

Zeitschrift: Helvetica Physica Acta
Band: 30 (1957)
Heft: V

Artikel: Fourth order magnetic moment of the electron
Autor: Petermann, A.
DOI: <https://doi.org/10.5169/seals-112823>

Nutzungsbedingungen

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften auf E-Periodica. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Das Veröffentlichen von Bildern in Print- und Online-Publikationen sowie auf Social Media-Kanälen oder Webseiten ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. [Mehr erfahren](#)

Conditions d'utilisation

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. La reproduction d'images dans des publications imprimées ou en ligne ainsi que sur des canaux de médias sociaux ou des sites web n'est autorisée qu'avec l'accord préalable des détenteurs des droits. [En savoir plus](#)

Terms of use

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. Publishing images in print and online publications, as well as on social media channels or websites, is only permitted with the prior consent of the rights holders. [Find out more](#)

Download PDF: 16.01.2026

ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>

Fourth order magnetic moment of the electron

by **A. Petermann.**

CERN. Theoretical Study Division. Institute for theoretical Physics. Copenhagen.

(17. VIII. 1957.)

In connection with the upper and lower bounds analysis done by the author¹⁾, which indicated a clear discrepancy with the Karplus and Kroll's result for the 4th order magnetic moment²⁾, we have performed an analytic evaluation of the five independent diagrams contributing to this moment in fourth order^{*)}. The results are the following:

$$\mu_I = \frac{\alpha^2}{\pi^2} \left(\frac{1}{6} + \frac{13}{36} \pi^2 + \frac{5}{4} \zeta(3) - \frac{5}{6} \pi^2 \text{Log } 2 \right) = -0.467 \frac{\alpha^2}{\pi^2}. \quad (1)$$

$$\mu_{II_a} = \frac{\alpha^2}{\pi^2} \left(\frac{11}{48} + \frac{\pi^2}{18} \right) = 0.778 \frac{\alpha^2}{\pi^2}. \quad (2)$$

$$\begin{aligned} \mu_{II_c} = \frac{\alpha^2}{\pi^2} \left(-\frac{67}{24} + \frac{\pi^2}{18} - \frac{1}{2} \zeta(3) + \frac{1}{3} \pi^2 \text{Log } 2 - \frac{1}{2} \text{Log } \frac{\lambda^2}{m^2} \right) = \\ -0.564 \frac{\alpha^2}{\pi^2} - \frac{1}{2} \frac{\alpha^2}{\pi^2} \text{Log } \frac{\lambda^2}{m^2}. \end{aligned} \quad (3)$$

$$\mu_{II_d} = \frac{\alpha^2}{\pi^2} \left(\frac{11}{24} - \frac{\pi^2}{18} + \frac{1}{2} \text{Log } \frac{\lambda^2}{m^2} \right) = -0.090 \frac{\alpha^2}{\pi^2} + \frac{1}{2} \frac{\alpha^2}{\pi^2} \text{Log } \frac{\lambda^2}{m^2}. \quad (4)$$

$$\mu_{II_e} = \frac{\alpha^2}{\pi^2} \left(\frac{119}{36} - \frac{\pi^2}{3} \right) = 0.016 \frac{\alpha^2}{\pi^2}. \quad (5)$$

$$\mu_{\text{total}}^{(4)} = \frac{\alpha^2}{\pi^2} \left(\frac{197}{144} + \frac{\pi^2}{12} + \frac{3}{4} \zeta(3) - \frac{1}{2} \pi^2 \text{Log } 2 \right) = -0.328 \frac{\alpha^2}{\pi^2}. \quad (6)$$

Compared with the values given in their original paper by KARPLUS and KROLL, one can see that two terms were in error: μ_I differs by

$$\frac{\alpha^2}{\pi^2} \frac{1}{32} = 0.031 \frac{\alpha^2}{\pi^2};$$

$$\mu_{II_c} \text{ by } \frac{\alpha^2}{\pi^2} \left(\frac{32}{3} - \frac{61}{8} \pi^2 + \frac{17}{2} \pi^2 \text{Log } 2 - \frac{109}{4} \zeta(3) \right) = 2.614 \frac{\alpha^2}{\pi^2}.$$

The three other terms check. The error in μ_I remained of course undetected in the upper and lower bound analysis owing to its small-

^{*)} The terminology of ref. 2 is used throughout this paper.

ness. But the large discrepancy in μ_{IIc} was that pin-pointed out in the previous paper.

A summary of the most important electromagnetic observables, the theoretical values of which are modified by the new value of the magnetic moment, is now given:

$$\text{Moment of the electron: } \frac{\mu_e}{\mu_0} = 1.0011596 = 1 + \frac{\alpha}{2\pi} - 0.328 \frac{\alpha^2}{\pi^2}.$$

FRANKEN and LIEBES' value for it: $\mu_e/\mu_0 = 1.001167 \pm 0.000005^*)$.
 g -factor of the μ -meson (electromagnetic):

$$2 (1.0011654) = 2 \left(1 + \frac{\alpha}{2\pi} + 0.75 \frac{\alpha^2}{\pi^2} \right).$$

Last Lederman's value: $2 (1.0021 \pm 0.0008)^*)$.

$$2^2 S_{1/2} - 2^2 P_{1/2} \text{ (Hydrogen): } (1057.94 \pm 0.15) \text{ Mc/s; observed: } (1057.77 \pm 0.10) \text{ Mc/s.}$$

$$2^2 S_{1/2} - 2^2 P_{1/2} \text{ (Deuterium): } (1059.22 \pm 0.15) \text{ Mc/s; observed: } (1059.00 \pm 0.10) \text{ Mc/s.}$$

Fine structure constant: $1/\alpha = 137.0384$; (previously: 137.0365).

The new fourth order correction given here is in agreement with:

a) The upper and lower bounds given by the author¹⁾.

b) A calculation using a different method, performed by C. SOMMERFIELD³⁾.

c) A recalculation done by N. M. KROLL and collaborators*).

The author thanks Prof. NIELS BOHR for the hospitality at the Institute.

References.

- ¹⁾ A. PETERMANN. Nuclear Physics **3**, 689 (1957) and Nuclear Physics in the press.
- ²⁾ R. KARPLUS and N. M. KROLL, Phys. Rev. **77**, 536 (1950).
- ³⁾ C. SOMMERFIELD, Phys. Rev. In the press.

*) Private Communication.