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## Quantum Theory in Real Hilbert Space II (Addenda and Errata)

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(15. II. 1961)

*Abstract:* A more concise demonstration of the necessity of the thermodynamic signature of the metric  $g^{\mu\nu}$  and of the pseudo-chronous character of  $\overset{\circ}{J}$  (corresponding to  $i = \sqrt{-1}$ ) is given (§ 4 of I<sup>1</sup>). Furthermore, the anti-unitary operator (Annex 3 of I) is wrong. A more concise demonstration in terms of observables is given, showing the perfect correspondence between RHS (Real Hilbert Space) and CHS (Complex Hilbert Space).

A more concise demonstration for the *thermodynamic signature* and the *pseudo-chronous character of  $\overset{\circ}{J}$*  is:

### **§ 4 bis. The Thermodynamic Signature of $g^{\mu\nu}$ and the Pseudo-Chronous Character of $J = \overset{\circ}{J}$**

For a *local scalar observable*, we have the identity ( $\alpha\beta\dots = 12\dots n$ ):

$$\begin{aligned} {}'F('x) &= F(x) = F(L^{-1}{}'x) = O^{-1} F('x) O, \\ L \leftarrow O: {}'x &= L x = \{{}'x'^\alpha = L'^\alpha_\alpha (x^\alpha + L^\alpha)\}, \\ g'^\mu{}'\nu &= L'^\mu_\mu L'^\nu_\nu g^{\mu\nu}. \end{aligned} \quad (4 \text{ bis. } 1)$$

We form the *bilocal observable*

$$\overset{\circ}{G}(x y) = \overset{\circ}{J} [F(x), F(y)] \quad (4 \text{ bis. } 2)$$

appearing in the UP:

$$\langle \Delta F(x)^2 \rangle_\Psi \langle \Delta F(y)^2 \rangle_\Psi \geq \frac{1}{4} \langle \overset{\circ}{G}(x y) \rangle_\Psi^2. \quad (4 \text{ bis. } 3)$$

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The hypothesis, that an *L-invariant vacuum state*  $\Psi^0$  exists

$$\langle \overset{\circ}{G}(x y) \rangle_{\Psi^0} = \overset{\circ}{f}(x y) = - \overset{\circ}{f}(y x) \quad (4 \text{ bis. } 4)$$

requires that

$$\overset{\circ}{f}'(x'y) = c(L) \overset{\circ}{f}(x y) = c(L) \overset{\circ}{f}(L^{-1}x L^{-1}y) \quad (4 \text{ bis. } 5)$$

*is the same function in every frame*, i.e.

$$\overset{\circ}{f}'(x'y) = \overset{\circ}{f}(x'y) \quad (4 \text{ bis. } 6)$$

because *vacuum is a frame-invariant concept*.

$c(L)$  is a (real) *number*, satisfying the representation condition

$$c(L_2) C(L_1) = c(L_2 L_1) . \quad (4 \text{ bis. } 7)$$

From the identity, following from (4 bis. 5) and (4 bis. 6)

$$c(L) \overset{\circ}{f}(L^{-1}x L^{-1}y) = \overset{\circ}{f}(x'y) = - \overset{\circ}{f}(y'x) \quad (4 \text{ bis. } 8)$$

follows, that  $\overset{\circ}{f}(xy)$  is, except for the factor  $c(L)$ , an invariant anti-symmetric function. Such a function exists certainly, if we choose a metric

$$\text{signat } (g^{\alpha\beta}) = \pm (11 \dots 1 - 1) . \quad (4 \text{ bis. } 9)$$

We call  $\{x^i\} = \vec{x}$  ( $i k \dots = 12 \dots d$ ,  $d = n - 1$ ) *space* and  $x^n = t$  *time* and define

$$\overset{\circ}{f}(x y) = \text{sig}(x^n - y^n) \cdot f((x - y)^2) = - \overset{\circ}{f}(y x) \quad (4 \text{ bis. } 10)$$

$$f(z^2) = 0 \quad \text{for } z = x - y = \text{spatial.}$$

We have therefore

$$\begin{aligned} c(L) \overset{\circ}{f}(L^{-1}x L^{-1}y) &= c(L) \text{sig}(L^{-1}{}^n{}_n(x'^n - y'^n)) \cdot f((x - y)^2) \\ &= c(L) \text{sig}(L'{}^n{}_n) \cdot \text{sig}(x'^n - y'^n) \cdot f((x - y)^2) \\ &= c(L) \text{sig}(L'{}^n{}_n) \cdot \overset{\circ}{f}(x'y) \equiv \overset{\circ}{f}(x'y), \end{aligned} \quad (4 \text{ bis. } 11)$$

i.e.

