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Autor: Olsen, J.L.
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Magnetoresistance and Size Effects in Indium at Low Temperatures

by **J. L. Olsen**

Institut für Kalorische Apparate und Kältetechnik,
Eidgenössische Technische Hochschule, Zürich

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Zusammenfassung. Messungen des elektrischen Widerstandes von sehr reinen Indium-Drähten mit Durchmessern zwischen 0,06 und 2,5 mm wurden bei der Temperatur des flüssigen Heliums durchgeführt. Aus der Durchmesserabhängigkeit des Widerstandes ohne angelegtes Magnetfeld wurde der Wert $\varrho_b l = 1{,}8 \times 10^{-11}$ Ohm cm^2 berechnet (ϱ_b ist der Widerstand des Metalls bei grossen Proben, l die freie Weglänge der Elektronen). Eine durchmesserabhängige Abweichung von der Matthiessenschen Regel wurde auch beobachtet.

Die magnetische Widerstandsänderung bei dicken Proben stimmt mit den Messungen von JUSTI und Mitarbeitern bis zu den höchsten von ihnen gemessenen Werten von H/ϱ überein (H = Magnetfeld, ϱ = Widerstand im Feld Null). Bei weiteren Felderhöhungen wird zunehmende Sättigung der Widerstandsänderung beobachtet. Dünne Proben in Quer- und Längsfeld zeigen Durchmesserabhängigkeiten der magnetischen Widerstandsänderung, die den von MACDONALD und von CHAMBERS in Natrium beobachteten Effekten analog sind. Aus diesen Effekten lässt sich abschätzen, dass in Indium $m\bar{v}$ ungefähr den dreifachen Wert freier Elektronen besitzt. (m = effective mass, $\bar{v}$ = mittlere Geschwindigkeit der Elektronen an der Fermi-Oberfläche.)

1. Introduction

There is an apparent disagreement between theoretical predictions and experimental results on the magnetoresistance of pure metals at low temperatures in high magnetic fields. Recent theoretical studies[1)2)] agree with older work[3)4)] in suggesting that only two types of behaviour should be found in high fields. For one group of metals the increase in resistance should approach a saturation value while for the other the resistance change should continue to increase quadratically with field far beyond any magnetic field values at present accessible experimentally. There is, however, an increasing body of experimental data on metals where the high field resistance change is neither quadratic in the field nor constant, but appears to vary linearly with field.

A change from quadratic to linear behaviour in high fields has recently been observed in sodium[5]), copper[6])[7]), silver[8]), and tin[9]). It has, however, been suggested by BOROVIK[10]) that this observed linear dependence only occurs in a limited field region and that it merely represents a transition between two quadratic regions. His extension of KAPITZA's[11]) data on cadmium appears to support this view. On the other hand the work of CHAMBERS[6]) and COTTI[7]) on copper, and work by LÜTHI[8]) on silver shows the linear region to be far wider than would be consistent with BOROVIK's argument.

The situation is thus unclear as far as metals which do not tend towards saturation is concerned. It seemed interesting also to investigate those few metals where saturation has hitherto been believed to occur. Recent indications[12]) from measurements in pulsed field up to 150000 Oersted that the resistance of aluminium after appearing to reach saturation may again increase linearly are being investigated in more detail*), and we have also thought it useful to examine the saturation region in indium. The easy availability of indium of remarkably high purity makes it possible to extend JUSTI's[13]) magnetoresistance data considerably without going to very high magnetic fields.

In the present paper we report on a series of such measurements in longitudinal and in transverse fields up to 10000 Oersted. The size dependence of the zero field resistivity allowed a determination of the electronic mean free paths. In the course of our investigation of this size effect, we found a deviation from Matthiessen's rule similar to that noted previously by ANDREW[14]) for tin. The effect in indium is, however, several times greater than that in tin. Size effects in the magnetoresistance corresponding to those reported by MACDONALD[15]) and by CHAMBERS[16]) for sodium were also observed.

2. Experimental

The specimens were extruded from indium obtained from Johnson Matthey & Co. and from Consolidated Mining and Smelting Co. of Canada (Tadanac brand)**). A list of the specimens investigated is shown in Table I.

Potential leads to the specimens were made by coldwelding of pieces of pure indium wire. The resistance measurements were made by simultaneous current and potential measurements. The potential was measured with a galvanometer amplifier of a type described by MACDONALD[17]) and having a sensitivity of ca. 600 mm/μV. The customary field and current

*) We are grateful to Dr. D. SHOENBERG for telling us of measurements which cast doubt on these results.

**) We are grateful to Dr. G. K. WHITE of the National Research Council Ottawa for presenting us with one of his specimens – our In I.

reversal procedures were carried out to eliminate the Hall voltage. For the thickest specimens this could be equivalent to about 20% of the apparent resistance drop in the specimens.

