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Photonic Decay of Hyperons and Suzuki-Sugawara Hamiltonian[1])

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Abstract. Suzuki-Sugawara Hamiltonian of the weak hadronic decay is tested by the two-body photonic decay of hyperons. The S-matrix element of the photonic decay of hadrons associated with the weak interaction is proved to be proportional to the equal-time commutator of the weak Hamiltonian and electromagnetic current in a proper approximation. The weak-axial parameters are adjusted to those obtained from semi-leptonic decay of hyperons while those for the magnetic form factors of electromagnetic interaction are adjusted to SU(6) result.

Including the neutral weak currents the relative decay rate of $\Sigma^+ \to p\gamma$ to its total rate is found to be $2.7 \cdot 10^{-4}$ which is comparable with the experimental $(3.7 \pm 0.8) \cdot 10^{-4}$. The decay assymmetry parameter α for the longitudinal polarization is given by $\alpha = 0.14$.

The corresponding study has been made for five unobserved decays.

1. Introduction

Sugawara [1] and Suzuki [2] (SS) have independently proposed that the weak hadronic decay should be described by the PC-even current-current type of the Cabibbo hadronic currents [3] with the Fermi coupling constants in the scheme of the algebra of currents [4], by the partially-conserved axial-vector current [5] and in the limit of soft-pion hypothesis [6]. Their work was extended by Hara, Nambu and Schechter [7] as to include the parity-conserving amplitude. The result of that paper was re-examined by Biswas, Kumar and Saxena [8] including the decuplet-baryon contribution. Those calculations are in a reasonable agreement with the experimental [9] and some other information.

In this paper we want to show that the same Hamiltonian with additional neutral currents explains consistently the experimental decay rate [10] of $\Sigma^+ \to p\gamma$. If we neglect the neutral currents its rate is about ten times slower than the experimental one. In the approximation used here the contribution from the parity-violating weak Hamiltonian $H_{p.v.}$ is not zero but small compared with the one obtained from the conserving one $H_{p.c.}$. We find therefore a small longitudinal polarization of the decay proton which is consistent with SU(3) prediction [11] and perturbation calculations [12]. The decay rates of five unobserved two-body photonic decay of hyperons are also estimated in the same approach. In these estimates the weak axial parameters are adjusted to those obtained from the semi-leptonic decay of hyperons [13] while those for the magnetic form factors of the electromagnetic interaction are adjusted to the SU(6) result [14].

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We assumed an unsubtracted dispersion relation, additional neutral Cabibbo currents to SS-Hamiltonian and an octet dominance hypothesis in conformity with previous analyses [7, 8][2]).

In Sec. 2 we shall show that the S-matrix element of photonic decay associated with the weak hadronic decay can be described as an equal-time commutator of the weak Hamiltonian and electromagnetic current in a proper approximation and apply it to the decay processes discussed above in Sec. 3. In the last section we summarize our result.

2. Photonic Decay associated with Weak Interaction

The weak hadronic-decay Hamiltonian (density) of SUZUKI and SUGAWARA $H_w(x)$ is defined by

$$H_w(x) = \frac{G}{\sqrt{2}} \left(J_\alpha(x)\, J^*_\alpha(x) + J^*_\alpha(x)\, J_\alpha(x)\right), \tag{1}$$

where G $(= 1.03 \cdot 10^{-5} m_p^{-2})$ is the Fermi coupling constant and $J_\alpha(x)$ is a hadronic current, transforming as an octet Cabibbo current

$$J_\alpha = (J^1_\alpha + i J^2_\alpha + J^3_\alpha) \cos\theta + (J^4_\alpha + i J^5_\alpha + J^6_\alpha + i J^7_\alpha) \sin\theta$$

and

$$J^i_\alpha = J^{V\,i}_\alpha + J^{A\,i}_\alpha \,. \tag{2}$$

Here superscripts i denote the SU(3) transformation property, V and A signify the vector and the axial-vector currents, and θ $(= 0.26)$ is the Cabibbo angle. It is natural to exclude an iso-singlet neutral current in conformity with the success of the iso-triplet current hypothesis [16]. Substituting (2) into (1) the right-hand side splits into four parts $H_{p.c.}$, $H_{p.v.}$, $H^0_{p.c.}$ and $H^0_{p.v.}$ where the first two are strangeness-changing parity-conserving and parity-violating parts of the Hamiltonian, proportional to the product $\cos\theta\, \sin\theta$, and the last two are strangeness-non-changing parts. We are concerned with the first two parts in this paper.