$$c(L) = \text{sig}(L'{}^n{}_n) . \quad (4 \text{ bis. } 12)$$

This means, that  $\overset{\circ}{f}(xy)$  is a *pseudo-chronous number*, characterising the *vacuum*. Therefore  $\overset{\circ}{G}(xy)$  must be a *pseudochronous observable* i.e.

$$\begin{aligned}
 & \breve{G}('x'y) = \text{sig}(L'^n_n) \breve{G}(x'y) = \text{sig}(L'^n_n) \breve{G}(L^{-1}'x L^{-1}'y) = \\
 & = O^{-1} \breve{G}('x'y) O = (O^{-1} \breve{J} O) [O^{-1} F('x) O, O^{-1} F('y) O] = \\
 & = (O^{-1} \breve{J} O) ['F('x), 'F('y)] = (O^{-1} \breve{J} O) [F(x), F(y)] = \\
 & = (O^{-1} \breve{J} O \breve{J}^{-1}) \breve{J} [F(x), F(y)] = (O^{-1} \breve{J} O \breve{J}^{-1}) \breve{G}(x'y)
 \end{aligned} \tag{4 bis. 13}$$

where we have made use of (4 bis. 1). Comparing the second and the last member, we have  $O^{-1} \breve{J} O \breve{J}^{-1} = \text{sig}(L'^n_n) \cdot 1$

$$O^{-1} \breve{J} O = \text{sig}(L'^n_n) \breve{J} \equiv 'J \tag{4 bis. 14}$$

which is equation (4.14) of I.  $\breve{J}$  is thus a *pseudochronous operator*. Writing  $L_{(\text{ochr})} \leftarrow O_{(\text{ochr})}$  and  $L_{(\text{pchr})} \leftarrow O_{(\text{pchr})}$  for *ortho-chronous* ( $\text{sig}(L'^n_n) = +1$ ) and *pseudo-chronous* ( $\text{sig}(L'^n_n) = -1$ ) *Lorentz-transformations*, we have

$$[O_{(\text{ochr})}, \breve{J}] = [O_{(\text{ochr})}, \breve{J}]_- = 0, \tag{4 bis. 15_-}$$

$$(O_{(\text{pchr})}, \breve{J}) = [O_{(\text{pchr})}, \breve{J}]_+ = 0. \tag{4 bis. 15_+}$$

We are left to show, that (4 bis. 9) are the only two possible signatures of  $g^{\mu\nu}$ . To show this, let us assume a metric, in which  $z = \{z^\alpha\}$  has  $d = n - r$  space components and  $r \leq 1/2 n$  time components  $\vec{t} = \{t^\alpha\} = \{z^\alpha\}$  ( $i, k, \dots = 1, 2, \dots, n - r, a, b, \dots = (n - r + 1), (n - r + 2), \dots, n$ ).

We have to build a function  $\breve{f}(z) = \breve{f}(\vec{x}, \vec{t})$  which is *invariant* under *homogeneous continuous transformations*  $L_{(\text{cont})}$  (= *proper Lorentz-transformations*) and *changes sign* under *reflections* (' $x = (PT)x = -x$ '). Thus, we look for a vector  $z_0 = \{z_0^\alpha\}$ , which changes sign under the transformation  $L = PT$  but does not change sign under any continuous transformation. If  $r > 1$ , it is evident, at least for *space-like* and *time-like vectors*  $z$ , that no such vector  $z_0$  exists, because every vector  $z = (\vec{t}, \vec{z})$  can be transformed into  $-z$  by rotations in  $\vec{z}$ -space and  $\vec{t}$ -space. For null-vectors ( $g_{\alpha\beta} z^\alpha z^\beta = 0$ ) we use the theorem<sup>2)</sup>, that in  $n$  dimensional space time with  $r \leq 1/2 n$ , there exist exactly  $r$  two-by-two orthogonal linearly independent null vectors  $z$ . Therefore, for  $r > 1$ , a rotation of this  $r$ -dimensional 'frame', transforms  $z$  into  $-z$  (for instance  $z$  on the 'frame'). Thus only in the case  $r = 1$ , time-like vectors and null vectors (on the light cone) exist, which change sign under  $L = PT$ .

