} \rangle = \frac{-e}{(2\,k_0)^{3/2}}\,e_\mu^{(\lambda)} \iint d^3x\,d^3y\,\langle \alpha\,\pi^i_{in} \mid [H_w(\boldsymbol{x},0),\,J_\mu^V(\boldsymbol{y},0)] \mid \beta \rangle\,e^{-i\boldsymbol{k}\cdot\boldsymbol{y}}$$
$$\times\, 2\,\pi\,\delta\,(q_0 + E_\alpha + k_0 - E_\beta)\,, \tag{1}$$

where π^i is the pion state with isospin index i and q_0 its energy, $H_w(\boldsymbol{x}, 0)$ is the Cabibbo current-current weak Hamiltonian, $e_\mu^{(\lambda)}$ is the polarization vector of the photon and $\boldsymbol{k}$ its momentum, and $J_\mu^V(y)$ is the electromagnetic current operator.

Making use of the reduction technique for the pion field operator, we find that the integrand of Eq. (1) is equal to

$$\langle \alpha\,\pi^i_{in} \mid [H_w(\boldsymbol{x},0),\,J_\mu^V(\boldsymbol{y},0)] \mid \beta \rangle = \frac{i}{\sqrt{2\,q_0}} \int d^4z\,e^{-iqz}\,(\mu^2 - \Box_z)\,.$$
$$\times\,\theta\,(-z_0)\,\langle \alpha \mid [[H_w(\boldsymbol{x},0),\,J_\mu^V(\boldsymbol{y},0)],\,\varphi^i(z)] \mid \beta \rangle, \tag{2}$$

where $\varphi^i(z)$ is the pion field operator, μ and q are its mass and four momenta. The PCAC relation runs as follows:

$$\varphi^i(z) = c\,\frac{\partial J_\nu^{A\,i}(z)}{\partial z_\nu}\,,\qquad c = \frac{g_r\,K^{NN\pi}(0)}{g_A\,m_N\,\mu^2}\,. \tag{3}$$

By substituting (3) into (2), performing the partial integration, making use of Jacobi identity and finally taking the soft pion limit, we arrive at the desired expression.

$$\langle \alpha \pi^i_{in} \mid [H_w(\boldsymbol{x},0),\,J_\mu^V(\boldsymbol{y},0)] \mid \beta \rangle = \frac{i\,c\,\mu^2}{\sqrt{2\,q_0}} \int d^3z$$
$$\times\,\Big[\langle \alpha \mid [H_w(\boldsymbol{x},0),\,[J_\mu^V(\boldsymbol{y},0),\,J_0^{A\,i}(\boldsymbol{z},0)]] \mid \beta \rangle + \langle \alpha \mid [[H_w(\boldsymbol{x},0),\,J_0^{A\,i}(\boldsymbol{z},0)],\,J_\mu^V(\boldsymbol{y},0)] \mid \beta \rangle$$
$$- \sum_{E_n = m(B_8)} \{\langle \alpha \mid H_w(\boldsymbol{x},0) \mid n \rangle \langle n \mid [J_\mu^V(\boldsymbol{y},0),\,J_0^{A\,i}(\boldsymbol{z},0)] \mid \beta \rangle - \langle \alpha \mid [J_\mu^V(\boldsymbol{y},0),\,J_0^{A\,i}(\boldsymbol{z},0)] \mid n \rangle$$
$$\times\,\langle n \mid H_w(\boldsymbol{x},0) \mid \beta \rangle + \langle \alpha \mid [H_w(\boldsymbol{x},0),\,J_0^{A\,i}(\boldsymbol{z},0)] \mid n \rangle \langle n \mid J^V(\boldsymbol{y},0) \mid \beta \rangle - \langle \alpha \mid J_\mu^V(\boldsymbol{y},0) \mid n \rangle$$
$$\times\,\langle n \mid [H_w(\boldsymbol{x},0),\,J_0^{A\,i}(\boldsymbol{z},0)] \mid \beta \rangle\}\Big]\,, \tag{4}$$

where the summation over E_n in the last term goes only over the masses of the octet baryons $m(B_8)$. The derivation of Eq. (4) from Eqs. (2) and (3) can be performed completely parallel to Eq. (6) of HARA et al. [14], viz. the first two terms and remaining ones on the right-hand side (r.h.s.) of Eq. (4) correspond to the s-wave and p-wave parts respectively in that paper. Substituting (4) into (1) we get the S-matrix element of our problem.

