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# IV.

# Theories Concerning Polarization Effects of Nucleons

# Nuclear Forces and Polarization Phenomena[1])

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Nucleon-nucleon scattering is an important source of information concerning nuclear forces. Additional and at times important information is obtainable from the binding energies of the simpler light nuclei and from nucleon-nucleus scattering. The large value of the pion-nucleon coupling constant in the pseudoscalar meson theory with pseudoscalar coupling (*PSps*) has made it impossible to obtain a practically convergent series for the effective potential and the pseudoscalar theory with pseudovector coupling is not renormalizable. Furthermore, a complete theory must take into account the interactions of $K$-mesons with nuclei which are still poorly understood. The fundamental approaches are thus not available for comparison with experiment.

It would be helpful for data interpretation if it were possible to use a potential describing the interaction between two nucleons. So far no satisfactory potential has been found. The more hopeful approach is, therefore, that of employing phase shifts supplemented by coupling parameters between states with the same total angular momentum $J\,\hbar$ but different orbital angular momenta $L\,\hbar$. This description is completely general for energies below meson production as long as only two nucleons are involved, the phase shifts and coupling parameters, phase-parameters for short, describing the scattering phenomena completely. The phase-parameters do not suffice, however, for systems involving more than two nucleons such as the triton and they similarly are only helpful but not definitive in their predictions in such problems as the photodisintegration of the deuteron which involve the participation of a third entity, a photon in this case. Relativistically the phase-parameters can be [1][2]) uniquely defined in the center of mass system, by describing the nucleons as Dirac particles with anomalous magnetic moments and employing the 'large-large' combinations of the Dirac spinor for two particles at distances larger than those at which the specifically nuclear forces have an appreciable value.

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[2]) Numbers in brackets refer to References, page 357.

At such distances the 'small' Dirac components are obtainable in terms of the 'large' ones, the particles being uncoupled to each other and the phase-parameters of the large components uniquely define the whole wave function. In the case of $p-p$ scattering there is a slight complication in the Coulomb interaction. For large distances, however, this can be treated [1] relativistically and without ambiguity by first order theory. Coulomb effects are minor at high energies and at low energies where they are important, the relativistic effects are small. The phase-parameters employed in the present discussion are defined in this sense.

In the analysis of low energy (a few MeV) $p-p$ and $p-n$ data, reasonable assumptions regarding the absence of serious effects of $L > 0$ make it possible to obtain reliable values of the ${}^1S_0$ phase shift $K_0$ and the parameters for the ${}^3S_1 + {}^3D_1$ system from measurements of the scattering cross section $\sigma$. At higher energies the number of phase-parameters to be considered is so large that their determination from $\sigma$ alone becomes impossible. The polarization of the scattered particles is of help in restricting the possibilities to a smaller number and in indicating qualitatively the presence or absence of phase shifts. The information derivable from measurements of polarization in the scattering from unpolarized targets is essentially of two types: polarization properties of one or another of the two particles participating in the collision (type A), or else the combined orientation of the spins of the two particles (type B). For a spin 1/2 particle all information concerning the statistical mixture of spin functions can be summarized in a single vector

$$\boldsymbol{P} = \overline{\langle \boldsymbol{\sigma} \rangle} \, .$$

Here $\boldsymbol{\sigma}$ is the vector Pauli spin operator, $\langle \; \rangle$ denotes the expectation value and $\overline{\phantom{x}}$ the statistical average over the statistical mixture of states. Disregarding particle identity all information derivable from experiments with unpolarized targets in category (A) is obviously contained in $\boldsymbol{P}$. In category (B) additional information regarding the correlation of the spin directions of particles and recoils is available. This classification is useless for polarized targets.

In a *double scattering* experiment the partly polarized nucleons produced in one scattering are scattered by a second unpolarized target. If the energy change due to recoil is small, the differential scattering cross section is

$$\sigma_\Omega^{(2)} = [\sigma_\Omega^{(2)}]_0 \, [1 + (\boldsymbol{P}_1 \cdot \boldsymbol{P}_2)] \, .$$

Here $[\sigma_\Omega^{(2)}]_0$ is the value of $\sigma_\Omega^{(2)}$ for an unpolarized beam, $\boldsymbol{P}_1$, $\boldsymbol{P}_2$ are respectively the polarization vectors produced by scattering unpolarized beams on the first and second targets. This relation has been proved in the work by WOLFENSTEIN [2], DALITZ [3] and of WOLFENSTEIN and ASHKIN [2]. The proof makes use of invariance of interaction energy to time reversal. The directions of $\boldsymbol{P}_1$ and $\boldsymbol{P}_2$ are perpendicular to the scattering planes so that

$$\boldsymbol{P}_1 = P_1\,\boldsymbol{n}_1,\ \boldsymbol{P}_2 = P_2\,\boldsymbol{n}_2$$