The text of I is correct, if we write, on page 748, under b):

'We pose...' instead of 'We try, posing...' and leave out, on p. 749, the text following (A-3.20). (A-3.20) is the correct formula, the text following (A-3.20) is erroneous. However, a more logical deduction, using only observables  $F^X = F^{XT} = F^{\alpha\beta\dots}(xy\dots)$  ( $X = \{\alpha\beta\dots xy\dots\}$ ) shall be given below in:

### Annex 3 bis. Unitary ( $\hat{U}$ ) and Anti-Unitary ( $\hat{V}$ ) Operators in CHS

We rewrite the operator identity of (I 3.9) in RHS

$$F'_{a'b}^X = L'^X_X O_{aa} F_{ab} O_{bb}^T; \quad F_{ab}^X = F_{(ab)}^X. \quad (\text{A-3 bis. 1})$$

For:

#### a) Orthochronous Lorentz Transformations

We decompose the RHS ( $a$ -space) in the direct product of a two-dimensional ( $(r)$  ( $i$ )-space) and a  $\omega_C = (1/2) \omega_R$  dimensional  $p$ -space, using the two-dimensional matrices (in ( $r$ ) ( $i$ )-space)

$$1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad j = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}. \quad (\text{A-3 bis. 2})$$

On account of (4 bis. 15\_), we have ( $O = O_{(\text{ochr})}$ ):

$$\begin{aligned} F'_{a'b}^X &= 1 \times F'_{(r)p'q}^X + j \times F'_{(i)p'q}^X \\ &= L'^X_X (1 \times O_{(r)p'p} + j \times O_{(i)p'p}) (1 \times F_{(r)pq}^X + j \times F_{(i)pq}^X) \cdot \\ &\quad \cdot (1 \times O_{(r)q'q}^T - j \times O_{(i)q'q}^T, \\ F_{(r)pq}^X &= F_{(r)(pq)}^X; \quad F_{(i)pq}^X = F_{(i)[pq]}^X \end{aligned} \quad (\text{A-3 bis. 3})$$

formula, which, may be rewritten in CHS, using the correspondence RHS  $\rightleftarrows$  CHS,  $A \rightleftarrows \hat{A}$  given in (I A-2.3)  $\rightleftarrows$  (I A-2.5) in the form

$$\hat{F}'_{p'q}^X = L'^X_X \hat{O}_{p'p} \hat{F}_{pq}^X \hat{O}_{q'q}^\dagger = L'^X_X \hat{U}_{p'p} \hat{F}_{pq}^X \hat{U}_{q'q}^\dagger \quad (\text{A-3 bis. 4})$$

with:

$$\hat{F}^\dagger = \hat{F} = \hat{F}^{T*} = \hat{F}^{*T},$$

$$\hat{U} = \hat{O}; \quad \hat{U}^\dagger \hat{U} = \hat{U} \hat{U}^\dagger = 1; \quad \hat{U}^\dagger = \hat{U}^{-1} \quad (\text{A-3 bis. 5})$$

where  $\widehat{A}^*$  is the *complex conjugate operator* ( $(\widehat{A}\widehat{B})^* = \widehat{A}^*\widehat{B}^*$ ) and  $\widehat{A}^\dagger = \widehat{A}^{*T} = \widehat{A}^{T*}$  is the *hermitian conjugate operator* ( $(\widehat{A}\widehat{B})^\dagger = \widehat{B}^\dagger\widehat{A}^\dagger$ ) in CHS. Explicitly written, and omitting the indices  $p q \dots = 12 \dots \omega_C$  referring to a frame in  $\omega_C$ -dimensional CHS, the identity

$$\widehat{F}'^\alpha{}^\beta \dots ('x'y\dots) = c(L) L'{}^\alpha_\alpha L'{}^\beta_\beta \dots \widehat{U} F^\alpha{}^\beta \dots (L^{-1} 'x L^{-1} 'y \dots) \widehat{U}^{-1}$$

holds, with  $c(L) = 1$ ,  $\text{sig}(\det(L'i_i))$  and  $\text{sig}(\det(L'{}^\alpha_\alpha))$  for *ortho* ( $\widehat{F}$ ), *pseudo-chronous* ( $\overset{\circ}{\widehat{F}}$ ), *pseudochorous* ( $\widehat{F}$ ) and *pseudo-*( $\overset{\circ}{\widehat{F}}$ ) observables. The *transformed operators* are given by