Table I
The Specimens

Specimen	Source	Diam. mm	$\varrho_{4.2}/\varrho_{293} \times 10^4$	$\varrho_0/\varrho_{293} \times 10^4$	$\varrho_0 \times 10^9$ (ohm cm)
In 1	GKW	0·4	2·00	1·54	1·36
In 2	JM	2·0	0·705	0·307	0·27
In 3	T	0·311	1·945	1·403	1·24
In 4	T	0·0855	3·66	2·99	2·63
In 5	T	0·57	1·29	0·82	0·72
In 6	T	2·54	0·92	0·52	0·46
In 7	JM	0·0604	4·38	3·59	3·16
In 8	JM	0·2	1·75	1·22	1·07
In 9	T	0·3	1·82	1·34	1·18
In 10	T	2·54	0·94	0·57	0·50

GKW: G. K. WHITE; JM: JOHNSON, MATTHEY; T: "Tadanac" brand.

The stability and linearity of such amplifiers can be improved by further amplification of the photocell current before using it in the feed back circuit. We have found the photocell and transistor amplifier circuit shown in Figure 1 very simple and convenient.

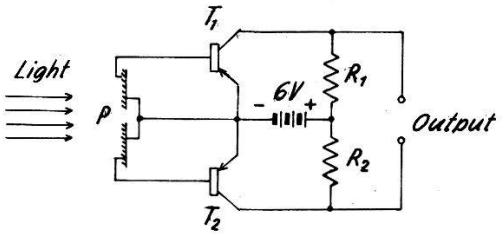

Fig. 1
Photocurrent amplifier circuit for galvanometer amplifier.
P: split photocell, T_1, T_2: Transistors (e.g. RCA type N 175)
R_1, R_2: 10 k Ohm.

An iron core electromagnet capable of producing 8000 Oersted was used for the transverse magnetic fields, and a liquid air cooled solenoid provided longitudinal magnetic fields up to 12000 Oersted.

3. Zero field resistance

The resistance in zero magnetic field was investigated over a range of temperatures. Below 3·37° K where indium becomes superconducting the zero field resistance had to be obtained by extrapolation from measure-

ments in fields greater than the critical. Near the superconducting transition temperature this can be done easily and reliably assuming a quadratic field dependance of the magnetoresistance. For the lower temperatures magnetoresistive saturation becomes important and some care is required in extrapolating.

The values of $\varrho_{4\cdot2}/\varrho_{293}$ and ϱ_0/ϱ_{293} where ϱ_T is the resistivity at temperature T °K are shown in Table I. It will be seen that there is a general tendency for the thinnest specimens to show the highest resistivity. In Figure 2 we show the temperature dependence of the resistivity below 4·2° K. To make identification of the curves easier we have plotted $(\varrho_T - \varrho_{4\cdot2})/\varrho_{293}$. We note that there is a very considerable deviation from Matthiessen's rule, and that the thinnest specimens show almost twice as great a resistivity change between 0° K and 4·2° K as do the thick specimens.

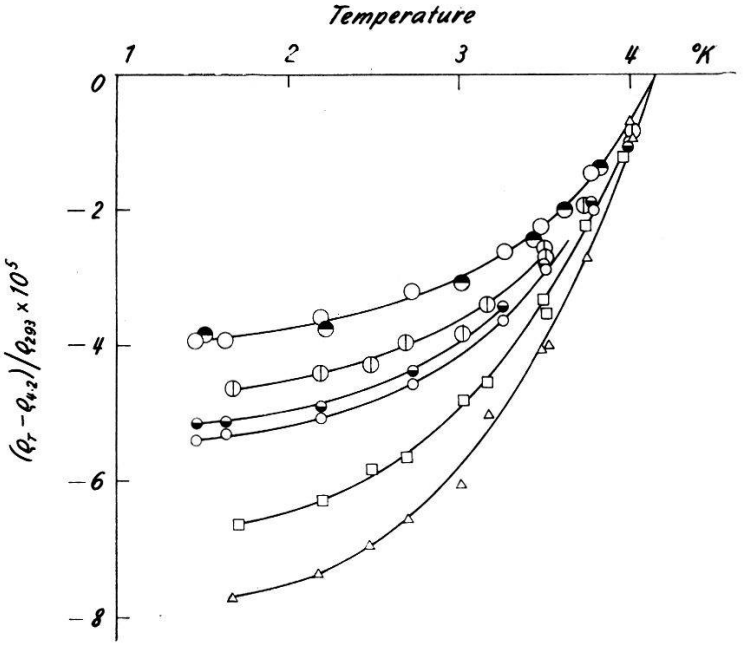

Fig. 2
Resistivity change in Indium below 4° K.