where the target characterizing quantities $P_1$ and $P_2$ are either the absolute values of $\boldsymbol{P}_1$ and $\boldsymbol{P}_2$ or their negatives. It has been pointed out by R. J. N. PHILLIPS [4] and independently by R. R. LEWIS [5] that on this basis, time reversal invariance could be tested more systematically in nucleon-nucleon scattering. Special tests for time reversal and other symmetries lie outside the scope of the present report. Reference may be made to the work of PHILLIPS and that of BELL and MANDL [6] for theory, ABASHIAN and HAFNER [7] for $p - p$ scattering experiments and to that of L. ROSEN and J. E. BROLLEY [8] who have tested the '$P - A$' relationship in a number of cases. The usual determination of $P = \pm\,|\boldsymbol{P}|$ by means of the asymmetry of double scattering rests on the validity of the formula quoted. The knowledge of $\boldsymbol{P}_2$, obtainable for instance by scattering from carbon gives for a known angle between $\boldsymbol{P}_1$ and $\boldsymbol{P}_2$ the value of $P_1$. A double scattering experiment is incapable of giving more than $P_1$ and its space direction which is known to be perpendicular to the first scattering plane. *Triple scattering* experiments determine what happens to the mean spin $\boldsymbol{P}/2$ if a beam is scattered. The first scattering produces a polarized beam with known $\boldsymbol{P}$, the second changes this $\boldsymbol{P}$ and the third determines the changed $\boldsymbol{P}$. The first and third scatterings serve as polarizer and analyzer respectively. Omitting subscripts on quantities referring to the second scattering, the relation for determining the depolarization parameter $D$ illustrated in figure 1 is

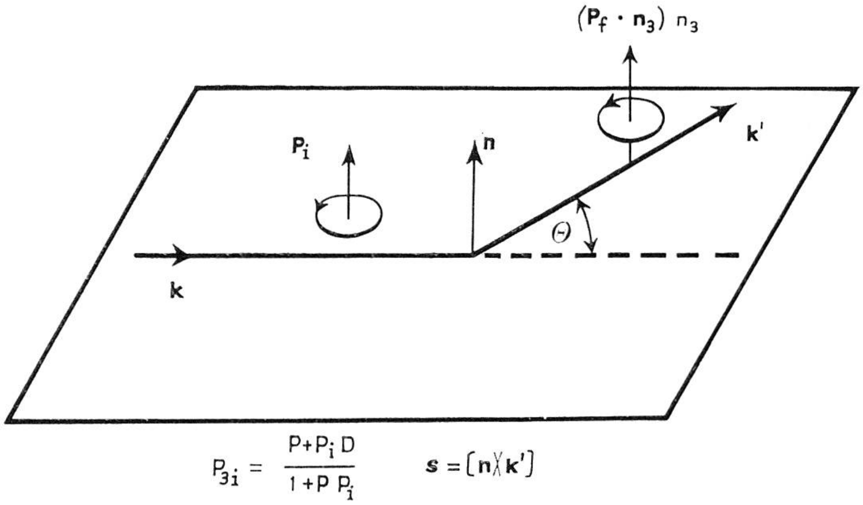


Figure 1

The three scattering planes are the same in this measurement and subscript $i$ indicates incidence. Thus $P_{3i}$ is the $P$ of the beam incident on the third scatterer, $P$ is the inherent $P$ of the second scatterer and $P_i$ is the polarization of the beam incident on the second scatterer.

In this and the following two figures $\boldsymbol{k}$ and $\boldsymbol{k}'$ are the initial and final propagation vectors for the second scattering while

$$\boldsymbol{s} = [\boldsymbol{n} \times \boldsymbol{k}'] .$$

The parameter $R$ is measured by having the first and third scattering planes perpendicular to the second. It determines the rotation of $\boldsymbol{P}$ caused by the second scattering, with $\boldsymbol{P}$ in the scattering plane and perpendicular to $\boldsymbol{k}$. In this case as shown in figure 2

$$P_{3i} = R\, P_i .$$

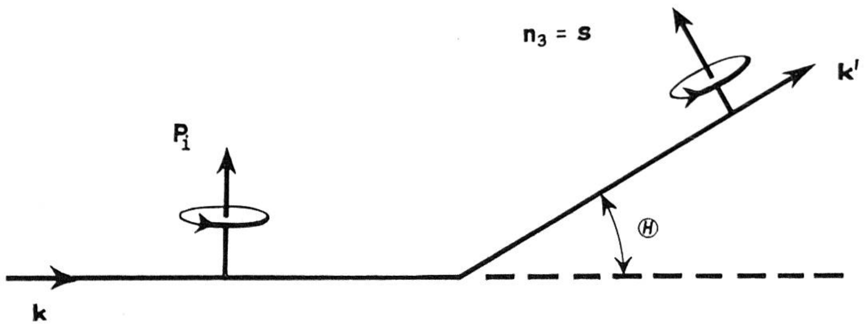


Figure 2

The parameter $A$ is measured by turning the polarization which is produced by the first scattering by means of a magnetic field to be directed along $\boldsymbol{k}$. The asymmetry of the third scattering again measures the polarization along $\boldsymbol{s}$, the third scattering plane being perpendicular to the second.

The measurement of the triple scattering parameter $A$ is illustrated in figure 3.

$$P_{3i} = A\, P_i .$$

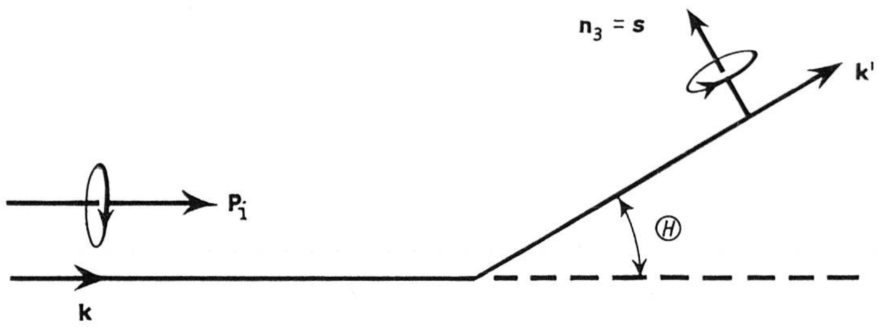


Figure 3

Category (B) type information is derived from spin- correlation experiments and is much less abundant. Double scattering is used in order to determine the spin correlation of the scattered and recoil particles. The theory of such experiments has been first treated by STAPP [9]. The two coefficients most commonly measured are $C_{nn}$ and $C_{KP}$. Both are concerned with unpolarized targets. The notation is