Our next task is to calculate the sub-commutators on the r.h.s. of Eq. (4). They contain the equal-time commutation relation between the time component of chiral SU(2) generators and the electromagnetic current, and between that and the weak Hamiltonian. The former will be considered as the SU(6) $\otimes$ SU(6) subgroup algebra of SU(12) (that part of the algebra may also be interpreted as SU(4) $\otimes$ SU(4) subalgebra of SU(8) since the hypercharge and isospin operators commute with each other) and the latter belongs to a similar one which was already studied by SUZUKI and SUGAWARA. In spite of its lengthy expression, Eq. (4) has a simple physical interpretation after the manipulations discussed below.

An interesting point to notice first refers to the relation

$$\iint d^3y\, d^3z\, [J_\mu^V(\boldsymbol{y}, 0),\, J_0^{A\,i}(\boldsymbol{z}, 0)] = \int d^3y\, J_\mu^A\left(\boldsymbol{y}, 0; \left[\frac{1}{2}\,\lambda_3 + \frac{1}{2}\,\frac{1}{\sqrt{3}}\,\lambda_8,\, \frac{1}{2}\,\lambda_i\right]\right), \tag{5}$$

where the commutator of SU(3) generators in the argument of the current operator on the r.h.s. shows the SU(3) transformation property of it. We have to substitute $J_0^{A(-)}(z)$, $J_0^{A(+)}(z)$ and $J_0^{A3}(z)$ respectively for the π^+, π^- and π^0 associated three-body photonic decays. The last field corresponds to $^1/_2\,\lambda_3$ in SU(3), so it commutes with the electromagnetic current and does not contribute to Eq. (5). Substitution of $J_0^{A(-)}$ and $J^{A(+)}$ on the l.h.s. of Eq. (5) leads to $-\,J_\mu^{A(-)}(\boldsymbol{y}, 0)$ and $J_\mu^{A(+)}(\boldsymbol{y}, 0)$ respectively on its r.h.s. From this, we can conclude that the first term in the square, the first and the second terms in the curly bracket on the r.h.s of Eq. (4) represent the inner bremsstrahlung photon-emission terms from the suddenly accelerated decay pions. Substituting the parity violating $H_{\text{p.v.}}$ and conserving $H_{\text{p.c.}}$ part of H_w respectively into the corresponding commutators we find that the A and B amplitude of HARA et al. may be multiplied by the velocity of the decay baryon $(p_1)_k/E_1$ and $-\,\langle\sigma_k\rangle$ (where $k = 1$, 2, 3 indicate the component of the photon polarization vector) instead of $\langle 1\rangle$ and $-\,\langle\boldsymbol{\sigma}\rangle\,\Delta\boldsymbol{p}/\Delta m$. In this discussion we have neglected the velocity square term of the decay baryon relative to the first and zeroth order term to it. From this study we find that the p-wave contribution in the two-body hadronic decay is magnified relative to the s-wave one in the three-body photonic decay which seems to be suited to disclose the problem raised in the introduction. We shall not try to discuss multiparticle effects, but shall content ourselves in this paper with replacing the three relevant terms on the r.h.s. of Eq. (4) by the experimental two-body hadronic decay amplitudes.

The commutator $[H_w(\boldsymbol{x}, 0),\, J_0^{Ai}(\boldsymbol{y}, 0)]$ in the second term in the square, the last two terms in the curly bracket on the r.h.s. of Eq. (4) have been already evaluated [10]. It leads, roughly speaking, to the interchange of $H_{\text{p.c.}}$ and $H_{\text{p.v.}}$ in H_w. Thus the relevant terms are replaced essentially by the two-body photonic decay amplitudes [8]. In that sense, these terms correspond to the inner bremsstrahlung photon emission from the suddenly annihilated and created baryons under consideration. There remains the question, in this approach, as how to deal with the charged pion associated decay where either the initial or the final baryon is not the one which appears in the two-body photonic decay so that there is no one to one correspondence between that and the amplitude under consideration. However, leaving it open may not damage our approximation when we want to find out the relative importance of the pionic and baryonic photon radiation, since we know the size of the magnetic and electric radiation in the baryon decays.

