Deuteron Triple-Scattering Experiments[1])

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Abstract. Such experiments are needed in general to determine the scattering matrix for deuterons on a spin-0 target. This kind of measurement is familiar in the study of nucleon scattering, but the deuteron spin 1 introduces unfamiliar features.

Introduction

This subject may be unpopular with experimenters, for even double scattering is difficult enough, but sooner or later it will have to be taken seriously.

The scattering amplitude for deuterons at a spinless target is a matrix $\mathfrak{M}$ in spin space, of the form [2]

$$\mathfrak{M} = a + ib\, N_i\, S_i + c\left(N_i\, N_j - \frac{1}{3}\delta_{ij}\right) S_{ij} + d\,(D_i\, D_i - E_i\, E_j)\, S_{ij}, \quad (1)$$

(assuming parity conservation and time-reversal invariance). S is the deuteron spin operator, and $S_{ij} = 1/2\ (S_i\, S_j + S_j\, S_i - 2/3\, \delta_{ij})$. $\boldsymbol{N}, \boldsymbol{D}$ and $\boldsymbol{E}$ are unit vectors in the directions $\boldsymbol{k}_i \times \boldsymbol{k}_f$, $\boldsymbol{k}_i + \boldsymbol{k}_f$ and $\boldsymbol{k}_f - \boldsymbol{k}_i$, respectively, where $\boldsymbol{k}_i$ and $\boldsymbol{k}_f$ are the initial and final deuteron relative momenta (i. e. momenta in the c. m. frame). The complex coefficients a, b, c, and d are functions of energy and scattering angle.

Single- and double-scattering provide five independent measurements at any angle [2, 3], whereas in general[2]) we need seven to determine $\mathfrak{M}$ (apart from the overall phase, which does not enter). More data may be found by triple-scattering experiments. These measure the effects of initial polarization on final polarization. Thus the second scattering is the interesting one: the first and third just produce and analyze polarization, and may be replaced by other processes with the same effects (e. g. polarized ion source, nuclear reaction).

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[1]) A fuller account of this work is found in ref. [1][3]).

[2]) If inelastic processes are negligible, unitarity arguments like those of ref. 4 give four integral constraints on a, b, c and d. Four measurements at all angles can then determine $\mathfrak{M}$, including the phase.

[3]) Numbers in brackets refer to References, page 428.

It would be tedious to list all possible triple-scattering measurements (various combinations of initial polarization and final analysis), and their values in terms of $\mathfrak{M}$. Anyway this is straightforward. Instead we briefly discuss how to control initial polarization, and how to analyze with a spin-0 scatterer.

Formalism

To describe the polarization of a deuteron beam we introduce the spherical tensors Ω_{JM} (see e. g. refs [3, 5], spin operators which transform like the spherical harmonics Y_{JM} and obey

$$\frac{1}{3} \text{ trace } \Omega_{JM}\, \Omega^*_{J'M'} = \delta_{JJ'}\, \delta_{MM'} \,. \tag{2}$$

If their expectation values are $\langle \Omega_{JM} \rangle$, the density matrix ϱ can be written

$$\varrho = \frac{1}{3} \left\{ 1 + \sum_{\substack{J=1,2 \\ M \leqslant J}} \langle \Omega_{JM} \rangle\, \Omega^*_{JM} \right\} , \tag{3}$$

where we have put $\Omega_{00} = 1$, and normalized trace $\varrho = 1$. The eight coefficients $\langle \Omega_{JM} \rangle$ specify the polarization completely.

If the density matrix is ϱ_i initially, it becomes $\varrho_f = \mathfrak{M}\, \varrho_i\, \mathfrak{M}^*$ after scattering. The final expectation value of any spin operator χ is then trace $\varrho_f \chi$ / trace ϱ_f. Thus the new quantities measured by triple-scattering are essentially terms, or sums of terms, like

$$\text{trace } \mathfrak{M}\, \Omega^*_{JM}\, \mathfrak{M}^*\, \Omega_{J'M'} \,, \tag{4}$$

with J, J' $\geqslant$ 0. Ω^*_{JM} and $\Omega_{J'M'}$, which refer to initial and final polarization, need not have the same axes: we prefer to take the axis OZ along the laboratory direction of motion in each case.

Control of Initial Polarization

For nucleons, polarization is simply a vector which can be rotated into any desired direction by magnetic fields. For deuterons, the polarization of a given beam cannot be adjusted arbitrarily, but some manipulation is still possible.