$$C_{nn} = \overline{\langle(\boldsymbol{\sigma}_1 \cdot \boldsymbol{n})\,(\boldsymbol{\sigma}_2 \cdot \boldsymbol{n})\rangle}\,, \quad \boldsymbol{K} = \frac{\boldsymbol{k}' - \boldsymbol{k}}{|\boldsymbol{k}' - \boldsymbol{k}|}\,,$$

$$C_{KP} = \langle(\boldsymbol{\sigma}_1 \cdot \boldsymbol{K})\,(\boldsymbol{\sigma}_2 \cdot \boldsymbol{P})\rangle\,, \quad \boldsymbol{P} = \frac{\boldsymbol{k}' + \boldsymbol{k}}{|\boldsymbol{k}' + \boldsymbol{k}|}\,,$$

with $\boldsymbol{k}$ and $\boldsymbol{k}'$ standing for the initial and final momenta of the scattered particle in the center of mass system.

The possibilities of deriving information from measurements at a given angle have been considered by SMORODINSKY, OEHME, STAPP, BROWN and KANELLOPOULOS, by PUSIKOV, RYNDIN and SMORODINSKY, by GOLOVIN *et al.* and most completely by R. J. N. PHILLIPS [9] who will report on these matters in more detail later in this session. The report by GAMMEL and BROLLEY is also closely concerned with this phase of the subject. There are 5 complex coefficients entering the expression for the scattering matrix $T$. Of the 10 real constants that determine the spin properties of the nucleons, one determines the common phase of all terms. The most complete investigation of spin conditions in scattering consists in the determination of the nine remaining parameters. Usual measurements furnish only $\sigma$, $P$, $D$, $R$, $A$ and in some cases $C_{nn}$ and $C_{KP}$ giving most frequently 2 in some cases 5 and rarely 6 or 7 pieces of information. In the references quoted experiments with polarized targets are considered. PHILLIPS gives twenty five linearly independent expressions which enter the different measurements. With only nine measurements he finds that the transition matrix $T$ (WOLFENSTEIN's $M$) is not fully determined. In practice it has not proved possible so far to analyze data by such general procedures.

According to PUSIKOV, RYNDIN and SMORODINSKY if scattering is elastic the unitary character of the scattering matrix makes the measurement of five quantities *at all angles* sufficient for the determination of $T$. This interesting fact may eventually prove useful. In practice the method involves the solution of an integral equation which has apparently not been applied to data analysis. The errors introduced by lack of knowledge of measured quantities at $\theta \simeq 0°$ and $\theta \simeq 180°$ are also apparently unknown. In a phase-parameter analysis nearly the equivalent of such information is supplied by reasonable hypotheses concerning phase-parameters with high $L$ and unitarity is automatically satisfied.

Although the systematic determination of $T$ by means of polarization measurements has not proved possible, they are, nevertheless, very useful in restricting the fits obtained through the introduction of phase parameters. Measurements of $P$ at 290 MeV, for example, have shown [10] that phase shifts for $L > 2$ are necessary and [11] that $^3F$ waves in addition to those arising from $^3P_2 - {}^3F_2$ coupling are needed at this energy.

Data analysis is carried out either with or without assumed potentials. The employment of potentials [12] has proved only moderately successful. In place of potentials the following methods have also been used: a) boundary value treatment [13], b) dispersion relations [14], c) phenomenological fits [15]. Information derived regarding phase-parameters has been compared [16] with nucleon-nucleus scattering data making use of various forms of the impulse approximation. While satisfactory agreement is obtained, such comparisons have not proved very informative regarding preferences for one or another nucleon-nucleon phenomenologic fit.

The $p-p$ phenomenologic fits indicate at 310 MeV differences in phase shifts for $^3P_{0,1,2}$ suggesting $\boldsymbol{L} \cdot \boldsymbol{S}$ interaction. If these differences are analyzed in terms of those expected as first order effects of the spin-orbit and tensor potentials, the necessity of including the former becomes apparent. It would be unjustifiable to conclude that there is present in the Hamiltonian a term of the $V_{LS}(r)\ (\boldsymbol{L} \cdot \boldsymbol{S})$ type because in higher orders the tensor interaction (a multiple of the usual $S_{12}$) can produce similar effects. The likelihood of the presence of spin-orbit interactions is, nevertheless, increased by this fact.

This situation has a direct connection with polarization. It may in fact be shown [17] that quite generally the first order effect of the tensor interaction gives no polarization as a consequence of the identity

$$\mathrm{Tr}\,\{(\sigma_{1z} + \sigma_{2z})\ S_{12}\} = 0\ .$$

If one were sure that higher order effects of $S_{12}$ are sufficiently small, the occurrence of $P \neq 0$ would, therefore, prove the existence of interaction terms in $\boldsymbol{L} \cdot \boldsymbol{S}$ within the limitations of the potential concept.