3. Decay Rate and Asymmetry Parameter

Using the formalism explained in the preceeding section, we now express our S-matrix element of the charged-pion associated three-body photonic decay by

$$\langle \alpha \pi^+ \gamma_{out} \mid \beta_{in} \rangle = \frac{-ie}{(2k_0)^{3/2}(2q_0)^{1/2}} \frac{c\mu^2}{\sqrt{2}} \frac{m_N^3}{3\pi^2} \frac{G}{\sqrt{2}} \cos\theta \sin\theta \, e_j^{(\lambda)}$$

$$\times \left[-(aA_h - A_p) \frac{(p_1)_j}{E_1} + (aB_h + B_p) < \sigma_j > \right] \cdot (2\pi)^4 \delta^4 (k + q + p_1 - p_2) \,, \qquad (6)$$

where m_N is the nucleon mass, G $(= 1.03 \cdot 10^{-5}\, m_p^{-2})$ is the Fermi coupling constant (muon beta decay coupling constant), θ $(= 0.26)$ is the Cabibbo angle, A_h, B_h and A_p, B_p are the $H^-_{\text{p.v.}}$, $H^-_{\text{p.c.}}$ two-body hadronic and the $H^-_{\text{p.c.}}$, $H^-_{\text{p.v.}}$ two-body photonic decay amplitudes respectively, and finally a is a constant defined by $a = 3\sqrt{2}\pi^2/m_N^3\, c\, \mu^2\, G \cos\theta \sin\theta$. A_h and B_h thus defined are dimensionless quantities which are related to the ones defined by Källén [19] and those in ref. [17] by $A_h = F = A\sqrt{2}/\mu$ and $B_h = F\varrho = B\sqrt{2}/\mu$. A_p and B_p are the ones defined by ref. [8] and must be normalized properly to the experimental values. In writing Eq. (6) we have assumed that the amplitudes $a\,A_h - A_p$ and $a\,B_h - B_p$ (hereafter we shall call them A and B amplitude respectively) are constants throughout the range of energies for that decay. The decay probability Γ is given by

$$\Gamma = \frac{1}{36\pi^5} \left(\frac{e^2}{4\pi}\right) \left(\frac{g_r^2}{4\pi}\right) K^{NN\pi}(0)^2 (1.03 \cdot 10^{-5})^2 \cos^2\theta \sin^2\theta$$

$$\times \int_{\mu}^{(q_0)max} \frac{q\, dq_0}{M - q_0} \left[\frac{M^2+\mu^2-m^2-2Mq_0}{M^2+\mu^2+m^2-2Mq_0} (aA_h - A_p)^2 + \frac{M^2+\mu^2+m^2-2Mq_0}{M^2+\mu^2-m^2-2Mq_0} (aB_h + B_p)^2 \right], \qquad (7)$$

where M, m and μ are masses of the decaying baryon, the decay baryon and the pion respectively, and $(q_0)_{max} = (M^2 + \mu^2 - m^2)/2M$. It is interesting to notice that the infrared divergence for the A amplitude is killed by the velocity factor of the decay baryon in Eq. (6), while the B amplitude square in Eq. (7) is multiplied by an infinite coefficient at that limit. Therefore the main contribution comes from the second term in the integrand of Eq. (7).

The energy dependent decay asymmetry parameter $\alpha(q_0)$ for the longitudinal polarization is given by

$$\alpha(q_0) = - \frac{2\, Re\, [(aA_h - A_p)^* (aB_h + B_p)]}{\frac{M^2+\mu^2-m^2-2Mq_0}{M^2+\mu^2+m^2-2Mq_0} (\mathrm{a}\mathrm{A}_h - \mathrm{A}_p)^2 + \frac{M^2+\mu^2+m^2-2Mq_0}{M^2+\mu^2-m^2-2Mq_0} (\mathrm{a}\mathrm{B}_h + \mathrm{B}_p)^2} . \qquad (8)$$

This relation is suited for the charged pion associated decay baryon, viz. the proton and Λ polarization in $\Lambda \to p\,\pi^- \gamma$ and $\Xi^- \to \Lambda\,\pi^- \gamma$ decays.

4. Numerical Results

At present, only a few photonic three-body decays of hyperons have been studied experimentally, so we shall first discuss them in the light of our theory. We found that the coefficient of the A amplitude square is so small that we can neglect its contribution for the decay rate in all the range of energies compared to that for the B amplitude.

In most cases considered in this paper the experimental aB_h amplitudes [17] are large compared to the theoretical B_p amplitude [8] normalized to $\Sigma^+ \to p\gamma$ decay rate [18] except for $\Sigma^- \to n\pi^-\gamma$ decay (if we neglect the experimental error).