$$\begin{aligned} ' \widehat{F}'^\alpha{}^\beta \dots ('x'y\dots) &= c(L) L'{}^\alpha_{(\text{ochr})\alpha} L'{}^\beta_{(\text{ochr})\beta} \dots \widehat{F}^\alpha{}^\beta \dots (L^{-1} 'x L^{-1} 'y \dots) \\ &= \widehat{U}^{-1} F^\alpha{}^\beta \dots ('x'y\dots) \widehat{U}. \end{aligned} \quad (\text{A-3 bis. 6})$$

They lead to the identity, for expectation values:

$$\begin{aligned} \langle \widehat{\Psi}, ' \widehat{F}^\alpha \dots ('x\dots) \widehat{\Psi} \rangle &= c(L) L'{}^\alpha_\alpha \dots \langle \widehat{\Psi}, \widehat{F}^\alpha \dots (L^{-1} 'x \dots) \widehat{\Psi} \rangle \\ &= \langle ' \widehat{\Psi}, \widehat{F}'^\alpha \dots ('x\dots) ' \widehat{\Psi} \rangle \end{aligned} \quad (\text{A-3 bis. 7})$$

with

$$' \widehat{\Psi} = \widehat{U} \Psi; \quad L_{(\text{ochr})} \leftarrow \widehat{U}_{(\text{ochr})}. \quad (\text{A-3 bis. 8})$$

### b) Pseudo-Chronous Lorentz-Transformations

We need an operator

$$K = k \times 1; \quad K^T = K = K^{-1}; \quad K^2 = 1 \quad (\text{A-3 bis. 8})$$

which transforms

$$' \overset{\circ}{J} = K^{-1} \overset{\circ}{J} K = - \overset{\circ}{J}; \quad \overset{\circ}{J} = j \times 1. \quad (\text{A-3 bis. 9} *)$$

Such an operator can be given in terms of the pseudoquaternions (I A-4.8). We chose, for example, the two-dimensional matrix in ( $r$ ) ( $i$ )-space

$$k = k^T = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}; \quad k^2 = 1, \quad k^T = k, \quad (k, j) = 0. \quad (\text{A-3 bis. 10})$$

\*) In CHS  $K \rightleftharpoons \widehat{K} : \widehat{K} i \widehat{K} = -i$  (A-3 bis 9\*).

The operator  $O_{(\text{pchr})} = O$  can now be given in terms of

$$\begin{aligned} O &= O' K; \quad O^T = O^{-1} = K^T O'^T = K O'^T; \quad [O', \bar{J}] = 0, \\ O'^T O' &= O' O'^T = 1 \end{aligned} \quad (\text{A-3 bis. 11})$$

and the identity (A-3 bis. 1) can be written, using the decomposition

$$\begin{aligned} F'_{a'b}^X &= 1 \times F'_{(r)p'q}^X + j \times F'_{(i)p'q}^X \\ &= L'^X_X (1 \times O'_{(r)p'p} + j \times O'_{(i)p'p}) (k \times 1) (1 \times F'_{(r)pq}^X + j \times F'_{(i)pq}^X) \cdot \\ &\quad \cdot (k \times 1) (1 \times O'^T_{(r)q'q} - j \times O'^T_{(i)q'q}) \\ &= L'^X_X (1 \times O'_{(r)p'p} + j \times O'_{(i)p'p}) (1 \times F'_{(r)pq}^X - j \times F'_{(i)pq}^X) \cdot \\ &\quad \cdot (1 \times O'^T_{(r)q'q} - j \times O'^T_{(i)q'q}). \end{aligned} \quad (\text{A-3 bis. 12})$$

We have the correspondence RHS  $\rightleftharpoons$  CHS.

$$1 \times F'_{(r)pq}^X - j \times F'_{(i)pq}^X \rightleftharpoons F'_{pq}^X - i F'_{pq}^X = \hat{F}'_{pq}^X. \quad (\text{A-3 bis. 13})$$