Some comparisons between calculated and observed quantities will be shown. The notation for phase-parameters will first be explained. For singlet states, non-relativistically the phase shift $K_L$ is specified in terms of the asymptotic form of the radial function $\mathfrak{F}_L/r$ by

$$\mathfrak{F}_L \sim \sin\left[kr - \frac{L\pi}{2} - \eta \ln 2\,kr + \arg \Gamma\,(L + 1 + i\,\eta) + K_L\right]$$

where $k = M\,v/2\,\hbar = 2\,\pi$ times wave number, $\eta = e^2/\hbar\,v$, $v =$ relative velocity, $r$ = relative distance of nucleons. In the case of two states with the same $J$ but different $L$, different notations are employed in the literature. The results will be presented in terms of the scattering matrix $U$ introduced so that the asymptotic forms at large $r$ for $r$ times the radial function are

$$\begin{cases} \left[-e^{-i\varphi_L} + U_{L,\,L}\, e^{i\varphi_L}\right] \mathfrak{Y}_\mu^{L,J} + U_{L,\,L+2}\, e^{-i\varphi_{L+2}}\, \mathfrak{Y}_\mu^{L+2,\,J}\,, \\ U_{L+2,\,L}\, e^{i\varphi_L}\, \mathfrak{Y}_\mu^{L,J} + \left[-e^{-i\varphi_{L+2}} + U_{L+2,\,L+2}\, e^{i\varphi_{L+2}}\right] \mathfrak{Y}_\mu^{L+2,\,J} \end{cases}$$

where

$$\varphi_L = kr - \eta \ln 2\,kr - \frac{L\,\pi}{2} + \arg\Gamma\,(L + 1 + i\,\eta), \quad J = L + 1\,.$$

Here the $\mathfrak{Y}_\mu^{L,J}$ are spin angular functions defined in a standard manner. These forms give for an ingoing wave of amplitude $-1$ in channel with orbital angular momentum $L$ an outgoing wave of amplitude $U_{L,L}$ in the same channel and an outgoing wave of amplitude $U_{L,\,L+2}$ in the channel with orbital momentum $L + 2$. Similarly the second line of the defining form gives the amplitudes in the case of incidence in channel $L + 2$. The matrix $U$ is frequently parametrized following BLATT and BIEDENHARN [18] in the form

$$U = \begin{pmatrix} c_\varepsilon^2\, e^{2i\,\delta_\alpha} + s_\varepsilon^2\, e^{2i\,\delta_\beta}, & c_\varepsilon\, s_\varepsilon\, (e^{2i\,\delta_\alpha} - e^{2i\,\delta_\beta}) \\ c_\varepsilon\, s_\varepsilon\, (e^{2i\,\delta_\alpha} - e^{2i\,\delta_\beta}), & s_\varepsilon^2\, e^{2i\,\delta_\alpha} + c_\varepsilon^2\, e^{2i\,\delta_\beta} \end{pmatrix},$$

$$c_\varepsilon \equiv \cos\varepsilon,\ s_\varepsilon \equiv \sin\varepsilon\,.$$

The quantities $\delta_\alpha$, $\delta_\beta$ have the significance of eigenphase-shifts while $\varepsilon$ determines the coupling between the two eigenstates. For eigenstates $\alpha, \beta$ the common phase shift in both channels is $\alpha$ or $\beta$ respectively. In the fits to nucleon-nucleon scattering made by the Yale group the parametrization has been

$$U = \begin{pmatrix} \sqrt{1 - \varrho^2}\, e^{2i\,\theta_1}, & i\,\varrho\, e^{i(\theta_1+\theta_2)} \\ i\,\varrho\, e^{i(\theta_1+\theta_2)}, & \sqrt{1 - \varrho^2}\, e^{2i\,\theta_2} \end{pmatrix}.$$

The second row and column refer to the higher of the two $L$. The two notations are connected by

$$\theta_1 + \theta_2 = \delta_\alpha + \delta_\beta, \qquad \tan\,(\theta_1 - \theta_2) = \cos 2\,\varepsilon \tan\,(\delta_\alpha - \delta_\beta),$$

$$\varrho = \sin\,(2\,\varepsilon)\sin\,(\delta_\alpha - \delta_\beta)\,;$$

this notation was introduced on account of the convenience of possible generalizations. It turned out to be very close to the 'nuclear bar' notation of the Berkeley group and is related to it by

$$\varrho = \sin 2\,\bar{\varepsilon}_J\,,\ \ \theta_1 = \bar{\delta}_{J-1}\,,\ \ \theta_2 = \bar{\delta}_{J+1}\,.$$

The letter $K$ is reserved for phase shifts of the singlet states and $\delta_J^L$ for phase shifts of triplet states with orbital and total angular momenta $L$ and $J$ in the uncoupled cases.

WOLFENSTEIN and ASHKIN [2] making use of invariance considerations have obtained the most general forms of the spin transition matrix $T$. The requirements were those of invariance of the Hamiltonian to time reversal, space reflections and space rotations. Their form is shown below and so are the relations between elements of $S$, the submatrix of $T$ referring to triplet states [19]. The first equation follows from time reversal and the last four from space reflections [20].