○ In 2	2·0 mm ∅	◓ In 6	2·5 mm ∅
ⓘ In 5	0·6 mm ∅	• In 8	0·2 mm ∅
○ In 3	0·3 mm ∅	□ In 4	0·1 mm ∅
△ In 7	0·06 mm ∅		

A detailed analysis of the conductivity of thin cylindrical wires has been given by DINGLE[18]) who gives tables ϱ/ϱ_b where ϱ is the observed resistivity and ϱ_b the bulk resistivity of the material. ϱ/ϱ_b depends on a/l where a is the diameter of the specimen and l is the bulk mean free path of the electrons. A further parameter p; the ratio of specular to diffuse scattering of the electrons at the specimen surface, also occurs. Work by ANDREW on mercury[14]) is consistent with $p = 0$ while MAC-

DONALD and SARGINSON[19]) find that in sodium this may not be the case. Our present measurements are most easily fitted to $p = 0$, and we shall assume $p = 0$ throughout this paper. It is worth noting that DINGLE's calculation with $p = 0$ differs by less than 5% from NORDHEIM's[20]) approximate formula

$$\varrho/\varrho_b = 1 + l/a\,. \tag{1}$$

Our problem is now to fit our measured data for ϱ and a to either DINGLE's table or to equation (1) using l and ϱ_b as variable parameters. In view of the observed deviation from Matthiessen's rule we must limit ourselves to making use of the data for 0° K. Since indium appears to self-anneal rapidly at room temperature it is quite reasonable to assume that ϱ_b is the same for all specimens pressed from a given batch of indium. This appears to be borne out by the results.

l is inversely proportional to ϱ_b so that according to (1) a plot of ϱ against l/a should yield a series of parallel straight lines each intersecting $l/a = 0$ at ϱ_b. Such a plot is shown in Figure 3. From this we find $\varrho_b = 4{\cdot}0 \times 10^{-10}$ ohm cm[1] and $l = 0{\cdot}045$ cm for the Tadanac material and

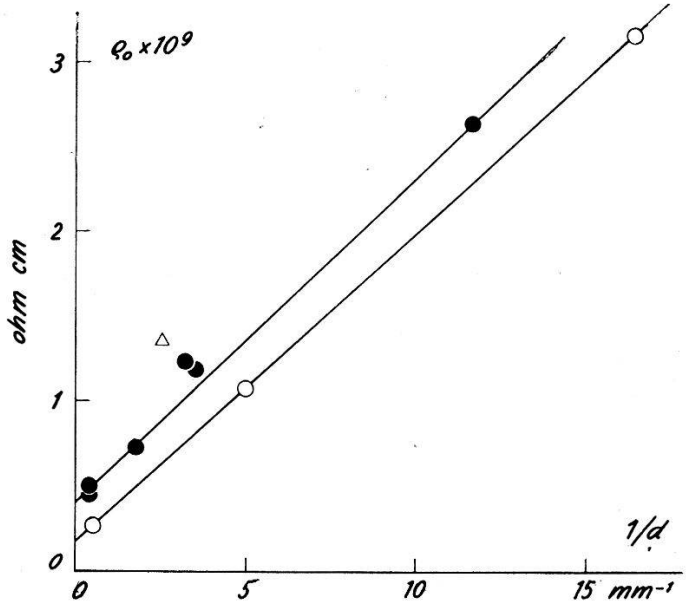

Fig. 3
Size effect in zero field resistivity at 0° K.
○ JOHNSON MATTHEY, ● TADANAC, △ In 1.

$\varrho_b = 1{\cdot}7 \times 10^{-10}$ ohm cm[1] and $l = 0{\cdot}105$ cm for the Johnson Matthey material. We thus obtain a value for the constant $\varrho_b l$ of $\varrho_b l = 1{\cdot}8 \times 10^{-11}$ ohm cm[2]. This is twice that reported recently by MEISSNER and ZDANIS[21]) but a careful examination of their results would appear to allow an interpretation of their results in terms of somewhat higher value for l than suggested in their publication. A somewhat similar discrepancy is seen when work of ANDREW[14]) and of MEISSNER[22]) on tin is compared.

We may note that according to the Sommerfeld theory of conduction $\varrho_b l$ for n free electrons per unit volume is given by

$$\varrho_b \, l = \left(\frac{3}{8\pi}\right)^{1/3} \frac{h}{\varepsilon^2 \, n^{2/3}} \, . \tag{2}$$

Here ε is the electronic charge and h is Planck's constant. Using this relation our measurements suggest

$$n/n_a = 0{\cdot}49$$

where n_a is the number of atoms per unit volume.