Now, using again the RHS  $\rightleftharpoons$  CHS correspondence (I A-2.3)  $\rightleftharpoons$  (I A-2.5), we may write (A-3 bis. 12) in CHS, omitting again the  $p$ -space indices,

$$\begin{aligned} \hat{F}'^X &= F'_{(r)}^X + i F'_{(i)}^X = L'^X_X \hat{O}' \hat{F}'^{X*} \hat{O}'^{-1} \\ &= L'^X_X \hat{U}' \hat{F}'^{X*} \hat{U}'^{-1}; \quad \hat{U}'^{-1} = U'^\dagger \end{aligned} \quad (\text{A-3 bis. 14})$$

or, explicitly

$$\hat{F}'^{\alpha'\beta} \dots ('x'y\dots) = c(L) L'^\alpha_\alpha L'^\beta_\beta \dots \hat{U}' \hat{F}'^{\alpha\beta\dots*} (L^{-1}x L^{-1}y\dots) \hat{U}'^{-1}. \quad (\text{A-3 bis. 15})$$

The transformed operator is defined by

$$\begin{aligned} \hat{F}'^\alpha \dots ('x\dots) &= c(L_{(\text{pchr})}) L'^\alpha_{(\text{pchr})\alpha} \dots \hat{F}'^\alpha \dots (L^{-1}x\dots) \\ &= \hat{U}'^{-1*} \hat{F}'^{\alpha\dots*} ('x\dots) \hat{U}'^*. \end{aligned} \quad (\text{A-3 bis. 16})$$

The identity for expectation values takes the form

$$\begin{aligned} \langle \hat{\Psi}, \hat{F}'^\alpha \dots ('x\dots) \hat{\Psi} \rangle &= c(L) L'^\alpha_\alpha \dots \langle \hat{\Psi}, \hat{F}'^\alpha \dots (L^{-1}x\dots) \hat{\Psi} \rangle \\ &= \langle \hat{\Psi}, \hat{F}'^\alpha \dots ('x\dots) \hat{\Psi} \rangle. \end{aligned} \quad (\text{A-3 bis. 17})$$

Defining the anti-unitary operator  $\widehat{V}$  by

$$'\widehat{\Psi} = \widehat{V} \widehat{\Psi} = \widehat{U}' \widehat{\Psi}^* \rightarrow ' \Psi_p = U'_{p,p} \Psi_p^*; \quad L_{(pchr)} \leftarrow \widehat{V}, \quad (\text{A-3 bis. 18})*$$

we have

$$\begin{aligned} \langle ' \widehat{\Psi}, \widehat{F}'^\alpha \dots ('x \dots) ' \Psi \rangle &= c(L) L'^\alpha_\alpha \dots \langle \widehat{\Psi}, \widehat{F}^\alpha \dots (L^{-1} 'x \dots) \widehat{\Psi} \rangle = \\ &= \widehat{\Psi}_p \widehat{U}'_{p,p}^T \widehat{F}'^\alpha_{p,q} ('x) \widehat{U}'_{q,q} \Psi_q^* = \widehat{\Psi}_q^* \widehat{U}'_{q,q}^T \widehat{F}'^\alpha_{q,p} ('x) \widehat{U}'_{p,p}^* \widehat{\Psi}_p = \\ &= \langle \widehat{\Psi}, \widehat{U}'^{\dagger *} \widehat{F}'^\alpha \dots ('x) \widehat{U}'^* \widehat{\Psi} \rangle. \end{aligned} \quad (\text{A-3 bis. 19})$$

By definition,  $F^\alpha \dots (x \dots)$  is an observable. Therefore it follows, from (A-3 bis. 5), that in (A-3 bis. 19), we have

$$\widehat{F}'^\alpha \dots ('x \dots) = \widehat{F}'^\alpha \dots ('x \dots) \quad (\text{A-3 bis. 20})$$

and, from (A-3 bis. 16) ( $\widehat{U}'^{\dagger *} = U'^{-1}$ )

$$\widehat{U}'^{\dagger *} \widehat{F}'^\alpha \dots ('x \dots) \widehat{U}'^* = c(L) L'^\alpha_\alpha \dots \widehat{F}^\alpha \dots (L^{-1} 'x \dots). \quad (\text{A-3 bis. 21})$$

Thus, formula (A-3 bis. 19) is identical with the second equation (A-3 bis. 17) and with (A-3 bis. 16).