The most general form is

$$T = A + B\,(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) + C\,\big((\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2)\cdot \boldsymbol{n}\big) + D\,\big((\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2)\cdot \boldsymbol{n}\big) +$$
$$+ E\,(\boldsymbol{\sigma}_1 \cdot \boldsymbol{K})\,(\boldsymbol{\sigma}_2 \cdot \boldsymbol{K}) + F\,(\boldsymbol{\sigma}_1 \cdot [\boldsymbol{n} \times \boldsymbol{K}])\,(\boldsymbol{\sigma}_2 \cdot [\boldsymbol{n} \times \boldsymbol{K}])$$

where

$$\boldsymbol{K} = \boldsymbol{k}_f - \boldsymbol{k}_i$$

and $A, B, C, D, E, F$ are constants. In the same manner they establish relations between elements of $S$, the $3 \times 3$ submatrix of $T$ refering to triplet states as follows

$$S_{1,1} - S_{0,0} - e^{2i\varphi}\,S_{1,-1} = 2^{1/2}\,(e^{-i\varphi}\,S_{0,1} + e^{i\varphi}\,S_{1,0})\cot\theta$$

which follows from time reversal and

$$S_{1,1} = S_{-1,-1},\ e^{-i\varphi}\,S_{-1,0} = -\,e^{i\varphi}\,S_{1,0},\ e^{-i\varphi}\,S_{0,1} =$$
$$= -\,e^{i\varphi}\,S_{0,-1},\ e^{2i\varphi}\,S_{1,-1} = e^{2i\varphi}\,S_{-1,1}$$

which involve the use of space reflections.

The explicit forms of the matrix elements are shown hereafter and so is the way in which the symmetry relations are satisfied by these relations as well. Only 4 of the 5 quantities $\alpha_1$, $\alpha_2$, $\alpha_3$, $\alpha_4$, $\alpha_5$ are linearly independent. Modifications needed when coupling between states with the same

$J$ and different $L$ is considered are not shown, being readily available in the literature.

$$k\,(S_{1,1} - S^c) = k\,(S_{-1,-1} - S^c) = \alpha_2\, e^{i\Phi} = e^{i\Phi} \sum_L \frac{1}{2}\, e_{L0}\,[(L+2)\, Q_{L,L+1} +$$

$$+ (2\,L+1)\, Q_{L,L} + (L-1)\, Q_{L,L-1}]\; P_L\,(\cos\theta)\,,$$

$$-\,k\,S_{0,1}\, e^{-i\varphi} = k\,S_{0,-1}\, e^{i\varphi} = -\,2^{-1/2}\,\alpha_1\,\sin\theta\, e^{i\Phi} =$$

$$= 2^{-1/2}\, e^{i\Phi} \sum_L e_{L0}\,[L\,(L+2)\, Q_{L,L+1} -$$

$$-\,(2\,L+1)\, Q_{L,L} - (L^2-1)\, Q_{L,L-1}]\; \frac{\sin\theta\, P_L'}{[L\,(L+1)]}\,,$$

$$k\,S_{-1,1}\, e^{-2i\varphi} = k\,S_{1,-1}\, e^{2i\varphi} = \alpha_3\,\sin^2\theta\, e^{i\Phi} =$$

$$= \frac{1}{2}\, e^{i\Phi} \sum_L e_{L0}\,[L\, Q_{L,L+1} - (2\,L+1)\, Q_{L,L} + (L+1)\, Q_{L,L-1}] \times$$

$$\times\; \frac{\sin^2\theta\, P_L''}{[L(L+1)]}\,,$$

$$k\,S_{1,0}\, e^{i\varphi} = -\,k\,S_{-1,0}\, e^{i\varphi} = 2^{-1/2}\,\alpha_4\,\sin\theta\, e^{i\Phi} = 2^{-1/2}\, e^{i\Phi} \times$$

$$\times \sum_L e_{L0}(Q_{L,L+1} - Q_{L,L-1})\,\sin\theta\, P_L'\,,$$

$$k\,(S_{0,0} - S^c) = \alpha_5\, e^{i\Phi} = e^{i\Phi} \sum_L e_{L0}\,[(L+1)\, Q_{L,L+1} + L\, Q_{L,L-1}]\; P_L\,,$$

$$e_{L0} = \exp\,[2\,i\,\sigma_{L,0}\,(\eta)],\; \sigma_{L,0}\,(\eta) = \sigma_L\,(\eta) - \sigma_o\,(\eta) = \sum_{s=1}^{L} \operatorname{arc\,tan} \frac{\eta}{s}$$

$$\Phi = \varrho - \eta\,\log 2\,\varrho + 2\,\sigma_0\,(\eta), \qquad Q_{L,J} = Q\,(\delta_{L,J})$$

$$Q(\delta) = \frac{e^{2i\delta} - 1}{2\,i} = e^{i\delta}\,\sin\delta, \qquad s = \sin\left(\frac{\theta}{2}\right).$$

The Wolfenstein-Ashkin relations are obviously satisfied by these explicit expressions and the time reversal condition corresponds to the identity

$$\alpha_2 - \alpha_5 - \alpha_3\,\sin^2\theta = (\alpha_1 + \alpha_4)\,\cos\theta$$

$$S^c = -\,\frac{\eta}{2\,ks^2}\, e^{i\,(\Phi - \eta\,\log s^2)}\,.$$

For an unpolarized incident beam the polarization compenent $P_1^y$ and the differential cross section $\sigma_\Omega$ are given by

$$P_1^y = \langle \sigma_1^y \rangle = \frac{1}{2^{3/2}\,\sigma_\Omega} \operatorname{Im} \sum_m (S_{1,m} - S_{-1,m})^* \, S_{0,m}$$

$$k^2 \, P_1^y \, \sigma_\Omega = \frac{1}{2} \sin\theta \cos\varphi \operatorname{Im} \{\alpha_1 (\alpha_2 + \alpha_c)^* - \alpha_1 \alpha_3^* \sin^2\theta + (\alpha_5 + \alpha_c) \, \alpha_4^*\}$$