The deviation from Matthiessen's rule shown in Figure 2 requires some comment. Equation (1) may be rewritten

$$\varrho = \varrho_b + \varrho_b \, l/a \, .$$

Since $\varrho_b l$ is a constant of the metal the second term on the right is independent of ϱ_b, and therefore a given change in bulk resistivity should be reflected in an equal change in the resistivity of a thin specimen. This suggests that at any rate to the degree of approximation provided by (1) Matthiessen's rule should hold. DINGLE's more accurate calculation would give rise to a slight deviation, but by no means large enough to explain the experimental results. A possible explanation appears to us to lie in the different character of the impurity scattering which causes the residual resistance, and of the phonon scattering which provides the temperature dependent part of the resistance. The impurity scattering is a large angle scattering and thus a single event suffices to remove the momentum given to an electron by the field. For phonon scattering, however, this is not the case and many collisions are needed to achieve this in the bulk metal. In thin specimens, on the other hand, a few electron phonon collisions leading to quite a small curvature of the electronic path may be sufficient to force an electron originally moving along the specimen axis into collision with the surface where diffuse scattering may take place. The phonons will thus be particularly effective in shortening the free path of those electrons which should be contributing significantly to the conductivity.

In this way the phonons will create more temperature dependent resistivity in thin specimens than they normally do in thick specimens.

4. Bulk Magnetoresistance

In Figure 4 we show measurements of the magnetoresistance in In 2 at 4·2° K where $a/l > 5$. The results are plotted in a reduced Kohler diagram against $H/\mathfrak{r}$ where $\mathfrak{r} = \varrho_T/\varrho_\theta$ and θ is the Debye temperature. We have taken $\theta = 100°$ K for Indium. Our results in a transverse mag-

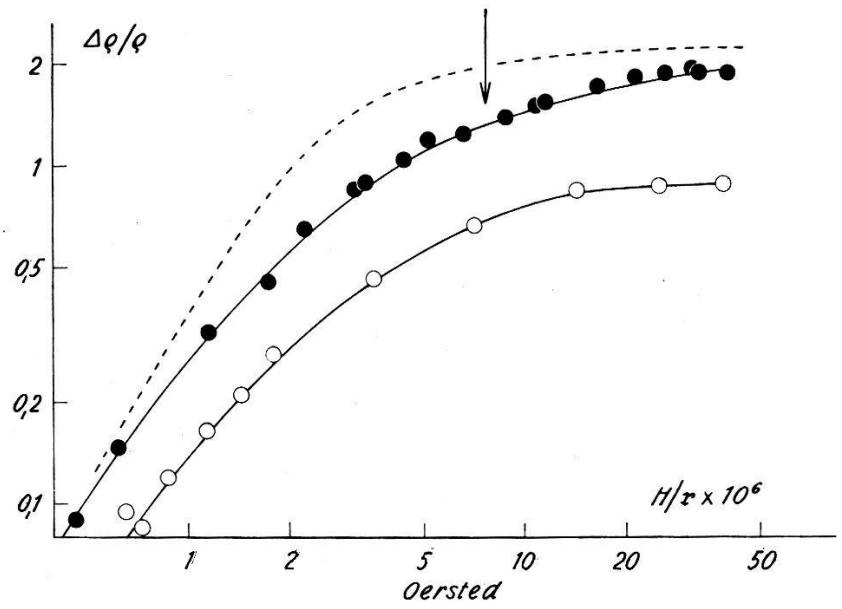

Fig. 4
Bulk Magnetoresistance In 2 at 4° K.
○ longitudinal, ● transverse, – – – equation (3), $\tau = \varrho_T/\varrho_{100}$.

netic field are in good agreement with JUSTI's[13]) curve as far as this goes (indicated by arrow). Beyond this point the approach to saturation continues and there is no indication of a tendency to increase above the saturation level. It is perhaps interesting to compare the shape of the experimental curve with that obtained from simple theoretical treatments such as that of SONDHEIMER and WILSON[3]) which assumes two overlapping energy bands of spherical symmetry. Such a treatment yields

$$\frac{\Delta\varrho}{\varrho} = \frac{A H^2}{1+BH^2} \tag{3}$$

where $\Delta\varrho$ is the resistivity change on applying a magnetic field H, and ϱ is the resistivity in zero field. A and B are constants depending on the relative numbers of electrons in the two bands and upon the relaxation times and effective masses. It will be seen that the transition from the quadratic to the constant region is much wider experimentally than suggested by this simple theory. Such a difference is not at all surprising and can be explained by the existence of a smeared out range of effective masses and relaxation times rather than just two well defined values of each as assumed in the simple theory.

No longitudinal measurements have been published so far by other workers. Our results are in keeping with the general trend of longitudinal results in many other metals in that $\Delta\varrho/\varrho$ for the longitudinal case is about half that for the transverse case.