Let us consider the decay of a strange particle β into a particle α and a photon γ

$$\beta \to \alpha\,\gamma \tag{3}$$

induced by H_w. The S-matrix element may be written[3]) in the lowest order of weak and electromagnetic interaction $H_\gamma(x)$ as

$$\begin{aligned}
\langle \alpha\, \gamma_{out} \mid \beta_{in} \rangle &= \frac{(-i)^2}{2!} \iint d^4x\, d^4x' \quad \langle \alpha\, \gamma_{in} \mid P(:H_w(x):\ :H_\gamma(x')':) \mid \beta_{in} \rangle \\
&= \frac{(-i)^3\, e}{2} \iint d^4x\, d^4x' \quad \langle \alpha\, \gamma_{in} \mid T(:H_w(x):\ :J^V_\nu(x'):\, A_\nu(x')) \mid \beta_{in} \rangle \\
&= \frac{(-i)^3\, ie}{2}\, \frac{1}{\sqrt{2\,\omega}}\, e^{(\lambda)}_\mu \iint d^4x\, d^4x' \int d^3y\, e^{-iky}\, \overleftrightarrow{\partial}_{y_0} \\
&\quad \cdot \langle \alpha_{in} \mid T(:H_w(x):\ :J^V_\nu(x'):\, A_\nu(x')) \cdot A_\mu(y)_{out} \mid \beta_{in} \rangle \,.
\end{aligned}$$

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[2]) The smallness of the decuplet contribution is largely due to the properties of the spin-projection operator of Rarita-Schwinger particle. A large cancellation was also noticed when it was treated as a virtual particle [15].

[3]) The notation used without explanation will be found in the standard text of Quantum Electrodynamics [17].

Here e is the electric charge, $e^{(\gamma)}$ is the polarization vector of the photon, ω is its energy, and use is made of $A_\mu(y)_{in}\, S = S\, A_\mu(y)_{out}$ (S is an S-matrix). Integrating over y_0 and dropping a non-contributing counter term we find

$$\langle \alpha\, \gamma_{out} \mid \beta_{in} \rangle = \frac{(-i)^2\, e}{2} \frac{1}{\sqrt{2\,\omega}}\, e_\mu^{(\lambda)}$$

$$\cdot \iint d^4x\, d^4x' \int d^4y\, \partial_{y_0} \left\{ e^{-iky}\, \overleftrightarrow{\partial}_{y_0} \langle \alpha_{in} \mid T(:H_w(x):\ :J_\nu^V(x'):) \mid \beta_{in} \rangle \cdot \frac{1}{2}\, \delta_{\mu\nu}\, D_F\,(y - x') \right\}$$

$$= \frac{ie}{2} \frac{1}{\sqrt{2\,\omega}}\, e_\mu^{(\lambda)} \iint d^4x\, d^4y\, e^{-iky} \langle \alpha_{in} \mid T(:H_w(x):\ :J_\mu^V(y):) \mid \beta_{in} \rangle \cdot \qquad (4)$$

Making use of the integral representation of $\theta(x)$ function and completeness relation in between H_w and J_μ^V we find

$$\langle \alpha\, \gamma_{out} \mid \beta_{in} \rangle = -\frac{e}{2} \frac{1}{\sqrt{2\,\omega}}\, e_\mu^{(\lambda)}\, (2\,\pi)^7\, \delta^4\, (k + p_\alpha - p_\beta)$$

$$\cdot \sum_n \Big[\langle \alpha_{in} \mid H_w(0) \mid n \rangle \langle n \mid J_\mu^V(0) \mid \beta \rangle \frac{\delta\,(\boldsymbol{p}_n - \boldsymbol{p}_\alpha)}{E_\alpha - E_n}$$

$$- \langle \alpha_{in} \mid J_\mu^V(0) \mid n \rangle \langle n \mid H_w(0) \mid \beta \rangle \frac{\delta\,(\boldsymbol{p}_n - \boldsymbol{p}_\beta)}{E_n - E_\beta} \Big] \cdot \qquad (5)$$

We now calculate the matrix element of an equal-time commutator of H_w and J_μ^V, following the method of Fubini, Furlan and Rossetti [18].