$$\alpha_c = -\frac{\eta}{2\,s^2} \, e^{-i\eta \log s^2}$$

$$4\,\sigma_\Omega = |T_{0\,0}^{\,s}|^2 + \sum_{\mu,\nu} |T_{\mu\nu}|^2 \, .$$

The statistical average of the expectation value of the component of the spin of one of the particles is readily obtainable as seen in the following. The calculation of other scattering parameters is also straightforward. The fits have been made [21] by employing a gradient search in the space of the phase-parameters to data at all available energies from 9 to 340 MeV and employing one pion exchange potential OPEP values for the higher $L$, a procedure initiated at selected energies by MORAVCSIK, CZIFFRA, MAC GREGOR and STAPP [15]. The number of phase-parameters searched for in the $p-p$ case was 11 if OPEP was used for $L \geq 5$, but in some searches OPEP parameters were used also for $\delta_3^F$, $\theta_4^F$, $\varrho_4$ and $K_4$. For $\tau = 0$ in the $n-p$ case the choice of OPEP parameters was similarly made.

In figure 4 there are shown preliminary versions of phase shifts $K_0$, $K_2$ obtained by fitting $p-p$ data, in figure 5 $K_4$ and $\varrho_4$ are similarly shown. The notation YRBl refers to searches using as a starting point the Signell-Marshak potential up to 150 MeV and the Stapp, Ypsilantis, Metropolis number 1 fit at 310 MeV, YLA to searches with the Gammel-Thaler potential as a start and YAVG to an average of the searched values. The dashed curves show preliminary error limits and the values shown in the graphs have been improved regarding accuracy of representing data. The width of error bands has also been considerably reduced.

In figure 6 is shown a comparison for $p-p$ of $P(\theta)$ and of triple scattering parameters $A(\theta)$ and $D(\theta)$ with experiment at various energies. At 147 MeV only the Harvard data are shown. The general tendency of the fits has been to give values intermediate between Harvard and Harwell but the improved YLA type fit favors Harvard. The notation 'not searched' in the figures means that the particular datum has not been included in the search. In figure 7 is shown a comparison with experiment of $P(\theta)$ for $n-p$ at 310 MeV. The figure also shows a curve for the same quantity computed for one of the Gammel-Thaler unpublished potential versions for $n-p$. The same figure shows comparison with data for $\sigma(\theta)$, $p-p$, at 250 MeV employing fit YLAM and the

Gammel-Thaler published potential. In figure 8 are shown plots of $P(\theta)$, $p-p$ at 20°, 45° and 80° as a function of energy and also $R(\theta)$, $p-p$ at 140, 210, 312 MeV as a function of angle. The plots against energy in these and other cases produce the impression that there may still be present some systematic errors which differ for various groups of observers. Many more comparisons with experiment are available in the work at Yale, only a few of the representative plots having been shown.

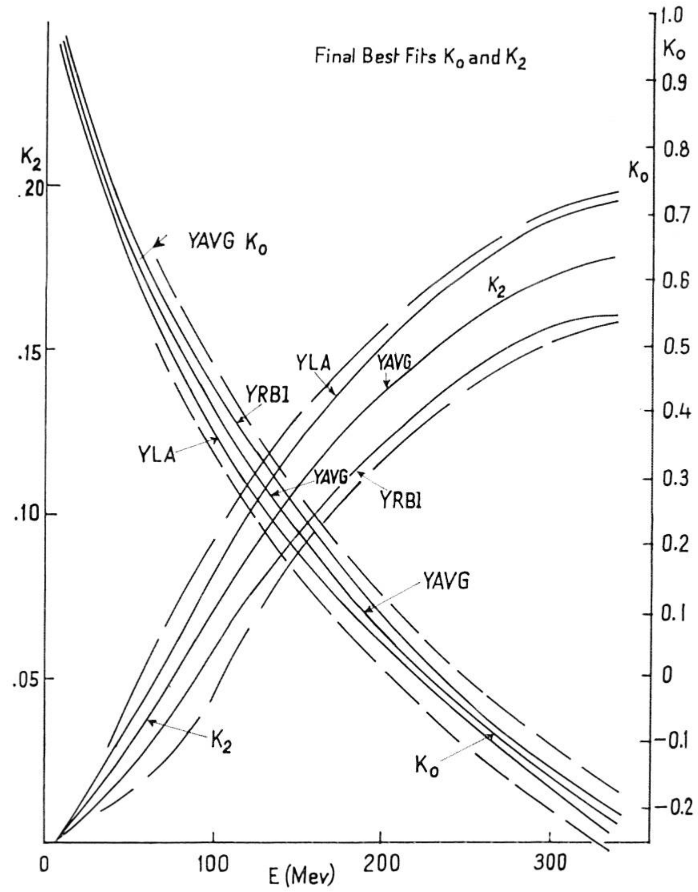


Figure 4

In a phase parameter adjustment it is helpful to know how $p$-waves behave when they are small. Although low energy (a few MeV) $p-p$ and $n-p$ data can be well represented by means of $s$-waves alone with due account of vacuum polarization, it has been shown by HULL and SHAPIRO [22] and confirmed by MAC GREGOR [23] that it is possible to represent the data by admitting $p$-waves in the analysis and that appreciable differences between the three $^3P$ phase shifts are admissible resulting in appreciable polarization. The fits YRBl, YLA and the others mentioned in this report give very small polarizations. Thus at 18.2 MeV and

$\theta = 50°$ the calculated values are 0.06% and 0.08% respectively for a modification of YRBl and another version of the YRB search procedure. These may be compared with the Blanpied [24] measured value of $(0.6 \pm 0.5)\%$ at 16.0 MeV at $\theta = 12.5°$ and the 3.3 MeV values of ALEXEFF and HAEBERLI [25] of $(0.08 \pm 0.16)\%$ at $\theta = 30°$, $(0.25 \pm 0.16)\%$ at 45°, $(0.59 \pm 0.24)\%$ at 53°. These values have not been included in the searches for phenomenologic fits reported on. There is likely to be difficulty in reconciling the larger values in these difficult experiments with potentials, currently in vogue.