Simple free electron theory provides an estimate of the ratio of electron mean free path l to orbital radius r in a magnetic field H.

$$\frac{l}{r} = \frac{H}{n\varepsilon c} \cdot \frac{1}{\varrho} \tag{4}$$

where c is the velocity of light, n the number of free electrons per unit volume, and ϱ is the resistivity. A numerical evaluation of (4) for 10000 Oersted gives $l/r = 40$ for the Tadanac specimens and $l/r = 95$ for the Johnson, Matthey specimens at 0° K.

5. Magnetoresistive size effects

Figures 5 and 6 show $\Delta\varrho/\varrho$ in transverse and longitudinal magnetic fields for a range of specimens of smaller diameters. There are obvious deviations from KOHLER's rule according to which

$$\Delta\varrho/\varrho = f(H/\varrho) \tag{5}$$

where ϱ is the resistivity in zero field and the function f is characteristic of a given metal. These deviations are small in low fields but become important in higher fields. It will be seen that the thinner the specimen the smaller the resistivity change, and in longitudinal fields $\Delta\varrho/\varrho$ even starts decreasing in the highest fields.

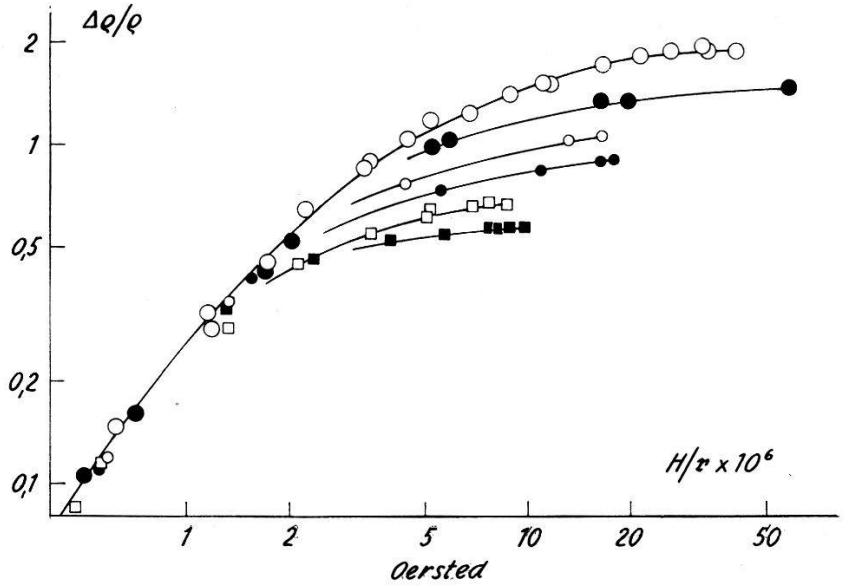

Fig. 5
Magnetoresistive size effect in transverse field.
○● In 2 2 mm ∅; o● In 3 0·3 mm ∅; □■ In 4 0·1 mm ∅
○ o □ 4·2° K; ● ● ■ 2° K

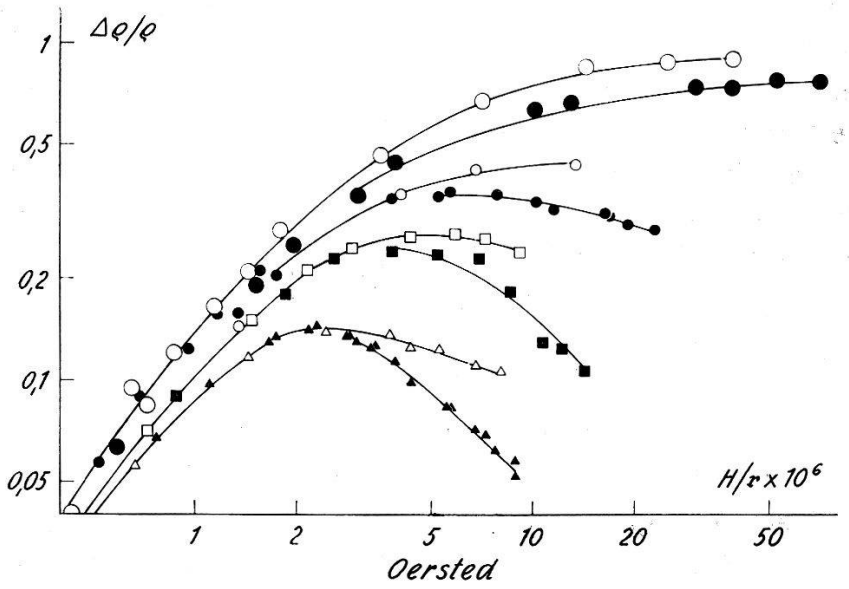

Fig. 6
Magnetoresistive size effect in longitudinal field.
○● In 2 2 mm ⌀; o● In 3 0·3 mm ⌀;
□■ In 4 0·1 mm ⌀; △▲ In 7 0·06 mm ⌀;
○ o □ △ 4·2° K; ● ● ■ ▲ 2° K.