Making use of the experimental two-body decay amplitude [17] we find that $\Gamma(\Sigma^+ \to n\pi^+\gamma) = 4 \cdot 10^{-14}$ MeV (up to $q = 100$ MeV/c) which is comparable to the value expected from the factor $\alpha = 1/137$ but is large compared to the observed $2 \cdot 10^{-16}$ MeV. If we use HARA et al.'s p-wave Σ_+^+ solution we get value $7 \cdot 10^{-15}$ MeV while BISWAS et al.'s solution gives $1 \cdot 10^{-16}$ MeV. These solutions [14, 15] are obtained by a best fit to the two-body hadronic decay amplitudes with an additional p-wave amplitude to SUZUKI-SUGAWARA theory. The experimental data we have made use of were obtained by assuming the p-wave dominant Σ_+^+ decay [17], so that our analysis presents some doubt about such a fit.

It would be interesting to measure the relative rate $\Gamma(\Sigma^+ \to p\pi^0\gamma)/\Gamma(\Sigma^+ \to n\pi^+\gamma)$ which is found equal to 10^{-4}, $5 \cdot 10^{-4}$ or $3 \cdot 10^{-2}$ by making use of the p-wave Σ_+^+ amplitude values in ref. [17], [14] and [15] respectively. Therefore the measurement of this rate will give an additional test for the correct interpretation of that amplitude.

One more reported decay rate [18] for $\Sigma^- \to n\pi^-\gamma$ is experimentally found equal to $\Gamma(\Sigma^- \to n\pi^-\gamma) - 4 \cdot 10^{-17}$ MeV while our calculated value is $1 \cdot 10^{-16}$ MeV (up to $q = 100$ MeV/c). Here the agreement is not bad (We recall the good agreement of experimental and theoretical values [14, 15] for Σ_-^- decay).

Further two relative rates can be calculated by our theory, viz.

$$\Gamma(\Lambda \to p\pi^-\gamma)/\Gamma(\Lambda \to \text{tot}) = 3 \cdot 10^{-3} \text{ (up to } q = 60 \text{ MeV/c) and}$$
$$\Gamma(\Xi^- \to \Lambda\pi^-\gamma)/\Gamma(\Xi^- \to \text{tot}) = 1 \cdot 10^{-3} \text{ (up to } q = 75 \text{ MeV/c)}.$$

Computing finally the decay asymmetry parameter $\alpha(q_0)$ of the decay proton for $\Lambda \to p\pi^-\gamma$ yields $+ 0.1\%$ at zero energy and decreases to zero at $(q_0)_{max}$, while that parameter for $\Xi^- \to \Lambda\pi^-\gamma$ starts from $-.1$ at zero energy and goes to $-.09$ at $q = 75$ MeV/c. In both cases the predicted asymmetry parameters are very small.

5. Discussion

Assuming current algebra and the soft pion limit, we have shown that the three-body photonic decay amplitude of the hyperon may be parametrized by the corresponding two-body decay amplitudes. Our approach may thus give a tool for clarifying at least partially the known difficulty of the p-wave solution in the two-body hadronic decays. There are only two pieces of experimental information on the branching ratios of the three-body photonic decays, $\Sigma^+ \to n\pi^+\gamma$ and $\Sigma^- \to n\pi^-\gamma$, which may not be considered to be conclusive. Thus our comparison will mainly be considered as an illustration. We may have over-simplified our approach in order to point out the main contributing terms. Yet it may be modified suitably when we need a more detailed treatment.

Some aspects however could be subjected to criticism. Gauge invariance, for instance, is only satisfied at the soft photon limit or when the higher power terms with respect to photon momentum are neglected.

If CP violation really has an electromagnetic origin [20][2]), some of the basis of our theory would have to be modified. Yet the uncertainty of the present physical situation [21] does not justify taking this problem into account.

We hope that the still unclarified experimental situation could be improved by performing the proposed three-body experiments, especially the suggested ratio of the neutral and the charged pion associated decays, in order to arrive at conclusive results on the inconsistent amplitude relations due to semiphenomenological theories.

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One of us (S. I.) has heard from private communications that similar topics are being worked upon by a student of Professor K. Nishijima and by Dr. S. Pakvasa.

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[2]) One of us (S. I.) is indebted to Professor J. S. Bell for informing him on the BNL (Princeton group) and CERN experiments on the charged and the neutral two pion decay rate of the long lived neutral kaon and for his discussion of its theoretical implication.