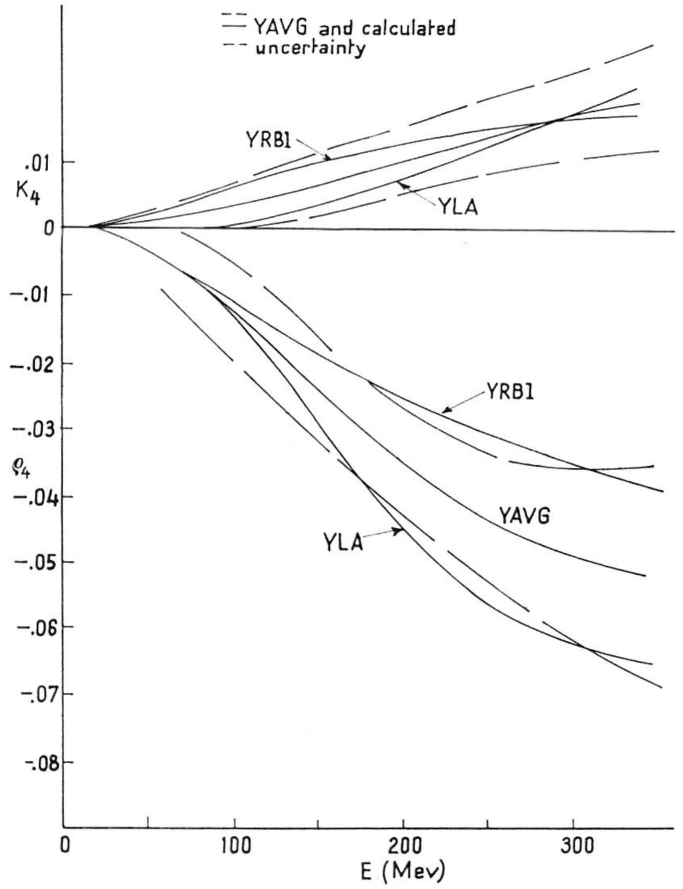


Figure 5

The polarization correlation coefficient $C_{nn}$ according to ALLABY, ASHMORE, DIDDENS and EADES [26] at $\theta = 90°$ and $E = 320$ is $0.75 \pm 0.11$. The lower limit of their standard error belt *i.e.*, 0.64 is in agreement with the calculated value 0.63(6) for fit YRBl but is appreciably higher than the expected value $\sim 0.52$ for fit YLAM. The latter fit is on the whole, however, the better of the two. The value of $C_{nn}$ (90°) obtained by ASHMORE, DIDDENS and HUXTABLE [27] at 382 MeV is $0.42 \pm 0.085$ and agrees better with YLAM calculation at 320 MeV but this agreement is at the wrong energy. Since $C_{nn}$ measurements are available only in a few cases, the disagreements just mentioned are not definite enough to give preference to YRBl over YLAM. Additional measurements of this parameter at more energies and angles would be helpful. According to Ashmore, Diddens, Huxtable and Skarsveg [28], denoting triplets and singlets by $t$ and $s$,

$$C_{nn} = \frac{\sigma_t - \sigma_s}{\sigma_t + \sigma_s} ,$$

an exact relation neglecting the relatively small Coulomb scattering. It should accordingly be possible to resolve the usual $\sigma$ into $\sigma_s$ and $\sigma_t$. The calculated $C_{KP}(90°)$ changes from $0.44 \pm 0.05$ for Set 1 of Cziffra, Mac Gregor, Moravcik and Stapp [14] to $0.49 \pm 0.09$ for Set 2 at 310 MeV and appears to be not sensitive to the choice of phase-parameter. The experimentally available value [27] of $0.83 \pm 0.10$ at 382 MeV is not truly comparable being at an appreciably different energy. The writer is not aware of a systematic set of calculations showing the possibilities of this quantity as a means of distinguishing between phase parameter sets.