A magnetoresistive size effect leading to a size dependent reduction of resistivity in a magnetic field was first observed in sodium by MacDonald[15]) in a transverse field and by Chambers[16]) in a longitudinal field. Calculations of this effect for a metal showing no bulk magnetoresistance have been carried out by MacDonald and Sarginson[19]) and by Chambers[16]) for the transverse and longitudinal cases respectively. It is reasonable to suppose that the effects observed by us have a similar origin.

The resistance decrease calculated by MacDonald and by Chambers arises because the mean time between collisions in a specimen where boundary scattering predominates can be increased by applying a magnetic field. This is because a sufficiently large field will make the electrons move in helical orbits which are smaller than the diameter of the wire. A portion of the electrons will therefore collide less often with the surface of the wire, and for them the mean time between collisions will be increased. The overall resistivity therefore decreases and in the limit it will reach the bulk resistivity. For real metals this bulk value will be subject to the normal bulk magnetoresistive resistance increase, and it becomes very difficult to distinguish between the various effects. In sodium, however, where the magnetoresistive size effect was first found the bulk magnetoresistance is very small, and the low field region can easily be analysed. The observation of similar effects in indium is possible because of the saturation of the bulk magnetoresistance in high fields.

*

An estimate of the combined effects of bulk magnetoresistance and of magnetoresistive size effect may be carried out as follows: We assume that Kohler's rule is obeyed for the bulk material, so that $\Delta\varrho/\varrho = f(H/\varrho)$ where the function $f(H/\varrho)$ is known. We also assume that the magnetoresistive size effect is known in the absence of bulk magnetoresistance. As remarked earlier theoretical treatments are available, and for the longitudinal effect CHAMBERS[16]) has given tables. We may write for this size and field dependent resistivity ϱ_s:

$$\varrho_s = \varrho_b\, C(l/r, a/l) \tag{5}$$

where ϱ_b is the bulk resistivity in the absence of a magnetic field and C is a calculated function of l/r and a/l (a = wire diameter, r = orbital radius, and l = mean free path).

In considering a suitable modification of Kohler's rule we may remember that H/ϱ occurs as a variable because it is a measure of the ratio of the mean free path and the radius of curvature of the electron orbit. In other words it is proportional to the ratio of the cyclotron frequency and the electronic relaxation time. Now simple treatments of the bulk magnetoresistance generally assume an unchanged relaxation time in the magnetic field. On the other hand it is clear that the relaxation time in thin specimens is reduced below that corresponding to the bulk resistivity by the size effect, and one must therefore use ϱ_s rather than ϱ_b for providing the correct relaxation time in Kohler's rule. It thus seems reasonable to change Kohler's rule from its usual form in the absence of size effect:

$$\frac{\varrho_H - \varrho_b}{\varrho_b} = f\left(\frac{H}{\varrho_b}\right) \tag{6}$$

where ϱ_H is the observed resistivity in a field H, to the following form:

$$\frac{\varrho_H - \varrho_s}{\varrho_s} = f\left(\frac{H}{\varrho_s}\right) \tag{7}$$

in the presence of size effect.

In low fields when ϱ_s has its zero field value this formula leads to the observed approximate applicability of Kohler's rule with the actual zero field resistance used as ϱ_s. In higher fields the agreement breaks down, and in the limit of very high fields when ϱ_s tends to ϱ_b we have

$$\varrho_H = \varrho_b\{1 + f(H/\varrho_b)\}. \tag{8}$$

For thin indium specimens one would thus expect the resistance initially to increase to a maximum value and then to decrease to the saturation value of the bulk resistivity. In sufficiently thin wires the resistance in very high fields might even be lower than that in zero field.

To compare equation (7) with experiment in somewhat more detail we need values of ϱ_s. Unfortunately these are difficult to evaluate for the transverse case, but for the longitudinal case we may use CHAMBERS[16]) tables. Once the bulk resistivity of the sample and the bulk magnetoresistance function f are known only the parameters l/a and l/r are required. l is related to the value of $\varrho_b\, l$ obtained from the measurements of § 3 while on Sommerfeld's theory

$$r = \frac{m\bar{v}\,c}{\varepsilon H} \tag{9}$$

where m is the mass of an electron and $\bar{v}$ is the mean velocity of electrons at the top of the Fermi distribution. It is obvious to choose for l/a a value fitting the zero field resistivity of the specimen and only $m\bar{v}$ has to be decided upon. This is a property of the metal and must be the same for all specimens.