$$\langle \alpha \Big| \Big[\int d^3x\, H_w(x, 0), \int d^3y\, J_\mu^V(y, 0) \Big] \Big| \beta \rangle = \int d^3x \int d^4y\, \langle \alpha \Big| \Big[H_w(x, 0)\,, \frac{\partial\, J_\mu^V(y)}{\partial\, y_0} \Big]$$

$$\mid \beta \rangle\, \theta\,(-\, y_0)\,.$$

Again making use of the completeness relation, performing the integration with respect to y and x, and replacing the energy differences arising from the numerators in the first and the second term of the commutator by the photon energy $E_n - E_\alpha = E_\beta - E_n' = +\,\omega$ we find finally

$$\langle \alpha \Big| \Big[\int d^3x\, H_w(\boldsymbol{x}, 0), \int d^3y\, J_\mu^V(\boldsymbol{y}, 0) \Big] \Big| \beta \rangle = (2\,\pi)^6\, \delta(\boldsymbol{p}_\alpha - \boldsymbol{p}_\beta)\, \omega$$

$$\cdot \sum_n \Big[\langle \alpha_{in} \mid H_w(0) \mid n \rangle \langle n \mid J_\mu^V(0) \mid \beta \rangle \frac{\delta\,(\boldsymbol{p}_n - \boldsymbol{p}_\alpha)}{E_\beta - E_n}$$

$$- \langle \alpha_{in} \mid J_\mu^V(0) \mid n \rangle \langle n \mid H_w(0) \mid \beta \rangle \frac{\delta\,(\boldsymbol{p}_n - \boldsymbol{p}_\beta)}{E_n - E_\alpha} \Big] \cdot \qquad (6)$$

Combining (5) and (6) and taking the limit $p_\alpha\ (= p_\beta) \to \infty$ we reach

$$-\frac{e}{2} \lim_{p_\alpha \to \infty} \frac{1}{\sqrt{2\,\omega}}\, e_\mu^{(\lambda)}\, (2\,\pi)\, \delta\,(\omega + E_\alpha - E_\beta) \frac{\langle \alpha \mid [\int d^3x\, H_\omega(\boldsymbol{x}, 0), \int d^3y J_\mu^V(\boldsymbol{y}, 0)] \mid \beta \rangle}{\delta\,(\boldsymbol{p}_\alpha - \boldsymbol{p}_\beta)}$$

$$= \lim_{p_\alpha \to \infty} \cdot \frac{\langle \alpha\, \gamma_{out} \mid \beta_{in} \rangle}{\delta\,(\boldsymbol{p}_\alpha + k - \boldsymbol{p}_\beta)}\,, \qquad (7)$$

or approximately[4])

$$\langle \alpha\, \gamma_{out} \mid \beta_{in} \rangle = -\frac{e}{(2\,\omega)^{3/2}}\, e_\mu^{(\lambda)} \langle \alpha \mid \left[\int d^3x\, H_w(\boldsymbol{x}, 0), \int d^3y\, J_\mu^V(\boldsymbol{y}, :0) \mid \beta \rangle\, e^{-i\boldsymbol{k}\cdot\boldsymbol{y}} \right. \cdot 2\,\pi\, \delta\,(\omega + E_\alpha - E_\beta)\,. \quad (8)$$

3. Application

Let us define the matrix elements of the effective Hamiltonian, Eq. (8), for the two-body photonic decay of hyperons by $A\ (\beta \to \alpha\gamma)$ and $B\ (\beta \to \alpha\gamma)$ for $H_{p.c.}$ and $H_{p.v.}$ respectively. We have

$$A\ (\beta \to \alpha\, \gamma) = -\frac{e}{(2\,\omega)^{3/2}}\, e_\mu^{(\lambda)} \langle\, \alpha \mid \left[\int d^3x\, H_{p.c.}\,(\boldsymbol{x}, 0), \int d^3y\, J_\mu^V(\boldsymbol{y}, 0) \right] \mid \beta \rangle \cdot e^{-iky}\,. \quad (9)$$

Redefining the right-hand side similar to ref. 1) and 8) we find

$$\begin{aligned} \langle \alpha \mid \left[\int d^3x\, H_w(\boldsymbol{x}, 0),\, d^3y\, J_\mu^V(\boldsymbol{y}, 0) \right] \mid \beta \rangle &= \langle B^{\nu_1} \mid [H_{p.c.}^{\nu_4\nu_5}(0),\, J_\mu^{V\,\nu_3}(0)] \mid B^{\nu_2} \rangle \\ &= \sum_n \langle B^{\nu_1} \mid H_{p.c.}^{\nu_4\nu_5}(0) \mid B^{\nu\,n} \rangle \langle B^{\nu\,n} \mid J_\mu^{V\,\nu_3}(0) \mid B^{\nu_2} \rangle \\ &\quad - \sum_n \langle B^{\nu_1} \mid J_\mu^{V\,\nu_3}(0) \mid B^{\nu\,n} \rangle \langle B^{\nu\,n} \mid H_{p.c.}^{\nu_4\nu_5}(0) \mid B^{\nu_2} \rangle\,, \end{aligned} \quad (10)$$