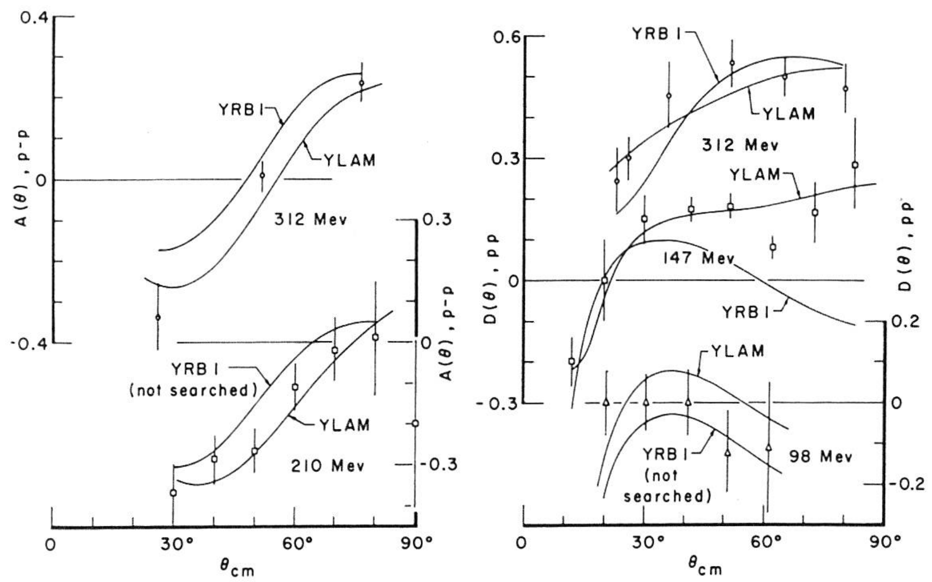


Figure 6

The successful searches with $p-p$ fits obtained from different starting points give essentially the same answer. The availability of triple scattering parameters, the information furnished by the interference with the Coulomb wave for $\sigma(\theta)$ and polarization as well as the relatively high accuracy of the measurements all contribute to this end. In the $n-p$ case there is much less uniqueness in end results. It would be helpful if triple scattering parameters and the polarization correlation could be measured in this case and if an increase in accuracy of $\sigma$ and $P$ could be achieved.

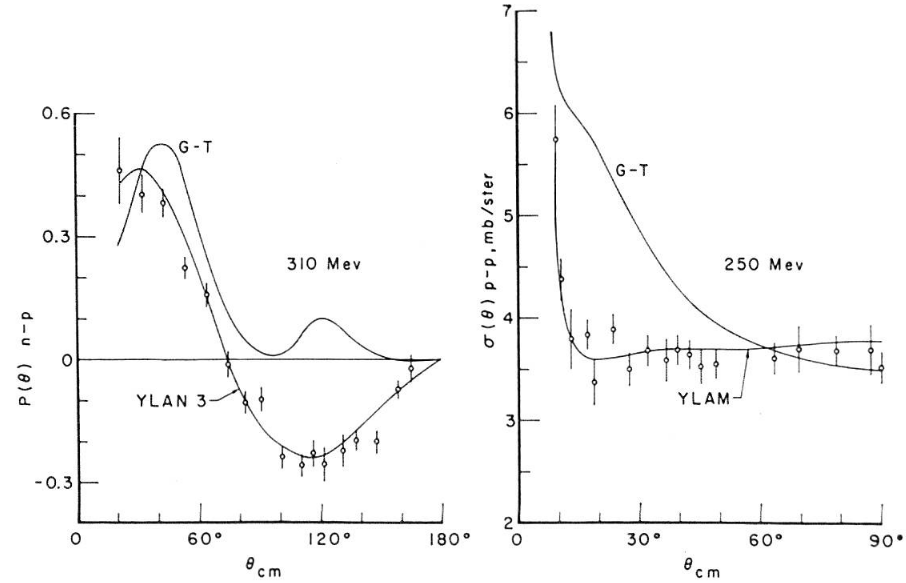


Figure 7

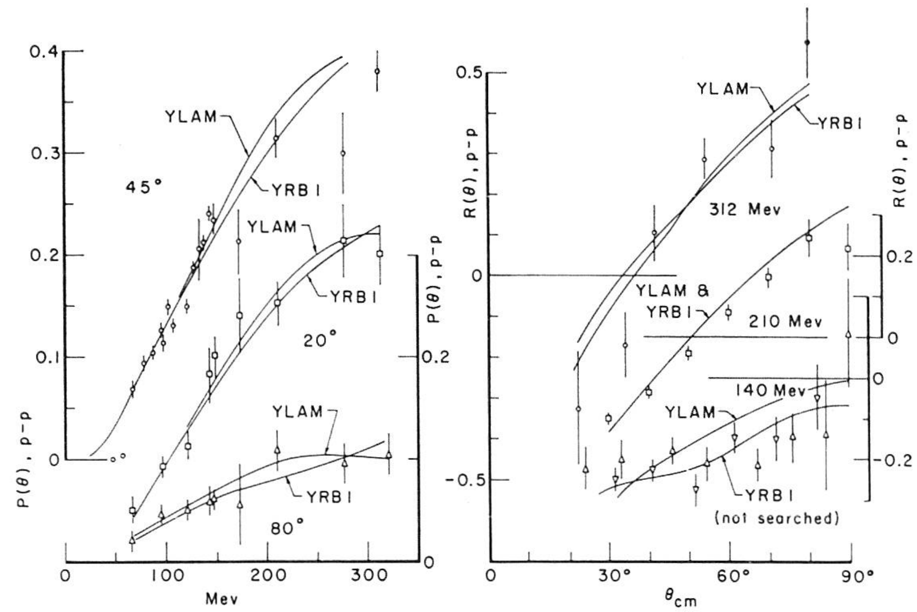


Figure 8

Measurements of nucleon polarization are also promising to be of value in the study of nuclear forces through measurements of polarization of protons and neutrons in the $d(\gamma, n)p$ reaction as first pointed out by ROSENTSVEIG [29]. At the lower energies the polarization measurement is concerned primarily with interference effects of E1 and M1 transitions and is especially simple regarding theoretical interpretation. One may hope that studies of this type will be especially illuminating regarding properties of $^3S_1 + {}^3D_1$, $^3P$ and $^1S_0$ states, electromagnetic properties combining here with the $n-p$ interaction properties.

The fits of the Yale group to nucleon-nucleon scattering quoted above are the result of collaboration with Messrs. HULL, LASSILA, PYATT, RUPPEL and DEGGES.

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