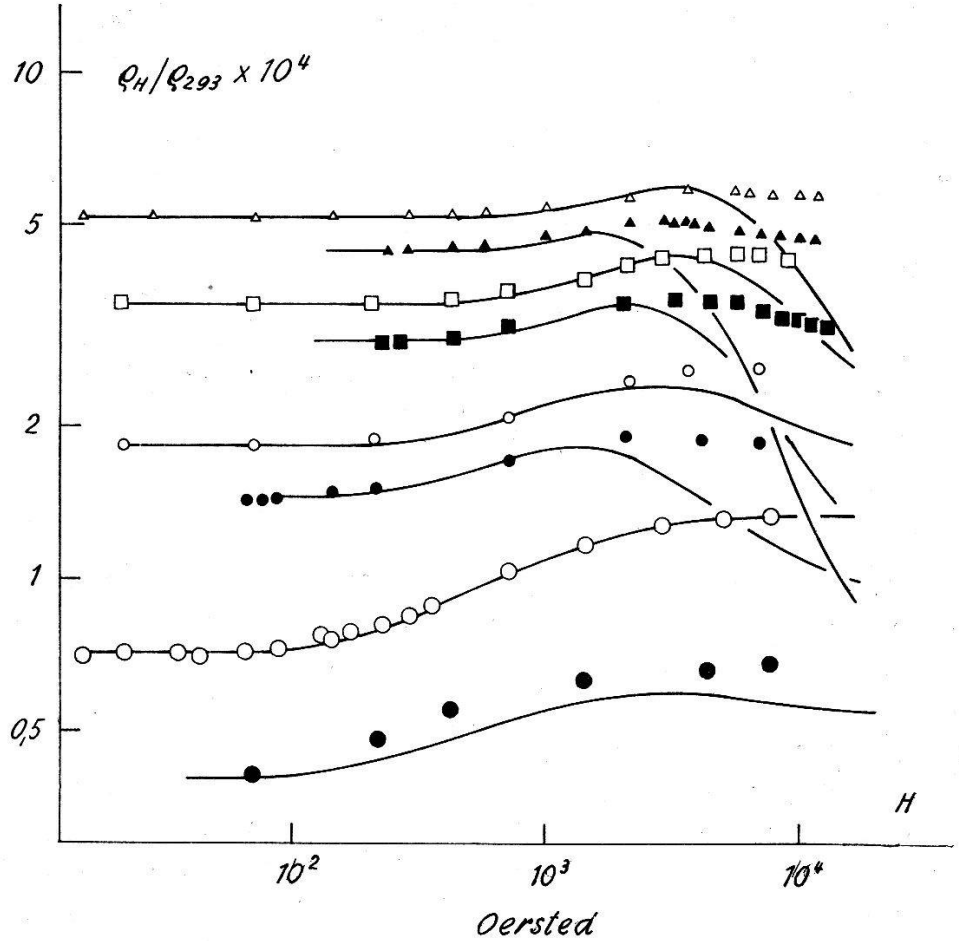

Fig. 7

Test of equation (7) for longitudinal case using $m\bar{v}$ = 2·7 × free electron value.

Experimental points: ○● In 2 2 mm ∅; ○ ● In 3 0·3 mm ∅;
□ ■ In 4 0·1 mm ∅; △ ▲ In 7 0·06 mm ∅;
○ ○ □ △ 4·2° K; ● ● ■ ▲ 2° K.
— calculated curves.

We find that no single value of $m\bar{v}$ fits all observations really well. Figures 7 and 8 therefore show the longitudinal case experimental results for a range of specimens compared with theoretical curves calculated using two different values of $m\bar{v}$. For ease in distinguishing between different specimens ϱ is plotted against H. Then (7) becomes