where SU(3) indices are specified by ν_i, B^{ν_2} and B^{ν_1} are the initial and the final baryons, and $J_\mu^{V\,\nu_3}$ is the electromagnetic current. The summation goes only over octet baryons which satisfy the $|\,\varDelta \boldsymbol{I}\,| = 1$ and 1/2 for $|\,\varDelta S\,| = 0$ and 1 respectively. The matrix element of $H_{p.c.}$ are evaluated by

$$\begin{aligned} \langle B^{\nu_1} \mid H_{p.c.}^{\nu_4\nu_5}(0) \mid B^{\nu\,n} \rangle &= \frac{G}{\sqrt{2}} \cos\theta \sin\theta \\ &\cdot \left[\sum_\nu \begin{pmatrix} 8 & 8 & 8f \\ \nu_4 & \nu & \nu_1 \end{pmatrix} \begin{pmatrix} 8 & 8 & 8f \\ \nu_5 & \nu_n & \nu \end{pmatrix} \langle B^{\nu_1} \,||\, J_\alpha^{V\,\nu_4} \,||\, B^\nu \rangle \langle B^\nu \,||\, J_\alpha^{V\,\nu} \,||\, B^{\nu\,n} \rangle \right. \\ &+ \sum_\nu \begin{pmatrix} 8 & 8 & 8d+8f \\ \nu_5 & \nu & \nu_1 \end{pmatrix} \begin{pmatrix} 8 & 8 & 8d+8f \\ \nu_4 & \nu_n & \nu \end{pmatrix} \langle B^{\nu_1} \,||\, J_\alpha^{A\,\nu_5} \,||\, B^\nu \rangle \langle B^\nu \,||\, J_\alpha^{A\,\nu_4} \,||\, B^{\nu\,n} \rangle \\ &\left. + \nu_4 \leftrightarrow \nu_5 \right], \end{aligned} \quad (11)$$

and similarly for $\langle B^{\nu\,n} \mid H_{p.c.}^{\nu_4\nu_5} \mid B^{\nu_2} \rangle$. Here $\begin{pmatrix} 8 & 8 & 8 \\ a & b & c \end{pmatrix}$ is the SU(3) Clebsch-Gordan coefficient [20] and $\langle B^c \,||\, J_\alpha^a \,||\, B^b \rangle$ is the reduced matrix element. Those for the $J_\mu^{V\,\nu_3}$ are defined by

$$\langle B^{\nu\,n} \mid J_\mu^{V\,\nu_3}(0) \mid B^{\nu_2} \rangle = \begin{pmatrix} 8 & 8 & 8d+8f \\ \nu_3 & \nu_2 & \nu_n \end{pmatrix} \langle B^{\nu\,n} \,||\, J_\mu^{V\,\nu_3} \,||\, B^{\nu_2} \rangle\,, \quad (12)$$

and similarly for $\langle B^{\nu_1} \mid J^{V\,\nu_3}(0) \mid B^{\nu\,n} \rangle$. The matrix element for $H_{p.v.}$, $B\ (\beta \to \alpha\gamma)$, can be calculated in a similar fashion by repeating the above procedure.

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[4]) Notice that photon-energy dependence obtained here reproduces the main feature of the infrared spectra in the three-body photonic decay of hyperons in the normal perturbation treatment [19], although we used a completely different approach. A detailed study of the three-body photonic decay of hyperons based on the present formalism gives a further insight on the relevant hadronic decays, F. Ghielmetti and S. Iwao (under preparation).

Except for the SU(3) reduced matrix elements[5]) the ones defined by (11) and (12) may be approximated by

$$\langle B(p_1) \,||\, J^V_\mu + J^A_\mu \,||\, B(p_2)\rangle = \bar{u}(p_1)\,[\gamma_\mu\,(F^V_1 + \gamma_5 F^A_1) - \frac{\sigma_{\mu\nu}\, q_\nu}{2\, m_1}\, F^V_2 + ic\, F^P_2 q_\mu \gamma_5]\, u(p_2)\,, \quad (13)$$

where $q = p_1 - p_2$, and $F^{V,A,P}(q^2)$ are defined by [21, 22]

$$F^{V,A,P}(q^2) = \frac{1}{\left(1 - \frac{q^2}{b}\right)^2}\,, \quad b = 36.2\,\mu_\pi^2, \quad (14)$$

