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Operational Meaning of Higher-Order Optical Coherence

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Abstract. The definition of higher-order optical coherence given by Glauber is interpreted in terms of the moments of the joint probability distribution of the numbers of photons to be counted in different modes. This makes it clear why normal-ordered operators need to be used for the former while not for the latter.

We consider coherence of light between modes of the radiation field whose harmonic oscillator annihilation and creation operators are a_i and a_i^+ . When expressed in terms of modes rather than space-time points, the definition given by GLAUBER¹⁾ is that light is coherent to the n^{th} order if for each mode i there is a complex number z_i such that

$$\langle a_{i_1}^+ \dots a_{i_m}^+ a_{i_{m+1}} \dots a_{i_{2m}} \rangle = z_{i_1}^* \dots z_{i_m}^* z_{i_{m+1}} \dots z_{i_{2m}} \quad (1)$$

for every $m \leq n$. We may be interested in coherence between modes characteristic of the light source or of devices such as slits which single out various parts of the beam. Coherence to the n^{th} order over a complete set of modes is nevertheless independent of the choice of modes.

We consider photons counted in modes whose annihilation and creation operators are b_r and b_r^+ . These modes are characteristic of the detectors. Their annihilation operators are complex linear combinations

$$b_r = \sum_i u_{ri} a_i \quad (2)$$

of the annihilation operators for the source or slit modes. The moments of the joint probability distribution of the numbers n_r of photons to be counted in the detection modes r are expectation values of products of the operators $b_r^+ b_r$ ²⁾. For example

$$\begin{aligned} \overline{n_r n_s} &= \langle b_r^+ b_r b_s^+ b_s \rangle = \delta_{rs} \langle b_r^+ b_r \rangle + \langle b_r^+ b_s^+ b_r b_s \rangle \\ &= \delta_{rs} \sum_{ij} u_{ri}^* u_{rj} \langle a_i^+ a_j \rangle + \sum_{ijklm} u_{ri}^* u_{sj}^* u_{rk} u_{sm} \langle a_i^+ a_j^+ a_k a_m \rangle. \end{aligned}$$

We go to normal-ordered operators, expand in the operators a_i and a_i^+ , and include only a_i and a_i^+ for excited modes.

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When light is coherent to the n^{th} order, the moments of order $\leq n$ of the joint probability distribution of the numbers n_r of photons to be counted in detection modes r are the same as for a classical field²⁾ with amplitude coefficients z_i in the modes i . This property is sufficient to establish n^{th} order coherence if the expectation values (1) can be determined by measuring the joint moments of the n_r for different choices of detection modes r and for different combinations of the modes i excited.

By considering photons counted in modes r that are different from the modes i between which we consider coherence, and by using the expansion (2), we get the joint moments of the n_r to involve all the expectation values (1), not only those having indices $i_1 \dots i_m$ paired with $i_{m+1} \dots i_{2m}$ which are joint moments of the numbers of photons to be counted in the modes i . Using normal-ordered operators allows us to include just the a_i and a_i^\dagger for excited modes. For the example of light coming through a system of slits, we include only a_i and a_i^\dagger for modes corresponding to the open slits and omit the a_i and a_i^\dagger for modes that are blocked off. Commutator terms from normal-ordering in the n^{th} order joint moments of the n_r bring in expectation values (1) for $m < n$.

This is analogous to the definition of first order coherence in the classical theory³⁾ where the mutual coherence function for two spacetime points is related by the linear propagation of the field to the intensities at different points.

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