$$\varrho_H = \varrho_s \{1 + f(H/\varrho_s)\}. \tag{10}$$

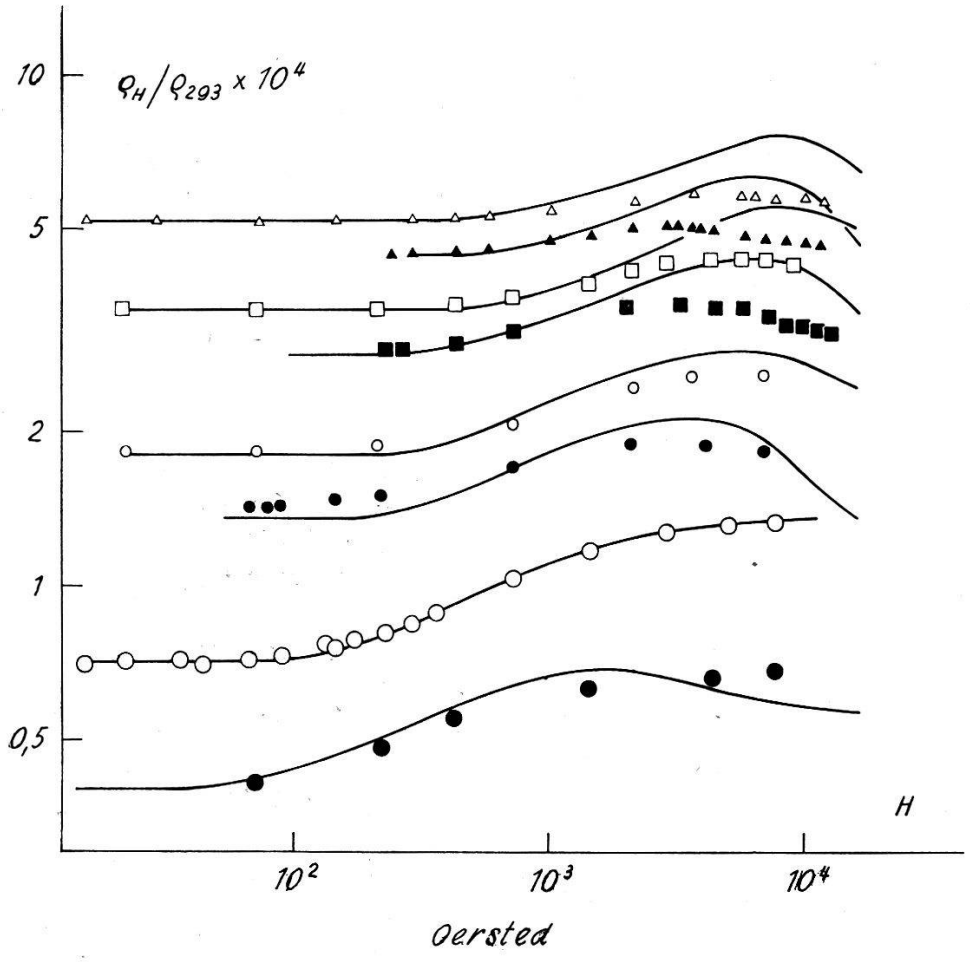

Fig. 8

Test of equation (7) for longitudinal case using $m\bar{v}$ = 11 × free electron value.
Experimental points:

○ ● In 2 2 mm ∅; o • In 3 0·3 mm ∅;
□ ■ In 4 0·1 mm ∅; △ ▲ In 7 0·06 mm ∅;
○ o □ △ 4·2° K; ● • ■ ▲ 2° K.
— calculated curves.

It will be seen that the agreement between theory and experiment remains highly unsatisfactory in detail. While it appears that a reasonable fit may be obtained in low fields by choosing $m\bar{v}$ to be approximately 3 times the free electron value no single choice of $m\bar{v}$ will make the curves coincide in high fields. At 10000 Oersted an increase of $m\bar{v}$ to 11 times

the free electron value will make the theoretical resistance roughly equal the experimentally observed one, but even so the two sets of curves will intersect at a large angle.

6. Discussion

It is of some interest to compare the results found in these experiments on indium with similar data for other metals. Data on $\varrho_b\, l$ exists for a number of substances both from size effect measurements such as ours and from anomalous skin effect measurements. Data on $m\bar{v}$ for sodium has been obtained by CHAMBERS from magnetoresistive size effect work. Table II contains a collection of such information taken from work by ANDREW[14]) and CHAMBERS[16])[24]). The theoretical values in the table are based on the assumption of one free electron per atom.

Table II

	$\varrho_b l \times 10^{11}$ (ohm cm^2)			$m\bar{v} \times 10^{20}$	
	size eff.	anomalous skin effect	theor.	exp.	theor.
In	1·8		1·21	30	11
Na	1·11		1·46	9·1	9·7
Cu		0·65	0·66		
Ag		1·08	0·84		
Au		1·18	0·83		
Sn	2·0	1·05	1·14		
Hg	3·6	2·7	1·10		
Al		1·6	0·82		

It will be seen that the differences between observed and theoretical values of $\varrho_b\, l$ are moderate for all the substances investigated but that anomalous skin effect and size effect measurements give different results. The discrepancies are probably no greater than might be expected in view of the wellknown deviations from spherical Fermi-surfaces.

Magnetoresistive size effect data on sodium yield a value for $m\bar{v}$ in low fields which is remarkably close to the free electron value, and even the value in indium seems quite reasonable. In indium, however, the theoretical curves cannot be made to fit the experimental data in higher fields. It is tempting to try to explain away this poor fit of the theoretical curves in high fields by postulating a wide range of values of effective momentum $m\bar{v}$ for different groups of electrons. On the other hand preliminary measurements here by P. COTTI on thin indium wires in pulsed magnetic fields up to 70000 Oersted indicate that a further rise in re-

sistivity may take place. This would mean a complete break-down of the theory sketched in § 4, and perhaps all that should be said now is that the present theory of galvanomagnetic phenomena has once more shown itself useful for low fields but less reliable in high fields.

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