In a magnetic field the spin axes rotate [3, 6], while the polarization relative to them is unchanged. Suppose these axes rotate right-handedly about the direction of motion (in a solenoidal field) through an angle ϕ. Then the density matrix *referred to the initial axes* is

$$\varrho = \frac{1}{3} \left\{ 1 + \sum_{J,M} \langle \Omega^{(i)}_{JM} \rangle\, \Omega^*_{JM}\, e^{iM\phi} \right\} , \tag{5}$$

where $\langle \Omega^{(i)}_{JM} \rangle$ are the initial expectation values.

Hence some components of polarization can in effect be eliminated by averaging over suitable values of ϕ. For instance, we can remove components with $M = \pm 1$ by averaging over $\phi = 0$ and π; $M = \pm 2$ by using $\phi = 0$ and $\pi/2$; both types simultaneously by using $\phi = 0$, $\pi/2$, π and $3\pi/2$.

Other effects can be achieved by rotating the spin axis OZ out of the direction of motion; this is harder, since the deuteron g-factor is close to 1.

Analysis with a Spin-O Scatterer

Consider a spin-0 scatterer which is relatively heavy, so that laboratory and c.m. frames are essentially the same (the case with recoil is similar). Let the incident polarization be given by $\langle \Omega_{JM} \rangle$, with spin axis OZ along the incident motion. Then the differential cross-section proves to be

$$I = I_0 \left\{ 1 + \sum_{J,M} (-1)^M e^{-iM\phi} \langle \Omega^{(0)}_{JM} \rangle \langle \Omega_{JM} \rangle \right\}, \tag{6}$$

where I_0 is the unpolarized cross-section, and $\langle \Omega^{(0)}_{JM} \rangle$ is the polarization that would have been set up in an initially unpolarized beam, referred to the outgoing direction as Z-axis and the normal $\boldsymbol{N}$ as Y-axis. ϕ is the angle between $\boldsymbol{N}$ and the Y-axis of the incident polarization, (measured from the latter, right-handedly about the incident motion).

Let us suppose $\langle \Omega^{(0)}_{JM} \rangle$ are known (from double-scattering [2, 3, 7]; they have the property $\langle \Omega^{(0)}_{JM} \rangle = (-1)^{J+M} \langle \Omega^{(0)}_{J-M} \rangle$. Then Eq. (6) can be re-written

$$\begin{aligned} I = I_0 \{ & 1 - \langle \Omega^{(0)}_{11} \rangle [\langle \Omega_{11} + \Omega_{1-1} \rangle \cos\phi - i \langle \Omega_{11} - \Omega_{1-1} \rangle \sin\phi] \\ & + \langle \Omega^{(0)}_{20} \rangle \langle \Omega_{20} \rangle \\ & - \langle \Omega^{(0)}_{21} \rangle [\langle \Omega_{21} - \Omega_{2-1} \rangle \cos\phi - i \langle \Omega_{21} + \Omega_{2-1} \rangle \sin\phi] \\ & + \langle \Omega^{(0)}_{22} \rangle [\langle \Omega_{22} + \Omega_{2-2} \rangle \cos 2\phi - i \langle \Omega_{22} - \Omega_{2-2} \rangle \sin 2\phi] \}. \end{aligned} \tag{7}$$

Clearly the terms in $\cos\phi$, $\cos 2\phi$, $\sin\phi$, $\sin 2\phi$ and the ϕ-indepe-pendent term can be separated by measurements at suitable values of the azimuth ϕ (holding the scattering angle fixed). And the contributions to $\sin\phi$ and $\cos\phi$ proportional to $\langle \Omega^{(0)}_{11} \rangle$ and $\langle \Omega^{(0)}_{21} \rangle$ can also be separated, by repeating the measurements at another scattering angle. Thus seven of the eight components of incident polarization can be measured in principle, though of course a particular scatterer may be insensitive to some components.

To measure the last component $\langle \Omega_{10} \rangle$, we need to rotate the spin axis OZ out of the direction of motion.

Conclusions

Triple-scattering or equivalent measurements are needed in general to determine $\mathfrak{M}$ for deuterons on spinless nuclei.

Unlike the nucleon case, it is not feasible to arrange for an arbitrary incident polarization at the scattering of interest. However, some control is possible, and certain components can be «averaged out».

Seven out of eight components of polarization can be analyzed with a calibrated spin-0 scatterer. The eighth requires a magnetic field.

Fortunately, only two new independent measurements[4]) are needed, beside single- and double-scattering data. It will not be necessary to arrange a great variety of incident polarizations, nor to measure many final components.

REFERENCES

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[4]) Plus perhaps one or two others not algebraically independent, to resolve ambiguities in the solution for $\mathfrak{M}$.