In order to show the physical significance of (A-3 bis. 16) let us consider the '*real scalar free field*'\*\*) considered as an observable:

$$w(x) = w^T(x); \quad \widehat{w}(x) = \widehat{w}^\dagger(x), \quad (\text{A-3 bis. 22})$$

expanded in plane waves (signat  $(g^{\alpha\beta}) = (11 \dots 1 - 1)$ ).

$$\begin{aligned} \widehat{w}(x) &= (2)^{-1/2} (2\pi)^{-d/2} \int d\sigma(\breve{k}) (\widehat{a}(\breve{k}) e^{i(\breve{k}, x)} + \widehat{a}^\dagger(\breve{k}) e^{-i(\breve{k}, x)}), \\ \breve{k}^2 + M^2 &= 0; \quad \breve{k}^n > |M|, \\ d\sigma(\breve{k}) &= (\breve{k}^n)^{-1} d^d \breve{k}; \quad d^d \breve{k} = \prod_{i=1}^d d\breve{k}^i, \end{aligned} \quad (\text{A-3 bis. 23}***)$$

\*)  $'\widehat{\Psi} = \widehat{U}' \widehat{K} \Psi = \widehat{U}' \widehat{\Psi}^*$ .

\*\*) 'Field Quantization in Real Hilbert Space' will be the object of a forthcoming paper in this journal (referred to as III).

\*\*\*) The surface integral is to be extended over the *positive shell* of the hyperboloid  $\breve{k}^2 + M^2 = 0$ .  $\breve{k} = \{\breve{k}^\alpha\}$  is therefore a *pseudochronous vector* (see III).

where, if the sum over plane waves is made denumerable,  $\widehat{a}(\vec{k})$  and  $\widehat{a}^\dagger(\vec{k})$  are the usual annihilation and creation operators of quanta in a state  $\vec{k}$ . Using (A-3 bis. 16) for the transformation  $L = PT$

$$'x = (PT)x = -x; \quad 'x'^\alpha = -x'^\alpha \quad (\text{A-3 bis. 24})$$

we have (A-3 bis. 16):

$$\begin{aligned} \widehat{w}'(x) &= \widehat{w}(x) \equiv \widehat{w}(-'x) \\ &= (2)^{-1/2} (2\pi)^{-d/2} \int d\sigma(\vec{k}) (\widehat{a}(\vec{k}) e^{-i(\vec{k}, x)} + \widehat{a}^\dagger(\vec{k}) e^{i(\vec{k}, x)}) \\ &= \widehat{U}'^{-1*} \widehat{w}^*(x) \widehat{U}'^* \\ &= (2)^{-1/2} (2\pi)^{-d/2} \int d\sigma(\vec{k}) ((\widehat{U}'^*{}^{-1} \widehat{a}^*(\vec{k}) \widehat{U}'^*) e^{-i(\vec{k}, x)} + \\ &\quad + \widehat{U}'^*{}^{-1} \widehat{a}^\dagger(\vec{k}) \widehat{U}'^*) e^{i(\vec{k}, x)}) . \end{aligned} \quad (\text{A-3 bis. 25})$$

Thus, we have

$$\widehat{U}'^{-1*} \widehat{a}^*(\vec{k}) \widehat{U}'^* = \widehat{a}(\vec{k}), \quad (\text{A-3 bis. 26})$$

$$\widehat{U}'^{-1*} \widehat{a}^\dagger(\vec{k}) \widehat{U}'^* = \widehat{a}^\dagger(\vec{k}). \quad (\text{A-3 bis. 26})$$

As the relations

$$[\widehat{a}(\vec{k}), \widehat{a}^\dagger(\vec{k}')] = \delta(\vec{k}, \vec{k}'); \quad \delta(\vec{k}, \vec{k}') = \vec{k}^n \delta(\vec{k} - \vec{k}'), \quad (\text{A-3 bis. 27})$$

$$[\widehat{a}(\vec{k}), \widehat{a}(\vec{k}')] = 0, \quad (\text{A-3 bis. 28})$$

are invariant, if we go over to the conjugate complex operators, an unitary matrix  $\widehat{U}'^*$  exists always, satisfying (A-3 bis. 26). In particular, if the annihilation and creation operators are chosen real,  $\widehat{U}'^* = 1$  is the unit operator.

### References

- <sup>1)</sup> E. C. G. STUECKELBERG, Helv. Phys. Acta 33, 727 (1960) to be referred to as I.
- <sup>2)</sup> R. JOST and M. GUENIN, unpublished.