and

$$|\, G_V\, c \,| \approx 7\, G_A\, m_\mu^{-1}\,.$$

Here $G_V = G\cos\theta$ is the vector constant, G_A is the axial constant, and m_μ is the muon mass. In addition to this momentum dependence we understood that the sign and size of the charge and the magnitude of the magnetic moment should be included in our definition of $F_{1,2}$ for the electromagnetic form factors. Assuming an unsubtracted dispersion relation the contribution from the octet baryon intermediate states can be estimated numerically. We find the relative decay rate of $\Sigma^+ \to p\gamma$ to its total rate as

$$\frac{\Gamma\,(\Sigma^+ \to p\gamma)}{\Gamma_{\text{tot}}(\Sigma^+)} = 2.6 \cdot 10^{-4}\,(+\,6.3 \cdot 10^{-6})\,, \quad (15)$$

where numbers out and inside the parentheses are the contribution from $H_{p.c.}$ and $H_{p.v.}$ respectively. In this estimate the magnetic moment of Σ^+ was assumed to that of SU(6) theory [14], and both neutral (charge-non-changing) and charged (charge-changing) weak currents were considered. If we neglect the neutral current we get $3.1 \cdot 10^{-5}$ $(+\,2.1 \cdot 10^{-7})$ corresponding to (15). These results can be understood physically.

In a type of unsubtracted dispersion integral met in our problem the important contribution comes from the low mass intermediate states. Since the charged current supplies a Σ^0 and a Λ while the neutral current is composed of a proton and a Σ^+ intermediate states the main contribution comes from the latter current. The experimental value [10] for the rate (15) is $(3.7 \pm 0.8) \cdot 10^{-4}$. We therefore conclude that

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[5]) In DE SWART sign convention [20] the SU(3) reduced matrix element D and F (for simplicity we represent them by D and F with proper indices) are normalized as follows:

electromagnetic form factor

charge, $D_c = 0, \quad -\frac{1}{\sqrt{3}}\, F_c = 1.$

magnetic, $+\frac{1}{\sqrt{15}}\, D_m = \frac{1}{3}\,, \quad -\frac{1}{\sqrt{3}}\, F_m = \frac{2}{3}\,.$

weak form factor

vector, $D' = 0, \quad \frac{1}{\sqrt{6}}\, F' = 1.$

axial, $-\sqrt{\frac{3}{10}}\, D = 0.74, \quad +\frac{1}{\sqrt{6}}\, F = 0.44.$

the neutral current must be included in the SS Hamiltonian. The decay assymmetry parameter α is given by

$$\alpha = 0.14 . \tag{16}$$

These results are consistent with previous investigations [11, 12].

Table 1

Relative Decay Rates of Two-body Photonic Decay of Hyperons

Process	$\Sigma^+ \to p\,\gamma$	$\Xi^- \to \Sigma^-\gamma$	$\Lambda \to n\,\gamma$	$\Sigma^0 \to n\,\gamma$	$\Xi^0 \to \Lambda\,\gamma$	$\Xi^0 \to \Sigma^0\,\gamma$
amplitude $H_{p.c.}$	-1.12	$+1.12$	.203	.351	.351	.203
amplitude $H_{p.v.}$	$-.020$	$-.007$	0.02	.004	$-.002$	$-.004$
relative rate Γ/Γ_{tot}	$2.7 \cdot 10^{-4}$	$2.2 \cdot 10^{-4}$	$2.3 \cdot 10^{-5}$	$<1.0 \cdot 10^{-9}$	$2.2 \cdot 10^{-5}$	$1.4 \cdot 10^{-5}$

Similar calculations can be done directly, or by making use of SU(3) amplitude relations of Lo [11] by estimating a few more decay processes. The results of calculation thus estimated are tabulated in Table 1. The lines 2 and 3 in Table 1 give the relative amplitudes for $H_{p.c.}$ and $H_{p.v.}$ in a proper normalization. The condition $p_1 \cdot e^{(\lambda)} = p_2 \cdot e^{(\lambda)} = k \cdot e^{(\lambda)} = 0$ in the CM frame simplifies our results. We see that the decay assymmetry parameters are too small to be observed and it would even be difficult to observe the so far unobserved five decay processes.

4. Discussion

We find that the Suzuki-Sugawara Hamiltonian with the additional neutral currents can successfully explain the observed photonic decay rate of $\Sigma^+ \to p\gamma$. The other five decay processes seem to be difficult to observe but may supply an additional confirmation of the theory in the future.

According to the recent Berkeley conference [23] the non-existence of the neutral hadron current is not established so that the result presented in this paper may be of interest.

It seems to us that the phenomenological theory of the weak interaction in the scheme of current algebra has progressed rather successfully but it is desirable to have a further improvement of the theory.

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